QUEEN'S UNIVERSITY AT KINGSTON
Department of Economics

ECONOMICS 351* - Fall Term 2003

Introductory Econometrics

Fall Term 2003 **FINAL EXAMINATION** M.G. Abbott

DATE: **Wednesday December 17, 2003**

TIME: **Three (3) hours (180 minutes); 9:00 a.m. - 12:00 noon**

INSTRUCTIONS: The examination is divided into two parts.

PART A contains two questions; students are required to **answer ONE** of the two questions 1 and 2 in Part A.

PART B contains two questions; students are required to **answer BOTH** questions 3 and 4 in Part B.

- Answer all questions in the exam booklets provided. Be sure **your *student number*** is printed clearly and legibly on the front page of all exam booklets used.
- **Do not write answers to questions on the front page of the first exam booklet**.
- **Please label clearly** each of your answers in the exam booklets with the appropriate number and letter.
- **Please write legibly.** GOOD LUCK! Happy Holidays!

The candidate is urged to submit with the answer paper a clear statement of any assumptions made if doubt exists as to the interpretation of any question that requires a written answer.

MARKING: The marks for each question are indicated in parentheses immediately above each question. **Total marks** for the exam **equal 200**.

PART A:	Questions 1 and 2 (30 marks for each question) Answer ***either one of*** **Questions 1 and 2**.	....	**30 marks**
PART B:	Questions 3 (90 marks) and 4 (80 marks) Answer ***all parts of*** **Questions 3 *and* 4**.	....	**170 marks**
TOTAL MARKS		....	**200 marks**

PART A (30 marks)

Instructions: Answer **EITHER ONE (1) of questions 1 and 2** in this part. Total marks for each question equal 30; marks for each part are given in parentheses.

(30 marks)

1. Consider the simple (two-variable) linear regression model for which the population regression equation can be written in conventional notation as:

$$Y_i = \beta_1 + \beta_2 X_i + u_i \qquad \textbf{(1)}$$

where Y_i and X_i are observable variables, β_1 and β_2 and unknown (constant) regression coefficients, and u_i is an unobservable random error term. The Ordinary Least Squares (OLS) sample regression equation corresponding to regression equation (1) is

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i \qquad (i = 1, ..., N) \qquad \textbf{(2)}$$

where $\hat{\beta}_1$ is the OLS estimator of the intercept coefficient β_1, $\hat{\beta}_2$ is the OLS estimator of the slope coefficient β_2, $\hat{u}_i$ is the OLS residual for the i-th sample observation, and N is sample size (the number of observations in the sample).

(20 marks)

(a) Stating explicitly all required assumptions, prove that the OLS slope coefficient estimator $\hat{\beta}_2$ is an unbiased estimator of the slope coefficient β_2. Include in your answer a definition of the unbiasedness property.

(10 marks)

(b) Write the formulas for two F statistics that can be used to test the null hypothesis H_0: $\beta_2 = 0$ against the alternative hypothesis H_1: $\beta_2 \neq 0$. Define all terms in the formulas you give.

(30 marks)

2. Consider the multiple linear regression model for which the population regression equation can be written in conventional notation as:

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \qquad \textbf{(1)}$$

where Y_i, X_{2i} and X_{3i} are observable variables; β_1, β_2 and β_3 are unknown (constant) regression coefficients; and u_i is an unobservable random error term. The Ordinary Least Squares (OLS) sample regression equation corresponding to regression equation (1) is

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i} + \hat{u}_i \qquad (i = 1, ..., N) \qquad \textbf{(2)}$$

where $\hat{\beta}_1$ is the OLS estimator of the intercept coefficient β_1, $\hat{\beta}_2$ is the OLS estimator of the slope coefficient β_2, $\hat{\beta}_3$ is the OLS estimator of the slope coefficient β_3, $\hat{u}_i$ is the OLS residual for the i-th sample observation, and N is sample size (the number of observations in the estimation sample).

(20 marks)

(a) State the Ordinary Least Squares (OLS) estimation criterion. Derive the OLS normal equations for regression equation (1) from the OLS estimation criterion.

(10 marks)

(b) Write an interpretive formula for $\mathrm{Var}(\hat{\beta}_2)$, the variance of the OLS slope coefficient estimator $\hat{\beta}_2$ in sample regression equation (2). Define all terms in the formula you give. Which of the following factors makes $\mathrm{Var}(\hat{\beta}_2)$ *smaller*?

(1) a larger estimation sample;

(2) more linear dependence between the sample values X_{2i} and X_{3i};

(3) less sample variation of the X_{2i} values;

(4) a larger error variance;

(5) more sample variation of the X_{2i} values;

(6) less linear dependence between the sample values X_{2i} and X_{3i};

(7) a smaller estimation sample;

(8) a smaller error variance.

PART B (170 marks)

Instructions: Answer ***all* parts of questions 3 and 4** in this part. Question 3 is worth a total of 90 marks. Question 4 is worth a total of 80 marks. Marks for each part are given in parentheses. **Show explicitly all formulas and calculations**.

(90 marks)

3. You are investigating the relationship between the annual salaries of company CEOs (Chief Executive Officers) and three of their determinants: annual sales revenues of the company, the market value of the company, and whether the CEO attended graduate school. You have sample data for 177 companies for the year 1990 on the following variables:

$SALARY_i$ = annual salary of the CEO of firm i, measured in *thousands* of dollars per year;
$SALES_i$ = annual sales revenues of firm i, measured in *millions* of dollars per year;
$MKTVAL_i$ = the market value of firm i, measured in *millions* of dollars;
$GRAD_i$ = an indicator variable defined such that $GRAD_i = 1$ if the CEO of firm i attended graduate school, and $GRAD_i = 0$ if the CEO of firm i did not attend graduate school.

Using the given sample data on 177 companies, your trusty research assistant estimates three different regression equations and reports the following estimation results (with estimated *standard errors* given in parentheses):

$$SALARY_i = \beta_1 + \beta_2 SALES_i + \beta_3 MKTVAL_i + \beta_4 GRAD_i + u_{1i} \qquad \textbf{(1)}$$

$\hat{\beta}_1 = 746.47$ (62.37) $\hat{\beta}_2 = 0.01631$ (0.01013) $\hat{\beta}_3 = 0.02600$ (0.009617) $\hat{\beta}_4 = -59.82$ (81.47)

$RSS_{(1)} = 49{,}814{,}493.2$ $TSS_{(1)} = 60{,}765{,}964.7$ N = 177

$$\ln SALARY_i = \alpha_1 + \alpha_2 \ln SALES_i + \alpha_3 \ln MKTVAL_i + \alpha_4 GRAD_i + u_{2i} \qquad \textbf{(2)}$$

$\hat{\alpha}_1 = 4.624$ (0.2547) $\hat{\alpha}_2 = 0.1609$ (0.03975) $\hat{\alpha}_3 = 0.1119$ (0.05062) $\hat{\alpha}_4 = -0.06057$ (0.07789)

$RSS_{(2)} = 45.1519$ $TSS_{(2)} = 64.6462$ N = 177

$$\ln SALARY_i = \gamma_1 + \gamma_2 SALES_i + \gamma_3 MKTVAL_i + \gamma_4 GRAD_i + u_{3i} \qquad \textbf{(3)}$$

$\hat{\gamma}_1 = 6.4427$ (0.06397) $\hat{\gamma}_2 = 0.0000243$ (0.0000104) $\hat{\gamma}_3 = 0.0000210$ (0.00000986) $\hat{\gamma}_4 = -0.03985$ (0.08355)

$RSS_{(3)} = 52.3977$ $TSS_{(3)} = 64.6462$ N = 177

3. (continued)

$RSS_{(n)}$ is the Residual Sum-of-Squares and $TSS_{(n)}$ is the Total Sum-of-Squares for sample regression equation (n), where n = 1, 2, 3. $\ln X_i$ denotes the natural logarithm of the variable X_i. Sample size N = 177. Estimated *standard errors* are given in parentheses below the coefficient estimates.

(15 marks)

(a) Interpret the slope coefficient estimate $\hat{\beta}_2$ in sample regression equation (1), the slope coefficient estimate $\hat{\alpha}_2$ in sample regression equation (2), and the slope coefficient estimate $\hat{\gamma}_2$ in sample regression equation (3). That is, explain in words what the numerical values of the slope coefficient estimates $\hat{\beta}_2$, $\hat{\alpha}_2$ and $\hat{\gamma}_2$ mean.

(15 marks)

(b) Interpret the slope coefficient estimate $\hat{\gamma}_4$ in sample regression equation (3); that is, explain in words what the numerical value of the slope coefficient estimate $\hat{\gamma}_4$ means. Use the estimation results for regression equation (3) to test the proposition that CEOs who have attended graduate school have lower mean salaries (i.e., lower mean values of $\ln SALARY$) than do CEOs who have not attended graduate school. Perform the test at the 5 percent significance level (i.e., for significance level $\alpha = 0.05$). State the null and alternative hypotheses, show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test.

(10 marks)

(c) Use the above estimation results to compute the value of R^2 for OLS sample regression equations (1), (2) and (3). Can these values of R^2 be used to compare the goodness-of-fit to the sample data of regression equations (1), (2) and (3)? Explain why or why not.

(10 marks)

(d) Use the estimation results for regression equation (1) to test the *individual* significance of each of the slope coefficient estimates $\hat{\beta}_2$ for $SALES_i$ and $\hat{\beta}_3$ for $MKTVAL_i$. For each test, state the null and alternative hypotheses, show how you calculate the required test statistic, and state its null distribution. Which of these two slope coefficient estimates are individually significant at the 5 percent significance level? Which of these two slope coefficient estimates are individually significant at the 1 percent significance level?

3. (continued)

(10 marks)
(e) Use the estimation results for regression equation (2) to test the *joint* significance of the slope coefficient estimates $\hat{\alpha}_2$, $\hat{\alpha}_3$ and $\hat{\alpha}_4$ at the 1 percent significance level (i.e., for significance level $\alpha = 0.01$). State the null and alternative hypotheses, show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test.

(10 marks)
(f) Use the estimation results for regression equation (2) to compute the two-sided 95 percent confidence interval for the slope coefficient α_3 of $\ln \text{MKTVAL}_i$. Use this confidence interval to perform a two-tail test of the hypothesis that $\alpha_3 = 0$ at the 5 percent significance level (i.e., for significance level $\alpha = 0.05$). State the null and alternative hypotheses, the decision rule you use, and the inference you would draw from the test.

(10 marks)
(g) Use the estimation results for regression equation (2) to perform a two-tail test of the proposition that $\alpha_2 = 0.2$ at the 5 percent significance level. Explain in words what this proposition means. State the null and alternative hypotheses. Show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test at the 5 percent significance level. Would your inference be the same at the 10 percent significance level?

(10 marks)
(h) Use the estimation results for regression equation (2) to perform a two-tail test of the null hypothesis $\alpha_2 = \alpha_3$ at the 5 percent significance level. Explain in words what this null hypothesis means. The estimated covariance of $\hat{\alpha}_2$ and $\hat{\alpha}_3$ is $\widehat{\text{Cov}}(\hat{\alpha}_2, \hat{\alpha}_3) = -0.001477$, and the Residual Sum-of-Squares value for the *restricted* OLS regression equation implied by the null hypothesis is RSS = 45.2402. State the null and alternative hypotheses, show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test at the 5 percent significance level.

(80 marks)

4. You are conducting an econometric investigation into the hourly wage rates of unionized and non-unionized female employees. The sample data consist of observations for 3286 female employees on the following variables:

W_i = the hourly wage rate of the i-th employee, in dollars per hour;

ED_i = years of formal education completed by the i-th employee, in years;

EXP_i = years of work experience accumulated by the i-th employee, in years;

UN_i = an indicator variable defined such that $UN_i = 1$ if the i-th employee is unionized, and $UN_i = 0$ if the i-th employee is non-unionized.

The regression model you propose to use is the log-lin (semi-log) regression equation

$$\ln W_i = \beta_1 + \beta_2 ED_i + \beta_3 EXP_i + \beta_4 ED_i^2 + \beta_5 EXP_i^2 + \beta_6 ED_i EXP_i + \beta_7 UN_i + \beta_8 UN_i ED_i$$
$$+ \beta_9 UN_i EXP_i + \beta_{10} UN_i ED_i^2 + \beta_{11} UN_i EXP_i^2 + \beta_{12} UN_i ED_i EXP_i + u_i \qquad \textbf{... (1)}$$

where the β_j (j = 1, 2, …, 12) are regression coefficients, $\ln W_i$ denotes the natural logarithm of the variable W_i, and u_i is a random error term.

Using the sample data described above, your research assistant computes OLS estimates of regression equation (1) and of three restricted versions of equation (1). For each of the sample regression equations estimated on the N = 3286 observations, the following table contains the OLS coefficient estimates (with estimated *standard errors* in parentheses below the coefficient estimates) and the summary statistics RSS (residual sum-of-squares), TSS (total sum-of-squares), and N (number of sample observations).

4. (continued)

<u>Question 4</u>: OLS Sample Regression Equations for $\ln W_i$
(*standard errors* in parentheses)

Regressors		(1)	(2)	(3)	(4)
Intercept	$\hat{\beta}_1$	0.5175 (0.2475)	0.6914 (0.2299)	0.7055 (0.2295)	0.7412 (0.2313)
ED_i	$\hat{\beta}_2$	0.08449 (0.02894)	0.06910 (0.02666)	0.06516 (0.02655)	0.05574 (0.02674)
EXP_i	$\hat{\beta}_3$	0.040026 (0.008062)	0.03417 (0.007574)	0.03566 (0.007524)	0.03771 (0.007580)
ED_i^2	$\hat{\beta}_4$	0.001601 (0.0008865)	0.001837 (0.0008112)	0.001957 (0.0008091)	0.002445 (0.0008130)
EXP_i^2	$\hat{\beta}_5$	–0.0003937 (0.0000890)	–0.0003664 (0.0000879)	–0.0004177 (0.0000841)	–0.0004624 (0.0000846)
ED_iEXP_i	$\hat{\beta}_6$	–0.001447 (0.0004434)	–0.001096 (0.0004094)	–0.001049 (0.0004063)	–0.001020 (0.0004095)
UN_i	$\hat{\beta}_7$	1.762 (0.7416)	–0.007131 (0.1059)	0.1709 (0.02311)	----
UN_iED_i	$\hat{\beta}_8$	–0.1526 (0.08174)	----	----	----
UN_iEXP_i	$\hat{\beta}_9$	–0.03941 (0.02491)	0.02097 (0.01065)	----	----
$UN_iED_i^2$	$\hat{\beta}_{10}$	0.002564 (0.002455)	----	----	----
$UN_iEXP_i^2$	$\hat{\beta}_{11}$	–0.0001760 (0.0002791)	–0.0005027 (0.0002447)	----	----
$UN_iED_iEXP_i$	$\hat{\beta}_{12}$	0.0032978 (0.001229)	----	----	----
	RSS =	698.894	700.839	701.744	713.442
	TSS =	905.589	905.589	905.589	905.589
	N =	3286	3286	3286	3286

<u>*Note:*</u> The symbol "----" means that the corresponding regressor was omitted from the estimated sample regression equation. The figures in parentheses below the coefficient estimates are the estimated *standard errors*. RSS is the Residual Sum-of-Squares, TSS is the Total Sum-of-Squares, and N is the number of sample observations.

4. (continued)

(10 marks)

(a) Compare the goodness-of-fit to the sample data of the four sample regression equations (1), (2), (3) and (4) in the table. Calculate the value of an appropriate goodness-of-fit measure for each of the sample regression equations (1), (2), (3) and (4) in the table. Which of the four sample regression equations provides the best fit to the sample data? Which of the four sample regression equations provides the worst fit to the sample data?

(10 marks)

(b) Use the estimation results for regression equation (3) in the above table to perform a test of the proposition that unionized employees of any given education and experience have higher average log-wages than non-unionized employees of the same education and experience. Perform the test at the 5 percent significance level (i.e., for significance level $\alpha = 0.05$). State the null hypothesis H_0 and the alternative hypothesis H_1. Show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test.

(10 marks)

(c) State the coefficient restrictions that regression equation (2) in the table imposes on regression equation (1). Explain in words what the restrictions mean. Use the estimation results given in the table to perform a test of these coefficient restrictions at the 5 percent significance level (i.e., for significance level $\alpha = 0.05$). State the null and alternative hypotheses, show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test. Would your inference be the same at the 1 percent significance level (i.e., for significance level $\alpha = 0.01$)? Based on the outcome of the test, which of the two regression equations would you choose, equation (1) or equation (2)?

(10 marks)

(d) State the coefficient restrictions that regression equation (3) in the table imposes on regression equation (1). Explain in words what the restrictions mean. Use the estimation results given in the table to perform a test of these coefficient restrictions at the 5 percent significance level (i.e., for significance level $\alpha = 0.05$). State the null and alternative hypotheses, show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test. Would your inference be the same at the 1 percent significance level (i.e., for significance level $\alpha = 0.01$)? Based on the outcome of the test, which of the two regression equations would you choose, equation (1) or equation (3)?

4. (continued)

(10 marks)

(e) State the coefficient restrictions that regression equation (4) in the table imposes on regression equation (1). Explain in words what the restrictions mean. Use the estimation results given in the table to perform a test of these coefficient restrictions at the 5 percent significance level (i.e., for significance level $\alpha = 0.05$). State the null and alternative hypotheses, show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test. Would your inference be the same at the 1 percent significance level (i.e., for significance level $\alpha = 0.01$)? Based on the outcome of the test, which of the two regression equations would you choose, equation (1) or equation (4)?

(15 marks)

(f) Write the expression (or formula) for the marginal effect of ED_i on $\ln W_i$ for *non-unionized* employees implied by regression equation (1). Use regression equation (1) to compute a test of the proposition that the marginal effect of ED_i on $\ln W_i$ for *non-unionized* employees is equal to zero for non-unionized employees with any given values of ED_i and EXP_i. State the coefficient restrictions on regression equation (1) implied by this proposition; that is, state the null hypothesis H_0 and the alternative hypothesis H_1. Write the *restricted* regression equation implied by the null hypothesis H_0. OLS estimation of this *restricted* regression equation yields a Residual Sum-of-Squares value of **RSS = 836.832**. Use this information, together with the results from OLS estimation of equation (1), to calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test. Choose an appropriate significance level for the test.

(15 marks)

(g) Write the expression (or formula) for the marginal effect of ED_i on $\ln W_i$ for *unionized* employees implied by regression equation (1). Use regression equation (1) to compute a test of the null hypothesis that the marginal effect of ED_i on $\ln W_i$ for *unionized* employees is equal to zero for unionized employees with any given values of ED_i and EXP_i. State the coefficient restrictions on regression equation (1) implied by this proposition; that is, state the null hypothesis H_0 and the alternative hypothesis H_1. Write the *restricted* regression equation implied by the null hypothesis H_0. OLS estimation of this *restricted* regression equation yields a Residual Sum-of-Squares value of **RSS = 722.988**. Use this information, together with the results from OLS estimation of equation (1), to calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test. Choose an appropriate significance level for the test.

Selected Formulas

<u>For the Simple (Two-Variable) Linear Regression Model</u>

$$Y_i = \beta_1 + \beta_2 X_i + u_i \qquad (i = 1, \ldots, N)$$

- Deviations from sample means are defined as:

$$y_i \equiv Y_i - \bar{Y}; \qquad x_i \equiv X_i - \bar{X};$$

where

$\bar{Y} = \Sigma_i Y_i / N = \dfrac{\Sigma_i Y_i}{N}$ is the sample mean of the Y_i values;

$\bar{X} = \Sigma_i X_i / N = \dfrac{\Sigma_i X_i}{N}$ is the sample mean of the X_i values.

- Formulas for the variance of the OLS intercept coefficient estimator $\hat{\beta}_1$ and the covariance of the OLS coefficient estimators $\hat{\beta}_1$ and $\hat{\beta}_2$ in the two-variable linear regression model:

$$\mathrm{Var}(\hat{\beta}_1) = \frac{\sigma^2 \Sigma_i X_i^2}{N \Sigma_i (X_i - \bar{X})^2} = \frac{\sigma^2 \Sigma_i X_i^2}{N \Sigma_i x_i^2};$$

$$\mathrm{Cov}(\hat{\beta}_1, \hat{\beta}_2) = -\bar{X}\left(\frac{\sigma^2}{\Sigma_i x_i^2}\right).$$

- Formulas for the variance of the conditional predictor $\hat{Y}_0 = \hat{\beta}_1 + \hat{\beta}_2 X_0$:

  - When $\hat{Y}_0$ is used as a mean predictor of $E(Y_0 \mid X_0) = \beta_1 + \beta_2 X_0$,

$$\mathrm{Var}(\hat{Y}_0^m) = \sigma^2\left[\frac{1}{N} + \frac{(X_0 - \bar{X})^2}{\Sigma_i x_i^2}\right].$$

  - When $\hat{Y}_0$ is used as an individual predictor of $Y_0 \mid X_0 = \beta_1 + \beta_2 X_0 + u_0$,

$$\mathrm{Var}(\hat{Y}_0) = \sigma^2 + \sigma^2\left[\frac{1}{N} + \frac{(X_0 - \bar{X})^2}{\Sigma_i x_i^2}\right].$$

Selected Formulas (continued)

For the Multiple (Three-Variable) Linear Regression Model

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \qquad (i = 1, \ldots, N)$$

- Deviations from sample means are defined as:

$$y_i \equiv Y_i - \overline{Y}; \qquad x_{2i} \equiv X_{2i} - \overline{X}_2; \qquad x_{3i} \equiv X_{3i} - \overline{X}_3;$$

where

$\overline{Y} = \Sigma_i Y_i / N = \dfrac{\Sigma_i Y_i}{N}$ is the sample mean of the Y_i values;

$\overline{X}_{2i} = \Sigma_i X_{2i} / N = \dfrac{\Sigma_i X_{2i}}{N}$ is the sample mean of the X_{2i} values;

$\overline{X}_{3i} = \Sigma_i X_{3i} / N = \dfrac{\Sigma_i X_{3i}}{N}$ is the sample mean of the X_{3i} values.

- The OLS slope coefficient estimators $\hat{\beta}_2$ and $\hat{\beta}_3$ in deviation-from-means form are:

$$\hat{\beta}_2 = \frac{\left(\Sigma_i x_{3i}^2\right)\left(\Sigma_i x_{2i} y_i\right) - \left(\Sigma_i x_{2i} x_{3i}\right)\left(\Sigma_i x_{3i} y_i\right)}{\left(\Sigma_i x_{2i}^2\right)\left(\Sigma_i x_{3i}^2\right) - \left(\Sigma_i x_{2i} x_{3i}\right)^2};$$

$$\hat{\beta}_3 = \frac{\left(\Sigma_i x_{2i}^2\right)\left(\Sigma_i x_{3i} y_i\right) - \left(\Sigma_i x_{2i} x_{3i}\right)\left(\Sigma_i x_{2i} y_i\right)}{\left(\Sigma_i x_{2i}^2\right)\left(\Sigma_i x_{3i}^2\right) - \left(\Sigma_i x_{2i} x_{3i}\right)^2}.$$

- Formulas for the variances and covariances of the slope coefficient estimators $\hat{\beta}_2$ and $\hat{\beta}_3$:

$$\mathrm{Var}\left(\hat{\beta}_2\right) = \frac{\sigma^2 \Sigma_i x_{3i}^2}{\left(\Sigma_i x_{2i}^2\right)\left(\Sigma_i x_{3i}^2\right) - \left(\Sigma_i x_{2i} x_{3i}\right)^2};$$

$$\mathrm{Var}\left(\hat{\beta}_3\right) = \frac{\sigma^2 \Sigma_i x_{2i}^2}{\left(\Sigma_i x_{2i}^2\right)\left(\Sigma_i x_{3i}^2\right) - \left(\Sigma_i x_{2i} x_{3i}\right)^2};$$

$$\mathrm{Cov}\left(\hat{\beta}_2, \hat{\beta}_3\right) = \frac{\sigma^2 \Sigma_i x_{2i} x_{3i}}{\left(\Sigma_i x_{2i}^2\right)\left(\Sigma_i x_{3i}^2\right) - \left(\Sigma_i x_{2i} x_{3i}\right)^2}.$$

Percentage Points of the t-Distribution

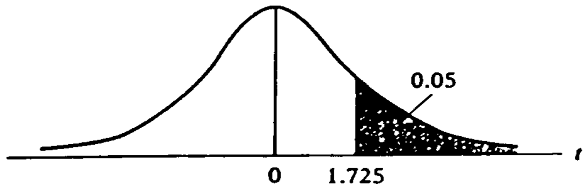
TABLE D.2
Percentage points of the t distribution

Example

$\Pr(t > 2.086) = 0.025$

$\Pr(t > 1.725) = 0.05$ for df = 20

$\Pr(|t| > 1.725) = 0.10$



df \ Pr	0.25 0.50	0.10 0.20	0.05 0.10	0.025 0.05	0.01 0.02	0.005 0.010	0.001 0.002
1	1.000	3.078	6.314	12.706	31.821	63.657	318.31
2	0.816	1.886	2.920	4.303	6.965	9.925	22.327
3	0.765	1.638	2.353	3.182	4.541	5.841	10.214
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090

Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 12, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

Source: Damodar N. Gujarati, *Basic Econometrics*, Third Edition. New York: McGraw-Hill, 1995, p. 809.

Selected Upper Percentage Points of the F-Distribution

TABLE D.3
Upper percentage points of the *F* distribution (*continued*)

df for denominator N_2	df for numerator N_1												
	Pr	1	2	3	4	5	6	7	8	9	10	11	12
22	.25	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
	.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12
24	.25	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37	1.36
	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21	2.18
	.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03
26	.25	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.36	1.35
	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.84	1.81
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96
28	.25	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79
	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12
	.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90
30	.25	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
	.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84
40	.25	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31
	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.73	1.71
	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
	.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66
60	.25	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.29
	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50
120	.25	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26
	.10	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.62	1.60
	.05	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.87	1.83
	.01	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34
200	.25	1.33	1.39	1.38	1.36	1.34	1.32	1.31	1.29	1.28	1.27	1.26	1.25
	.10	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66	1.63	1.60	1.57
	.05	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80
	.01	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.34	2.27
∞	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.24
	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75
	.01	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18

Source: Damodar N. Gujarati, *Basic Econometrics,* Third Edition. New York: McGraw-Hill, 1995, p. 814.