ECON 351* -- NOTE 16

Tests of Exclusion Restrictions on Regression Coefficients: Formulation and Interpretation

- The **population regression equation (PRE)** for the general multiple linear regression model takes the form:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_k X_{ki} + u_i \qquad \textbf{(1)}$$

where u_i is an iid (independently and identically distributed) random error term.

PRE (1) constitutes the *unrestricted* model for tests of exclusion restrictions.

- ***Exclusion restrictions*** set one or more regression coefficients equal to zero.

Common types of exclusion restrictions:

1. *One* slope coefficient equals zero.

Tests of the ***individual* significance** of **a *single* slope coefficient**.

2. *All* slope coefficients jointly equal zero.

Tests of the ***joint* significance** of ***all* slope coefficients**.

3. *Some* slope coefficients equal zero.

Tests of the ***joint* significance** of ***a subset of* slope coefficients**.

1. Tests of Exclusion Restrictions: Formulation

<u>TEST 1</u>: *<u>One</u>* slope coefficient equals zero.

♦ The ***null*** **hypothesis** is:

H_0: $\beta_j = 0$; the slope coefficient of regressor X_j equals zero.

♦ The ***alternative*** **hypothesis** is:

H_1: $\beta_j \neq 0$; the slope coefficient of regressor X_j is not equal to zero.

- **Question addressed by this test :**

  Is the regressor X_j relevant in explaining the dependent variable Y (controlling for the effects on Y of all the other included regressors)?

  Is the explanatory variable X_j ***individually*** **relevant** in explaining the dependent variable Y?

  Does the explanatory variable X_j have an ***individually*** **significant marginal effect** on the dependent variable Y?

  Does the true PRF for the dependent variable Y include X_j?

- **Names (labels) for this test:**

  a test of the ***individual relevance of the explanatory variable X_j***.

  a test of the ***individual significance of the slope coefficient for X_j.***

<u>TEST 2</u>: *<u>All</u>* slope coefficients *jointly* equal zero.

- The ***null*** **hypothesis** is:

  $$\mathbf{H_0:}\quad \beta_j = 0 \quad \forall\ j = 1, 2, ..., k.$$

  $$\beta_1 = 0 \textit{ and } \beta_2 = 0 \textit{ and } ... \beta_k = 0.$$

  - All K – 1 slope coefficients equal zero; i.e., the slope coefficients are jointly equal to zero.

- The ***alternative*** **hypothesis** is:

  $$\mathbf{H_1:}\quad \beta_j \neq 0 \qquad j = 1, 2, ..., k.$$

  $$\beta_1 \neq 0 \textit{ and/or } \beta_2 \neq 0 \textit{ and/or } \beta_3 \neq 0 \ ... \textit{ and/or } \beta_k \neq 0.$$

  - At least one of the slope coefficients does not equal zero.

- **Question addressed by this test :**

  Is the regression model given by PRE (1) relevant in explaining the dependent variable Y?

  Are the k = K – 1 explanatory variables in PRE (1) ***jointly*** **relevant** in explaining the dependent variable Y?

- **Names (labels) for this test:**

  a test of the ***overall significance of the regression model***

  a test of the ***joint significance of the slope coefficients***

<u>TEST 3</u>: *<u>Some</u>* regression coefficients equal zero.

<u>Example 1</u>:

Suppose the ***unrestricted*** **model** is given by the PRE

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \qquad (i = 1, ..., N) \qquad \textbf{(1)}$$

so that K = 4.

♦ The ***null*** **hypothesis** is:

H_0: $\beta_j = 0 \quad \forall \; j = 2, 3.$ $\qquad \beta_2 = 0$ *and* $\beta_3 = 0$.

- The two slope coefficients β_2 and β_3 ***both*** equal zero; i.e., the slope coefficients β_2 and β_3 are ***jointly*** **equal to zero**.

♦ The ***alternative*** **hypothesis** is:

H_1: $\beta_j \neq 0 \qquad j = 2, 3.$ $\qquad \beta_2 \neq 0$ *and/or* $\beta_3 \neq 0$.

- At least one of the slope coefficients β_2 and β_3 does not equal zero.

- **Question addressed by this test :**

  Are the explanatory variables X_2 and X_3 jointly relevant in explaining the dependent variable Y?

  Does the true PRF for the dependent variable Y include both the explanatory variables X_2 and X_3?

- **Names (labels) for this test:**

  a test of the ***joint relevance of the regressors X_2 and X_3***

  a test of the ***joint significance of the slope coefficients for X_2 and X_3***

Example 2: A model of North American car prices from *Stata Tutorials*

Suppose the ***unrestricted* model** is given by the PRE

$$\text{price}_i = \beta_0 + \beta_1\text{weight}_i + \beta_2\text{weight}_i^2 + \beta_3\text{mpg}_i + u_i \qquad (i = 1, ..., N) \qquad \textbf{(2)}$$

The ***marginal effect*** of the variable **weight$_i$** on the dependent variable **price$_i$** is obtained by taking the partial derivative of the regression function in (1) with respect to weight_i:

$$\frac{\partial \text{price}_i}{\partial \text{weight}_i} = \beta_1 + 2\beta_2\text{weight}_i \qquad (i = 1, ..., N).$$

A ***sufficient condition*** **for the marginal effect of weight$_i$ *to equal zero*** for all observations is that **the coefficients β_1 and β_2 *equal zero***.

- The ***null* hypothesis** is:

  H$_0$: $\beta_j = 0 \quad \forall \; j = 1, 2; \qquad \beta_1 = 0$ *and* $\beta_2 = 0$

  - The two slope coefficients **β_1 *and* β_2 *both*** equal zero; i.e., the slope coefficients β_1 and β_2 are ***jointly* equal to zero**.
  - The ***marginal effect* on price$_i$ of weight$_i$ is *zero***.

- The ***alternative* hypothesis** is:

  H$_1$: $\beta_j \neq 0 \qquad j = 1, 2; \qquad \beta_1 \neq 0$ *and/or* $\beta_2 \neq 0$

  - At least one of the slope coefficients β_1 and β_2 does not equal zero.
  - The ***marginal effect* on price$_i$ of weight$_i$** is not equal zero.

- **Question addressed by this test :**

  Is the explanatory variable weight$_i$ relevant in determining price$_i$?

  Does the true PRF for the dependent variable price$_i$ include both the regressors weight$_i$ and weight_i^2?

2. The Restricted and Unrestricted Models

2.1 Definitions

- The ***unrestricted* model** is the PRE that corresponds to, or is implied by, the ***alternative* hypothesis H_1**.

  It is the PRE that is presumed to be true if the null hypothesis **H_0 is *false***.

- The ***restricted* model** is the PRE that corresponds to, or is implied by, the ***null* hypothesis H_0**.

  It is the PRE that is presumed to be true if the null hypothesis **H_0 is *true***.

  It is obtained by ***substituting the coefficient restrictions*** specified by the null hypothesis H_0 ***into the unrestricted model***.

2.2 Examples

Suppose the ***unrestricted*** **model** is given by the PRE

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \qquad (i = 1, ..., N) \qquad \textbf{(1)}$$

The number of free (unrestricted) regressions coefficients in (1) is **K = 4**.

- **Test 1:** a test of the ***individual*** **significance of *one* slope coefficient**.

- The ***null*** **hypothesis** is:

  H_0: $\beta_3 = 0$; the slope coefficient of regressor X_3 equals zero.

- The ***alternative*** **hypothesis** is:

  H_1: $\beta_3 \neq 0$; the slope coefficient of regressor X_3 is not equal to zero.

- The ***unrestricted*** **model corresponding to the *alternative* hypothesis H_1** is simply PRE (1):

  $$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \qquad (i = 1, ..., N) \qquad \textbf{(1)}$$

  The number of free (unrestricted) regression coefficients in model (1) is $\mathbf{K = K_1 = 4}$.

- The ***restricted*** **model corresponding to the *null* hypothesis H_0** is obtained by setting $\beta_3 = 0$ in the unrestricted model (1):

  $$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i \qquad (i = 1, ..., N) \qquad \textbf{(3)}$$

  The number of free (unrestricted) regression coefficients in model (3) is $\mathbf{K_0 = 3}$.

- ***Number of coefficient restrictions*** **specified by the null hypothesis H_0 is** $\mathbf{q = K - K_0 = K_1 - K_0 = 4 - 3 = 1}$.

❑ **<u>Test 2</u>:** a test of the ***joint* significance of *all* slope coefficients**.

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \qquad (i = 1, ..., N) \qquad \textbf{(1)}$$

♦ The ***null* hypothesis** is:

H_0: $\beta_j = 0 \quad \forall\ j = 1, 2, 3$; $\qquad \beta_1 = 0$ *and* $\beta_2 = 0$ *and* $\beta_3 = 0$.

- All of the K–1 = k = 4–1 = 3 slope coefficients equal zero; i.e., the slope coefficients are jointly equal to zero.

♦ The ***alternative* hypothesis** is:

H_1: $\beta_j \neq 0 \qquad j = 1, 2, 3$; $\qquad \beta_1 \neq 0$ *and/or* $\beta_2 \neq 0$ *and/or* $\beta_3 \neq 0$.

- At least one of the 3 slope coefficients does not equal zero.

- The ***<u>unrestricted</u>* model corresponding to the *alternative* hypothesis H_1** is simply PRE (1):

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \qquad (i = 1, ..., N) \qquad \textbf{(1)}$$

The number of free (unrestricted) regression coefficients in the unrestricted model (1) is $\mathbf{K = K_1 = 4}$.

- The ***<u>restricted</u>* model corresponding to the *null* hypothesis H_0** is obtained by setting $\beta_1 = 0$ and $\beta_2 = 0$ and $\beta_3 = 0$ in the unrestricted model (1):

$$Y_i = \beta_0 + u_i \qquad (i = 1, ..., N) \qquad \textbf{(4)}$$

The number of free (unrestricted) regression coefficients in the restricted model (4) is $\mathbf{K_0 = 1}$.

- ***Number of coefficient restrictions* specified by the null hypothesis H_0 is $\mathbf{q = K - K_0 = K_1 - K_0 = 4 - 1 = 3}$**.

❑ **<u>Test 3</u>:** a test of the ***joint* significance of *two* of the slope coefficients**.

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \qquad (i = 1, ..., N) \qquad \textbf{(1)}$$

♦ The ***null* hypothesis** is:

H_0: $\beta_j = 0 \quad \forall\ j = 2, 3;$ $\qquad \beta_2 = 0$ *and* $\beta_3 = 0$.

- The two slope coefficients β_2 and β_3 ***both*** equal zero; i.e., the slope coefficients β_2 and β_3 are ***jointly* equal to zero**.

♦ The ***alternative* hypothesis** is:

H_1: $\beta_j \neq 0 \qquad j = 2, 3;$ $\qquad \beta_2 \neq 0$ *and/or* $\beta_3 \neq 0$.

- At least one of the slope coefficients β_2 and β_3 does not equal zero.

• The ***<u>unrestricted</u>* model corresponding to the *alternative* hypothesis H_1** is simply PRE (1):

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \qquad (i = 1, ..., N) \qquad \textbf{(1)}$$

The number of free (unrestricted) regression coefficients in the unrestricted model (1) is $\mathbf{K = K_1 = 4}$.

• The ***<u>restricted</u>* model corresponding to the *null* hypothesis H_0** is obtained by setting $\beta_2 = 0$ and $\beta_3 = 0$ in the unrestricted model (1):

$$Y_i = \beta_0 + \beta_1 X_{1i} + u_i \qquad (i = 1, ..., N) \qquad \textbf{(5)}$$

The number of free (unrestricted) regression coefficients in the restricted model (5) is $\mathbf{K_0 = 2}$.

• ***Number of coefficient restrictions* specified by the null hypothesis H_0 is $\mathbf{q = K - K_0 = K_1 - K_0 = 4 - 2 = 2}$**.