(70 marks)

4. You are conducting an econometric investigation into the selling prices of houses in a single urban area in the years 1993 and 2003. You have available for this purpose a random sample of houses that were sold in the years 1993 and 2003. The sample data consist of observations for these houses on the following variables:

$PRICE_i$ = the selling price of the i-th house, in *thousands of dollars*;

$HSIZE_i$ = the living area of the i-th house, in *hundreds of square feet*;

LOT_i = lot size of the i-th house, in *hundreds of square feet*;

AGE_i = the age of the i-th house at the time it was sold, *in years*;

$VIEW_i$ = an indicator variable defined such that $VIEW_i = 1$ if the i-th house has a view, and $VIEW_i = 0$ if the i-th house does not have a view;

$D03_i$ = an indicator variable defined such that $D03_i = 1$ if the i-th house was sold in 2003, and $D03_i = 0$ if the i-th house was sold in 1993.

The regression model you are asked to estimate is given by the population regression equation

$$\begin{aligned} PRICE_i = {} & \beta_0 + \beta_1 HSIZE_i + \beta_2 HSIZE_i^2 + \beta_3 LOT_i + \beta_4 AGE_i + \beta_5 HSIZE_i AGE_i + \beta_6 VIEW_i \\ & + \beta_7 D03_i + \beta_8 D03_i HSIZE_i + \beta_9 D03_i HSIZE_i^2 + \beta_{10} D03_i LOT_i \\ & + \beta_{11} D03_i AGE_i + \beta_{12} D03_i HSIZE_i AGE_i + \beta_{13} D03_i VIEW_i + u_i \qquad \textbf{... (1)} \end{aligned}$$

where the β_j (j = 0, 1, 2, …, 13) are regression coefficients and u_i is a random error term.

State the null hypothesis H_0 and alternative hypothesis H_1 of the statistical test that you would perform on regression equation (1) to assess the evidence for each of the following empirical propositions. In addition, for each hypothesis test, state which of the following tests you would use: (1) a two-tail t-test; (2) a left-tail t-test; (3) a right-tail t-test; or (4) an F-test.

Regression function for house prices implied by equation (1):

$$\begin{aligned} E(PRICE_i \mid \bullet) = {} & \beta_0 + \beta_1 HSIZE_i + \beta_2 HSIZE_i^2 + \beta_3 LOT_i + \beta_4 AGE_i + \beta_5 HSIZE_i AGE_i + \beta_6 VIEW_i \\ & + \beta_7 D03_i + \beta_8 D03_i HSIZE_i + \beta_9 D03_i HSIZE_i^2 + \beta_{10} D03_i LOT_i \\ & + \beta_{11} D03_i AGE_i + \beta_{12} D03_i HSIZE_i AGE_i + \beta_{13} D03_i VIEW_i \end{aligned}$$

Regression function for mean house prices in 2003, for which $\mathbf{D03_i = 1}$:

$$\begin{aligned} E(PRICE_i \mid D03_i = 1) = {} & \beta_0 + \beta_1 HSIZE_i + \beta_2 HSIZE_i^2 + \beta_3 LOT_i + \beta_4 AGE_i + \beta_5 HSIZE_i AGE_i \\ & + \beta_6 VIEW_i + \beta_7 + \beta_8 HSIZE_i + \beta_9 HSIZE_i^2 + \beta_{10} LOT_i \\ & + \beta_{11} AGE_i + \beta_{12} HSIZE_i AGE_i + \beta_{13} VIEW_i \end{aligned}$$

Regression function for mean house prices in 1993, for which $\mathbf{D03_i = 0}$:

$$E(PRICE_i \mid D03_i = 1) = \beta_0 + \beta_1 HSIZE_i + \beta_2 HSIZE_i^2 + \beta_3 LOT_i + \beta_4 AGE_i + \beta_5 HSIZE_i AGE_i + \beta_6 VIEW_i$$

Regression function for mean house prices in 2003, for which **$D03_i = 1$**:

$$\begin{aligned} E(PRICE_i \mid D03_i = 1) = {} & \beta_0 + \beta_1 HSIZE_i + \beta_2 HSIZE_i^2 + \beta_3 LOT_i + \beta_4 AGE_i + \beta_5 HSIZE_i AGE_i \\ & + \beta_6 VIEW_i + \beta_7 + \beta_8 HSIZE_i + \beta_9 HSIZE_i^2 + \beta_{10} LOT_i \\ & + \beta_{11} AGE_i + \beta_{12} HSIZE_i AGE_i + \beta_{13} VIEW_i \end{aligned}$$

Regression function for mean house prices in 1993, for which **$D03_i = 0$**:

$$E(PRICE_i \mid D03_i = 1) = \beta_0 + \beta_1 HSIZE_i + \beta_2 HSIZE_i^2 + \beta_3 LOT_i + \beta_4 AGE_i + \beta_5 HSIZE_i AGE_i + \beta_6 VIEW_i$$

Conditional mean price difference between 2003 and 1993:

$$\begin{aligned} & E(PRICE_i \mid D03_i = 1) - E(PRICE_i \mid D03_i = 0) = \\ & \quad \beta_7 + \beta_8 HSIZE_i + \beta_9 HSIZE_i^2 + \beta_{10} LOT_i + \beta_{11} AGE_i + \beta_{12} HSIZE_i AGE_i + \beta_{13} VIEW_i \end{aligned}$$

$$E(PRICE_i \mid \bullet) = \beta_0 + \beta_1 HSIZE_i + \beta_2 HSIZE_i^2 + \beta_3 LOT_i + \beta_4 AGE_i + \beta_5 HSIZE_i AGE_i + \beta_6 VIEW_i$$
$$+ \beta_7 D03_i + \beta_8 D03_i HSIZE_i + \beta_9 D03_i HSIZE_i^2 + \beta_{10} D03_i LOT_i$$
$$+ \beta_{11} D03_i AGE_i + \beta_{12} D03_i HSIZE_i AGE_i + \beta_{13} D03_i VIEW_i$$

Marginal effect of $HSIZE_i$ on $PRICE_i$:

$$\frac{\partial E(PRICE_i \mid \bullet)}{\partial HSIZE_i} = \beta_1 + 2\beta_2 HSIZE_i + \beta_5 AGE_i + \beta_8 D03_i + 2\beta_9 D03_i HSIZE_i + \beta_{12} D03_i AGE_i$$

$$= \beta_1 + 2\beta_2 HSIZE_i + \beta_5 AGE_i + \beta_8 + 2\beta_9 HSIZE_i + \beta_{12} AGE_i$$
$$= (\beta_1 + \beta_8) + 2(\beta_2 + \beta_9) HSIZE_i + (\beta_5 + \beta_{12}) AGE_i \qquad \text{in 2003}$$

$$= \beta_1 + 2\beta_2 HSIZE_i + \beta_5 AGE_i \qquad \text{in 1993}$$

Marginal effect of AGE_i on $PRICE_i$:

$$\frac{\partial E(PRICE_i \mid \bullet)}{\partial AGE_i} = \beta_4 + \beta_5 HSIZE_i + \beta_{11} D03_i + \beta_{12} D03_i HSIZE_i$$

$$= \beta_4 + \beta_5 HSIZE_i + \beta_{11} + \beta_{12} HSIZE_i = (\beta_4 + \beta_{11}) + (\beta_5 + \beta_{12}) HSIZE_i \qquad \text{in 2003}$$

$$= \beta_4 + \beta_5 HSIZE_i \qquad \text{in 1993}$$

$$
\begin{aligned}
E(PRICE_i \mid \bullet) = \beta_0 &+ \beta_1 HSIZE_i + \beta_2 HSIZE_i^2 + \beta_3 LOT_i + \beta_4 AGE_i + \beta_5 HSIZE_i AGE_i + \beta_6 VIEW_i \\
&+ \beta_7 D03_i + \beta_8 D03_i HSIZE_i + \beta_9 D03_i HSIZE_i^2 + \beta_{10} D03_i LOT_i \\
&+ \beta_{11} D03_i AGE_i + \beta_{12} D03_i HSIZE_i AGE_i + \beta_{13} D03_i VIEW_i
\end{aligned}
$$

Marginal effect of LOT_i on $PRICE_i$:

$$\frac{\partial E(PRICE_i \mid \bullet)}{\partial LOT_i} = \beta_3 + \beta_{10} D03_i$$

$$= \beta_3 + \beta_{10} \qquad \text{in 2003}$$

$$= \beta_3 \qquad \text{in 1993}$$

4. (continued)

(7 marks)
(a) The marginal effect of house size on price was zero in 1993.

Marginal effect of HSIZE_i on PRICE_i:

$$\frac{\partial E(\text{PRICE}_i \mid \bullet)}{\partial \text{HSIZE}_i} = \beta_1 + 2\beta_2 \text{HSIZE}_i + \beta_5 \text{AGE}_i + \beta_8 \text{D03}_i + 2\beta_9 \text{D03}_i \text{HSIZE}_i + \beta_{12} \text{D03}_i \text{AGE}_i$$

$$= \beta_1 + 2\beta_2 \text{HSIZE}_i + \beta_5 \text{AGE}_i + \beta_8 + 2\beta_9 \text{HSIZE}_i + \beta_{12} \text{AGE}_i$$

$$= (\beta_1 + \beta_8) + 2(\beta_2 + \beta_9)\text{HSIZE}_i + (\beta_5 + \beta_{12})\text{AGE}_i \qquad \text{in 2003}$$

$$= \beta_1 + 2\beta_2 \text{HSIZE}_i + \beta_5 \text{AGE}_i \qquad \text{in 1993}$$

H_0: $\beta_1 = 0$ and $\beta_2 = 0$ and $\beta_5 = 0$

H_1: $\beta_1 \neq 0$ and/or $\beta_2 \neq 0$ and/or $\beta_5 \neq 0$

Test: **F-test**

(7 marks)
(b) The relationship of house size to price exhibited increasing marginal returns in 2003.

H_0: $\beta_2 + \beta_9 = 0$ or $2(\beta_2 + \beta_9) = 0$

H_1: $\beta_2 + \beta_9 > 0$ or $2(\beta_2 + \beta_9) > 0$

Test: **right-tail t-test**

Marginal effect of AGE_i on $PRICE_i$:

$$\frac{\partial E(PRICE_i \mid \bullet)}{\partial AGE_i} = \beta_4 + \beta_5 HSIZE_i + \beta_{11} D03_i + \beta_{12} D03_i HSIZE_i$$

$$= \beta_4 + \beta_5 HSIZE_i + \beta_{11} + \beta_{12} HSIZE_i = (\beta_4 + \beta_{11}) + (\beta_5 + \beta_{12}) HSIZE_i \quad \text{in 2003}$$

$$= \beta_4 + \beta_5 HSIZE_i \quad \text{in 1993}$$

(7 marks)

(c) The marginal effect of house age on price was the same in 2003 as it was in 1993.

H_0: $\beta_{11} = 0$ and $\beta_{12} = 0$

H_1: $\beta_{11} \neq 0$ and/or $\beta_{12} \neq 0$

Test: **F-test**

(7 marks)

(d) Holding constant house size, lot size and view status, house prices in 2003 were unrelated to house age.

H_0: $\beta_4 + \beta_{11} = 0$ and $\beta_5 + \beta_{12} = 0$

H_1: $\beta_4 + \beta_{11} \neq 0$ and/or $\beta_5 + \beta_{12} \neq 0$

Test: **F-test**

4. (continued)

(7 marks)
(e) House size and house age were substitutable for one another in determining house prices in 2003.

Marginal effect of AGE_i on $PRICE_i$:

$$\frac{\partial E(PRICE_i \mid \bullet)}{\partial AGE_i} = \beta_4 + \beta_5 HSIZE_i + \beta_{11} D03_i + \beta_{12} D03_i HSIZE_i$$

$$= \beta_4 + \beta_5 HSIZE_i + \beta_{11} + \beta_{12} HSIZE_i = (\beta_4 + \beta_{11}) + (\beta_5 + \beta_{12}) HSIZE_i \quad \text{in 2003}$$

H_0: $\beta_5 + \beta_{12} = 0$

H_1: $\beta_5 + \beta_{12} < 0$

Test: ***left-tail* t-test**

4. (continued)

(7 marks)

(f) The marginal effect of house size on price was constant in 2003.

Marginal effect of $\mathbf{HSIZE_i}$ on $\mathbf{PRICE_i}$:

$$\frac{\partial E(PRICE_i \mid \bullet)}{\partial HSIZE_i} = \beta_1 + 2\beta_2 HSIZE_i + \beta_5 AGE_i + \beta_8 D03_i + 2\beta_9 D03_i HSIZE_i + \beta_{12} D03_i AGE_i$$

$$= \beta_1 + 2\beta_2 HSIZE_i + \beta_5 AGE_i + \beta_8 + 2\beta_9 HSIZE_i + \beta_{12} AGE_i$$

$$= (\beta_1 + \beta_8) + 2(\beta_2 + \beta_9) HSIZE_i + (\beta_5 + \beta_{12}) AGE_i \qquad \text{in 2003}$$

$$= \beta_1 + 2\beta_2 HSIZE_i + \beta_5 AGE_i \qquad \text{in 1993}$$

H_0: $\beta_2 + \beta_9 = 0$ and $\beta_5 + \beta_{12} = 0$

H_1: $\beta_2 + \beta_9 \neq 0$ and/or $\beta_5 + \beta_{12} \neq 0$

Test: **F-test**

4. (continued)

(7 marks)
(g) The marginal effect of lot size on price was zero in both 1993 and 2003.

Marginal effect of $\mathbf{LOT_i}$ on $\mathbf{PRICE_i}$:

$$\frac{\partial E(PRICE_i \mid \bullet)}{\partial LOT_i} = \beta_3 + \beta_{10} D03_i$$

$$= \beta_3 + \beta_{10} \qquad \text{in 2003}$$

$$= \beta_3 \qquad \text{in 1993}$$

H_0: $\beta_3 = 0$ and $\beta_{10} = 0$ OR $\beta_3 = 0$ and $\beta_3 + \beta_{10} = 0$

H_1: $\beta_3 \neq 0$ and/or $\beta_{10} \neq 0$ OR $\beta_3 \neq 0$ and/or $\beta_3 + \beta_{10} \neq 0$

Test: **F-test**

4. (continued)

(7 marks)

(h) The mean price of houses with a view was equal to the mean price of houses without a view in both 1993 and 2003.

Regression function for mean house prices in 2003, for which $\mathbf{D03_i = 1}$:

$$E(PRICE_i \mid D03_i = 1) = \beta_0 + \beta_1 HSIZE_i + \beta_2 HSIZE_i^2 + \beta_3 LOT_i + \beta_4 AGE_i + \beta_5 HSIZE_i AGE_i$$
$$+ \beta_6 VIEW_i + \beta_7 + \beta_8 HSIZE_i + \beta_9 HSIZE_i^2 + \beta_{10} LOT_i$$
$$+ \beta_{11} AGE_i + \beta_{12} HSIZE_i AGE_i + \beta_{13} VIEW_i$$

Regression function for mean house prices in 1993, for which $\mathbf{D03_i = 0}$:

$$E(PRICE_i \mid D03_i = 1) = \beta_0 + \beta_1 HSIZE_i + \beta_2 HSIZE_i^2 + \beta_3 LOT_i + \beta_4 AGE_i + \beta_5 HSIZE_i AGE_i + \beta_6 VIEW_i$$

Marginal effect of $\mathbf{VIEW_i}$ in 2003 = $\beta_6 + \beta_{13}$

Marginal effect of $\mathbf{VIEW_i}$ in 1993 = β_6

H_0: $\beta_6 = 0$ and $\beta_{13} = 0$ OR $\beta_6 = 0$ and $\beta_6 + \beta_{13} = 0$

H_1: $\beta_6 \neq 0$ and/or $\beta_{13} \neq 0$ OR $\beta_6 \neq 0$ and/or $\beta_6 + \beta_{13} \neq 0$

Test: **F-test**

4. (continued)

(7 marks)

(i) The mean price of houses in 1993 was equal to the mean price of similar houses in 2003.

Conditional mean price difference between 2003 and 1993:

$$E(PRICE_i \mid D03_i = 1) - E(PRICE_i \mid D03_i = 0) =$$
$$\beta_7 + \beta_8 HSIZE_i + \beta_9 HSIZE_i^2 + \beta_{10} LOT_i + \beta_{11} AGE_i + \beta_{12} HSIZE_i AGE_i + \beta_{13} VIEW_i$$

H_0: $\beta_7 = 0$ and $\beta_8 = 0$ and $\beta_9 = 0$ and $\beta_{10} = 0$ and $\beta_{11} = 0$ and $\beta_{12} = 0$ and $\beta_{13} = 0$

H_1: $\beta_7 \neq 0$ or $\beta_8 \neq 0$ or $\beta_9 \neq 0$ or $\beta_{10} \neq 0$ or $\beta_{11} \neq 0$ or $\beta_{12} \neq 0$ or $\beta_{13} \neq 0$

Test: **F-test**

4. (continued)

(7 marks)

(j) For houses with 2500 square feet of living area and lots of 6000 square feet that were 10 years old and had a view, mean price in 2003 was **greater than** mean price in 1993.

Evaluate 2003-1993 mean price difference

$$E(PRICE_i \mid D03_i = 1) - E(PRICE_i \mid D03_i = 0) =$$

$$\beta_7 + \beta_8 HSIZE_i + \beta_9 HSIZE_i^2 + \beta_{10} LOT_i + \beta_{11} AGE_i + \beta_{12} HSIZE_i AGE_i + \beta_{13} VIEW_i$$

at: $HSIZE_i = 25$; $LOT_i = 60$; $AGE_i = 10$; $VIEW_i = 1$.

H_0: $\beta_7 + \beta_8 25 + \beta_9 (25)^2 + \beta_{10} 60 + \beta_{11} 10 + \beta_{12} (25)(10) + \beta_{13} = 0$

or

$\beta_7 + \beta_8 25 + \beta_9 625 + \beta_{10} 60 + \beta_{11} 10 + \beta_{12} 250 + \beta_{13} = 0$

H_1: $\beta_7 + \beta_8 25 + \beta_9 (25)^2 + \beta_{10} 60 + \beta_{11} 10 + \beta_{12} (25)(10) + \beta_{13} > 0$

or

$\beta_7 + \beta_8 25 + \beta_9 625 + \beta_{10} 60 + \beta_{11} 10 + \beta_{12} 250 + \beta_{13} > 0$

Test: ***right-tail*** **t-test**