ECON 351* -- NOTE 15

Marginal Effects of Explanatory Variables: Constant or Variable?

1. Constant Marginal Effects of Explanatory Variables: A Starting Point

Nature: A continuous explanatory variable has a ***constant*** **marginal effect** on the dependent variable if it enters the regressor set only linearly and additively.

Model 1:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i \qquad \textbf{(1)}$$

- Model 1 contains only two ***explanatory variables*** – X_1 and X_2 – and two ***regressors***.

- The population regression function, or conditional mean function, $f(X_{1i}, X_{2i})$ in Model 1 takes the form

$$E(Y_i \mid X_{1i}, X_{2i}) = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i}.$$

- The ***marginal*** **effects on Y of the two explanatory variables X_1 and X_2** in equation (1) are obtained analytically by partially differentiating Y, or the conditional mean of Y given X_1 and X_2, with respect to each of the explanatory variables X_1 and X_2.

  1. The **marginal effect of X_1 in Model 1** is:

$$\frac{\partial Y_i}{\partial X_{1i}} = \frac{\partial E\left(Y_i \mid X_{1i}, X_{2i}\right)}{\partial X_{1i}} = \beta_1 = \text{a } \textit{constant}$$

  2. The **marginal effect of X_2 in Model 1** is:

$$\frac{\partial Y_i}{\partial X_{2i}} = \frac{\partial E\left(Y_i \mid X_{1i}, X_{2i}\right)}{\partial X_{2i}} = \beta_2 = \text{a } \textit{constant}$$

***Example of Model 1*:**

$$price_i = \beta_0 + \beta_1 wgt_i + \beta_2 mpg_i + u_i \qquad \textbf{(1*)}$$

where

$price_i$ = the price of the i-th car (in US dollars);
wgt_i = the weight of the i-th car (in pounds);
mpg_i = the miles per gallon (fuel efficiency) for the i-th car (in miles per gallon).

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. * Model 1: constant marginal effects of wgt and mpg
. regress price wgt mpg

      Source |       SS       df       MS              Number of obs =      74
-------------+------------------------------           F(  2,    71) =   14.74
       Model |   186321280     2  93160639.9           Prob > F      =  0.0000
    Residual |   448744116    71  6320339.67           R-squared     =  0.2934
-------------+------------------------------           Adj R-squared =  0.2735
       Total |   635065396    73  8699525.97           Root MSE      =  2514.0

------------------------------------------------------------------------------
       price |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         wgt |   1.746559   .6413538     2.72   0.008      .467736    3.025382
         mpg |  -49.51222   86.15604    -0.57   0.567    -221.3025     122.278
       _cons |   1946.069    3597.05     0.54   0.590    -5226.244    9118.382
------------------------------------------------------------------------------
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2. Variable Marginal Effects and Interaction Terms: Squares and Cross Products of Continuous Explanatory Variables

Nature: ***Interactions*** **between two *continuous* variables** refer to products of pairs of explanatory variables.

- If X_{ji} and X_{hi} are two continuous explanatory variables, the interaction term between them is the product $X_{ji}X_{hi}$.

- The interaction of the variable X_{ji} with itself is simply the product $X_{ji}X_{ji} = X_{ji}^2$.

- Inclusion of these regressors in a linear regression model allows for ***variable* – or *nonconstant* – marginal effects** of the explanatory variables on the conditional mean of the dependent variable Y.

Usage: ***Interaction terms*** between ***continuous* variables** allow the marginal effect of one explanatory variable to be a linear function of both itself and other explanatory variables.

Model 2:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i}^2 + \beta_4 X_{2i}^2 + \beta_5 X_{1i} X_{2i} + u_i \qquad \textbf{(2)}$$

- Model 2 contains only **two** ***explanatory variables*** – X_1 and X_2 – but **five** ***regressors***.

- Formally, the **population regression function** $E(Y_i \mid X_{1i}, X_{2i}) = f(X_{1i}, X_{2i})$ in PRE (2) can be derived as a second-order Taylor series approximation to the function $f(X_{1i}, X_{2i})$. A second-order Taylor series approximation to the population regression function $f(X_{1i}, X_{2i})$ takes the form

$$E(Y_i \mid X_{1i}, X_{2i}) = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i}^2 + \beta_4 X_{2i}^2 + \beta_5 X_{1i} X_{2i}.$$

Marginal Effects in Model 2:

- The ***marginal*** **effects on Y of the two explanatory variables X_1 and X_2** in equation (2) are obtained analytically by partially differentiating Y, or the conditional mean of Y given X_1 and X_2, with respect to each of the explanatory variables X_1 and X_2.

  The **population regression function for Model 2** is:

$$E(Y_i | X_{1i}, X_{2i}) = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i}^2 + \beta_4 X_{2i}^2 + \beta_5 X_{1i} X_{2i}$$

1. The **marginal effect of X_1 in Model 2** is:

$$\frac{\partial Y_i}{\partial X_{1i}} = \frac{\partial E(Y_i | X_{1i}, X_{2i})}{\partial X_{1i}} = \beta_1 + 2\beta_3 X_{1i} + \beta_5 X_{2i}$$

= a linear function of *both* X_{1i} and X_{2i}

2. The **marginal effect of X_2 in Model 2** is:

$$\frac{\partial Y_i}{\partial X_{2i}} = \frac{\partial E(Y_i | X_{1i}, X_{2i})}{\partial X_{2i}} = \beta_2 + 2\beta_4 X_{2i} + \beta_5 X_{1i}$$

= a linear function of *both* X_{1i} and X_{2i}

Squares of Continuous Explanatory Variables

Purpose: Allow for ***increasing*** **or** ***decreasing*** **marginal effects** of an explanatory variable on the dependent variable -- sometimes called ***increasing*** **or** ***decreasing*** **marginal returns**.

Determining whether the *marginal* effect of X_1 is *increasing* or *decreasing*

- Whether the ***marginal* effect of X_1 is *increasing* or *decreasing*** – i.e., whether X_1 exhibits ***increasing* or *decreasing* marginal returns** – is determined by the sign of the regression coefficient β_3 on the regressor X_{1i}^2 in Model 2.

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i}^2 + \beta_4 X_{2i}^2 + \beta_5 X_{1i} X_{2i} + u_i \qquad \textbf{(2)}$$

- We previously saw that the ***marginal* effect of X_1 in Model 2** is given by the ***first-order* partial derivative** of Y_i, or $E(Y_i \mid X_{1i}, X_{2i})$, with respect to X_{1i}:

$$\frac{\partial Y_i}{\partial X_{1i}} = \frac{\partial E\left(Y_i \mid X_{1i}, X_{2i}\right)}{\partial X_{1i}} = \beta_1 + 2\beta_3 X_{1i} + \beta_5 X_{2i}$$

- To determine whether the ***marginal* effect of X_1 in Model 2** is ***increasing* or *decreasing* in X_1**, we need to examine the ***second-order* partial derivative** of Y_i, or $E(Y_i \mid X_{1i}, X_{2i})$, with respect to X_{1i}:

$$\frac{\partial^2 Y_i}{\partial X_{1i}^2} = \frac{\partial}{\partial X_{1i}} \frac{\partial E\left(Y_i \mid X_{1i}, X_{2i}\right)}{\partial X_{1i}} = \frac{\partial^2 E\left(Y_i \mid X_{1i}, X_{2i}\right)}{\partial^2 X_{1i}^2} = 2\beta_3$$

1. The ***marginal* effect of X_1** is ***increasing* in X_1** – meaning X_1 exhibits ***increasing* marginal returns** – when

$$\frac{\partial^2 Y_i}{\partial X_{1i}^2} = \frac{\partial^2 E\left(Y_i \mid X_{1i}, X_{2i}\right)}{\partial^2 X_{1i}^2} = 2\beta_3 > 0 \qquad \text{i.e., when } \beta_3 > 0$$

2. The ***marginal* effect of X_1** is ***decreasing* in X_1** – meaning X_1 exhibits ***decreasing* marginal returns** – when

$$\frac{\partial^2 Y_i}{\partial X_{1i}^2} = \frac{\partial^2 E\left(Y_i \mid X_{1i}, X_{2i}\right)}{\partial^2 X_{1i}^2} = 2\beta_3 < 0 \qquad \text{i.e., when } \beta_3 < 0$$

Determining whether the *marginal* effect of X_2 is *increasing* or *decreasing*

- Whether the ***marginal*** **effect of X_2 is *increasing*** **or *decreasing*** – i.e., whether X_2 exhibits ***increasing*** **or *decreasing*** **marginal returns** – is determined by the sign of the regression coefficient β_4 on the regressor X_{2i}^2 in Model 2.

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i}^2 + \beta_4 X_{2i}^2 + \beta_5 X_{1i} X_{2i} + u_i \qquad \textbf{(2)}$$

- We previously saw that the ***marginal*** **effect of X_2 in Model 2** is given by the ***first-order*** **partial derivative** of Y_i, or $E(Y_i \mid X_{1i}, X_{2i})$, with respect to X_{2i}:

$$\frac{\partial Y_i}{\partial X_{2i}} = \frac{\partial E\left(Y_i \mid X_{1i}, X_{2i}\right)}{\partial X_{2i}} = \beta_2 + 2\beta_4 X_{2i} + \beta_5 X_{1i}$$

- To determine whether the ***marginal*** **effect of X_2 in Model 2** is ***increasing*** **or *decreasing*** **in X_2**, we need to examine the ***second-order*** **partial derivative** of Y_i, or $E(Y_i \mid X_{1i}, X_{2i})$, with respect to X_{2i}:

$$\frac{\partial^2 Y_i}{\partial X_{2i}^2} = \frac{\partial}{\partial X_{2i}} \frac{\partial E\left(Y_i \mid X_{1i}, X_{2i}\right)}{\partial X_{2i}} = \frac{\partial^2 E\left(Y_i \mid X_{1i}, X_{2i}\right)}{\partial^2 X_{2i}^2} = 2\beta_4$$

1. The ***marginal*** **effect of X_2** is ***increasing*** **in X_2** – meaning X_2 exhibits ***increasing*** **marginal returns** – when

$$\frac{\partial^2 Y_i}{\partial X_{2i}^2} = \frac{\partial^2 E\left(Y_i \mid X_{1i}, X_{2i}\right)}{\partial^2 X_{2i}^2} = 2\beta_4 > 0 \qquad \text{i.e., when } \beta_4 > 0$$

2. The ***marginal*** **effect of X_2** is ***decreasing*** **in X_2** – meaning X_2 exhibits ***decreasing*** **marginal returns** – when

$$\frac{\partial^2 Y_i}{\partial X_{2i}^2} = \frac{\partial^2 E\left(Y_i \mid X_{1i}, X_{2i}\right)}{\partial^2 X_{2i}^2} = 2\beta_4 < 0 \qquad \text{i.e., when } \beta_4 < 0$$

Products of Two Continuous Explanatory Variables

Purpose: Allow for relationships of ***complementarity*** **or** ***substitutability*** **between X_1 and X_2** in determining Y.

- Whether **X_1 and X_2 are *complementary*** **or** ***substitutable*** is determined by the ***sign*** **of the regression coefficient β_5** on the interaction term $X_{1i}X_{2i}$ in Model 2.

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i}^2 + \beta_4 X_{2i}^2 + \beta_5 X_{1i} X_{2i} + u_i \qquad \textbf{(2)}$$

- The ***marginal*** **effects of X_1 and X_2 in Model 2** are given by the ***first-order*** **partial derivatives** of Y_i, or $E(Y_i \mid X_{1i}, X_{2i})$, with respect to X_{1i} and X_{2i}:

$$\frac{\partial Y_i}{\partial X_{1i}} = \frac{\partial E\left(Y_i \mid X_{1i}, X_{2i}\right)}{\partial X_{1i}} = \beta_1 + 2\beta_3 X_{1i} + \beta_5 X_{2i}$$

$$\frac{\partial Y_i}{\partial X_{2i}} = \frac{\partial E\left(Y_i \mid X_{1i}, X_{2i}\right)}{\partial X_{2i}} = \beta_2 + 2\beta_4 X_{2i} + \beta_5 X_{1i}$$

- To determine whether the ***marginal*** **effect of X_1 (X_2) in Model 2** is ***increasing*** **or** ***decreasing*** **in X_2 (X_1)**, we need to examine the ***second-order*** **cross partial derivative** of Y_i, or $E(Y_i \mid X_{1i}, X_{2i})$, with respect to X_{1i} and X_{2i}:

$$\frac{\partial^2 Y_i}{\partial X_{2i}\partial X_{1i}} = \frac{\partial^2 E\left(Y_i \mid X_{1i}, X_{2i}\right)}{\partial X_{2i}\partial X_{1i}} = \beta_5$$

1. The ***marginal*** **effect of X_1** is ***increasing*** **in X_2** (or the ***marginal*** **effect of X_2** is ***increasing*** **in X_1**) -- meaning **X_1 and X_2 are *complementary*** -- when

$$\frac{\partial^2 Y_i}{\partial X_{2i}\partial X_{1i}} = \frac{\partial^2 E\left(Y_i \mid X_{1i}, X_{2i}\right)}{\partial X_{2i}\partial X_{1i}} = \beta_5 > 0$$

2. The ***marginal*** **effect of X_1** is ***decreasing*** **in X_2** (or the ***marginal*** **effect of X_2** is ***decreasing*** **in X_1**) -- meaning **X_1 and X_2 are *substitutable*** -- when

$$\frac{\partial^2 Y_i}{\partial X_{2i}\partial X_{1i}} = \frac{\partial^2 E\left(Y_i \mid X_{1i}, X_{2i}\right)}{\partial X_{2i}\partial X_{1i}} = \beta_5 < 0$$

***Example of Model 2*:**

$$\text{price}_i = \beta_0 + \beta_1 \text{wgt}_i + \beta_2 \text{mpg}_i + \beta_3 \text{wgt}_i^2 + \beta_4 \text{mpg}_i^2 + \beta_5 \text{wgt}_i \text{mpg}_i + u_i . \qquad \textbf{(2*)}$$

where

price_i = the price of the i-th car (in US dollars);
wgt_i = the weight of the i-th car (in pounds);
wgt_i^2 = the square of wgt_i;
mpg_i = the miles per gallon (fuel efficiency) for the i-th car (in miles per gallon);
mpg_i^2 = the square of mpg_i;
wgt_impg_i = the product of wgt_i and mpg_i for the i-th car.

```
. * Model 2: variable marginal effects of wgt and mpg
. regress price wgt mpg wgtsq mpgsq wgtmpg

      Source |       SS       df       MS              Number of obs =      74
-------------+------------------------------           F(  5,    68) =   12.84
       Model |   308384833     5  61676966.6           Prob > F      =  0.0000
    Residual |   326680563    68  4804125.93           R-squared     =  0.4856
-------------+------------------------------           Adj R-squared =  0.4478
       Total |   635065396    73  8699525.97           Root MSE      =  2191.8

------------------------------------------------------------------------------
       price |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         wgt |  -31.88985   9.148215    -3.49   0.001    -50.14483   -13.63487
         mpg |  -3549.495   1126.464    -3.15   0.002    -5797.318   -1301.672
       wgtsq |   .0034574   .0008629     4.01   0.000     .0017355    .0051792
       mpgsq |   38.74472   12.62339     3.07   0.003     13.55514    63.93431
      wgtmpg |   .5421927   .1971854     2.75   0.008     .1487154    .9356701
       _cons |   92690.55   25520.53     3.63   0.001     41765.12      143616
------------------------------------------------------------------------------
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Hypothesis Tests on the Marginal Effects of *wgt* and *mpg*

$$\text{price}_i = \beta_0 + \beta_1 \text{wgt}_i + \beta_2 \text{mpg}_i + \beta_3 \text{wgt}_i^2 + \beta_4 \text{mpg}_i^2 + \beta_5 \text{wgt}_i \text{mpg}_i + u_i \qquad \textbf{(2*)}$$

Test 1: The **marginal effect of wgt_i on $price_i$ is *zero*** for all cars.

- The **marginal effect of wgt_i on $price_i$ in Model 2*** is:

$$\frac{\partial\, \text{price}_i}{\partial\, \text{wgt}_i} = \frac{\partial E\left(\text{price}_i \mid \text{wgt}_i, \text{mpg}_i\right)}{\partial\, \text{wgt}_i} = \beta_1 + 2\beta_3 \text{wgt}_i + \beta_5 \text{mpg}_i$$

- ***Null and Alternative Hypotheses***

  H_0: $\beta_1 = 0$ *and* $\beta_3 = 0$ *and* $\beta_5 = 0$ specifies **three** coefficient restrictions

  H_1: $\beta_1 \neq 0$ *and/or* $\beta_3 \neq 0$ *and/or* $\beta_5 \neq 0$

- ***Unrestricted Model Corresponding to H_1***: regression equation (2*)

- ***Restricted Model Corresponding to H_0***: set $\beta_1 = 0$ *and* $\beta_3 = 0$ *and* $\beta_5 = 0$ in (2*).

$$\text{price}_i = \beta_0 + \beta_2 \text{mpg}_i + \beta_4 \text{mpg}_i^2 + u_i$$

```
. * Test 1: Test hypothesis that marginal effect of wgt equals zero for all cars
. test wgt wgtsq wgtmpg

 ( 1)  wgt = 0.0
 ( 2)  wgtsq = 0.0
 ( 3)  wgtmpg = 0.0

       F(  3,    68) =    6.42
            Prob > F =    0.0007
```

<u>Test 2</u>: The **marginal effect of *mpg_i* on *$price_i$* is *zero*** for all cars.

$$price_i = \beta_0 + \beta_1 wgt_i + \beta_2 mpg_i + \beta_3 wgt_i^2 + \beta_4 mpg_i^2 + \beta_5 wgt_i mpg_i + u_i \qquad \textbf{(2*)}$$

- The **marginal effect of *mpg_i* on *$price_i$* in Model 2*** is:

$$\frac{\partial\, price_i}{\partial\, mpg_i} = \frac{\partial E\left(price_i \mid wgt_i, mpg_i\right)}{\partial\, mpg_i} = \beta_2 + 2\beta_4 mpg_i + \beta_5 wgt_i$$

- ***Null and Alternative Hypotheses***

H_0: $\beta_2 = 0$ *and* $\beta_4 = 0$ *and* $\beta_5 = 0$ specifies **three** coefficient restrictions

H_1: $\beta_2 \neq 0$ *and/or* $\beta_4 \neq 0$ *and/or* $\beta_5 \neq 0$

- ***Unrestricted Model Corresponding to H_1***: regression equation (2*)

- ***Restricted Model Corresponding to H_0***: set $\beta_2 = 0$ *and* $\beta_4 = 0$ *and* $\beta_5 = 0$ in (2*).

$$price_i = \beta_0 + \beta_1 wgt_i + \beta_3 wgt_i^2 + u_i$$

```
. * Test 2: Test hypothesis that marginal effect of mpg equals zero for all cars
. test mpg mpgsq wgtmpg

 ( 1)  mpg = 0.0
 ( 2)  mpgsq = 0.0
 ( 3)  wgtmpg = 0.0

       F(  3,    68) =    4.03
            Prob > F =    0.0106
```

<u>Test 3</u>: The **marginal effect of *wgt_i* on *$price_i$* is *constant***.

$$price_i = \beta_0 + \beta_1 wgt_i + \beta_2 mpg_i + \beta_3 wgt_i^2 + \beta_4 mpg_i^2 + \beta_5 wgt_i mpg_i + u_i \qquad \textbf{(2*)}$$

- The **marginal effect of *wgt_i* on *$price_i$* in Model 2*** is:

$$\frac{\partial\, price_i}{\partial\, wgt_i} = \frac{\partial E\left(price_i \mid wgt_i, mpg_i\right)}{\partial\, wgt_i} = \beta_1 + 2\beta_3 wgt_i + \beta_5 mpg_i$$

$$= \beta_1 \text{ (a constant) if } \beta_3 = 0 \textit{ and } \beta_5 = 0$$

- ***Null and Alternative Hypotheses***

H_0: $\beta_3 = 0$ *and* $\beta_5 = 0$ specifies **two** coefficient restrictions

H_1: $\beta_3 \neq 0$ *and/or* $\beta_5 \neq 0$

- ***Unrestricted Model Corresponding to H_1***: regression equation (2*)

- ***Restricted Model Corresponding to H_0***: set $\beta_3 = 0$ *and* $\beta_5 = 0$ in (2*).

$$price_i = \beta_0 + \beta_1 wgt_i + \beta_2 mpg_i + \beta_4 mpg_i^2 + u_i$$

```
. * Test 3: Test hypothesis that marginal effect of wgt is constant
. test wgtsq wgtmpg

 ( 1)  wgtsq = 0.0
 ( 2)  wgtmpg = 0.0

       F(  2,    68) =    8.80
            Prob > F =    0.0004
```

<u>Test 4</u>: The **marginal effect of *mpg_i* on *$price_i$* is *constant***.

$$price_i = \beta_0 + \beta_1 wgt_i + \beta_2 mpg_i + \beta_3 wgt_i^2 + \beta_4 mpg_i^2 + \beta_5 wgt_i mpg_i + u_i \qquad \textbf{(2*)}$$

- The **marginal effect of *mpg_i* on *$price_i$* in Model 2*** is:

$$\frac{\partial\, price_i}{\partial\, mpg_i} = \frac{\partial E\left(price_i \mid wgt_i, mpg_i\right)}{\partial\, mpg_i} = \beta_2 + 2\beta_4 mpg_i + \beta_5 wgt_i$$

$$= \beta_2 \text{ (a constant) if } \beta_4 = 0 \textit{ and } \beta_5 = 0$$

- ***Null and Alternative Hypotheses***

H_0: $\beta_4 = 0$ *and* $\beta_5 = 0$ specifies **two** coefficient restrictions

H_1: $\beta_4 \neq 0$ *and/or* $\beta_5 \neq 0$

- ***Unrestricted Model Corresponding to H_1***: regression equation (2*)

- ***Restricted Model Corresponding to H_0***: set $\beta_4 = 0$ *and* $\beta_5 = 0$ in (2*).

$$price_i = \beta_0 + \beta_1 wgt_i + \beta_2 mpg_i + \beta_3 wgt_i^2 + u_i$$

```
. * Test 4: Test hypothesis that marginal effect of mpg is constant
. test mpgsq wgtmpg

 ( 1)  mpgsq = 0.0
 ( 2)  wgtmpg = 0.0

       F(  2,    68) =    4.75
            Prob > F =    0.0117
```

<u>Test 5</u>: The **marginal effects of <u>both</u> *wgt_i* and *mpg_i* on *$price_i$* are *constant***.

$$price_i = \beta_0 + \beta_1 wgt_i + \beta_2 mpg_i + \beta_3 wgt_i^2 + \beta_4 mpg_i^2 + \beta_5 wgt_i mpg_i + u_i \qquad \textbf{(2*)}$$

- The **marginal effect of *wgt_i* on *$price_i$* in Model 2*** is:

$$\frac{\partial\, price_i}{\partial\, wgt_i} = \frac{\partial E(price_i \mid wgt_i, mpg_i)}{\partial\, wgt_i} = \beta_1 + 2\beta_3 wgt_i + \beta_5 mpg_i$$

$$= \beta_1 \text{ (a constant) if } \beta_3 = 0 \textit{ and } \beta_5 = 0$$

- The **marginal effect of *mpg_i* on *$price_i$* in Model 2*** is:

$$\frac{\partial\, price_i}{\partial\, mpg_i} = \frac{\partial E(price_i \mid wgt_i, mpg_i)}{\partial\, mpg_i} = \beta_2 + 2\beta_4 mpg_i + \beta_5 wgt_i$$

$$= \beta_2 \text{ (a constant) if } \beta_4 = 0 \textit{ and } \beta_5 = 0$$

- ***Null and Alternative Hypotheses***

H_0: $\beta_3 = 0$ *and* $\beta_4 = 0$ *and* $\beta_5 = 0$ specifies **three** coefficient restrictions

H_1: $\beta_3 \neq 0$ *and/or* $\beta_4 \neq 0$ *and/or* $\beta_5 \neq 0$

- ***Unrestricted Model Corresponding to H_1***: regression equation (2*)

- ***Restricted Model Corresponding to H_0* is Model 1***: set $\beta_3 = 0$ *and* $\beta_4 = 0$ *and* $\beta_5 = 0$ in (2*).

$$price_i = \beta_0 + \beta_1 wgt_i + \beta_2 mpg_i + u_i \qquad \textbf{(1*)}$$

```
. * Test 5: Test hypothesis that marginal effects of wgt and mpg are constants
. test wgtsq mpgsq wgtmpg

 ( 1)  wgtsq = 0
 ( 2)  mpgsq = 0
 ( 3)  wgtmpg = 0

       F(  3,    68) =    8.47
            Prob > F =    0.0001
```