QUEEN'S UNIVERSITY AT KINGSTON
Department of Economics

ECONOMICS 351* - Section A

Introductory Econometrics

Fall Term 2002 **FINAL EXAMINATION** M.G. Abbott

DATE: **Friday December 13, 2002**

TIME: **Three (3) hours (180 minutes); 2:00 p.m. - 5:00 p.m.**

INSTRUCTIONS: The examination is divided into two parts.

PART A contains two questions; students are required to answer **ONE** of the two questions 1 and 2 in Part A.

PART B contains three questions; students are required to answer **ALL THREE** of the questions 3, 4 and 5 in Part B.

- Answer all questions in the exam booklets provided. Be sure **your *student number*** is printed clearly and legibly on the front page of all exam booklets used.
- **Do not write answers to questions on the front page of the first exam booklet**.
- **Please label clearly** each of your answers in the exam booklets with the appropriate number and letter.
- **Please write legibly.** GOOD LUCK! Happy Holidays!

The candidate is urged to submit with the answer paper a clear statement of any assumptions made if doubt exists as to the interpretation of any question that requires a written answer.

MARKING: The marks for each question are indicated in parentheses immediately above each question. **Total marks** for the exam **equal 200**.

PART A: Questions 1 and 2 (30 marks for each question) .... **30 marks**
Answer ***either one of*** **Questions 1 and 2**.

PART B: Questions 3 (80 marks), 4 (50 marks) and 5 (40 marks) .... **170 marks**
Answer ***all parts of*** **Questions 3, 4 *and* 5**.

TOTAL MARKS .... **200 marks**

PART A (30 marks)

Instructions: Answer **EITHER ONE (1) of questions 1 and 2** in this part. Total marks for each question equal 30; marks for each part are given in parentheses.

(30 marks)

1. Consider the simple (two-variable) linear regression model for which the population regression equation can be written in conventional notation as:

$$Y_i = \beta_1 + \beta_2 X_i + u_i \qquad \textbf{(1)}$$

where Y_i and X_i are observable variables, β_1 and β_2 and unknown (constant) regression coefficients, and u_i is an unobservable random error term. The Ordinary Least Squares (OLS) sample regression equation corresponding to regression equation (1) is

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i \qquad (i = 1, ..., N) \qquad \textbf{(2)}$$

where $\hat{\beta}_1$ is the OLS estimator of the intercept coefficient β_1, $\hat{\beta}_2$ is the OLS estimator of the slope coefficient β_2, $\hat{u}_i$ is the OLS residual for the i-th sample observation, and N is sample size (the number of observations in the sample).

(20 marks)

(a) Give a general definition of a t-statistic. Starting from this definition, derive the t-statistic for $\hat{\beta}_2$ in OLS sample regression equation (2). State the assumptions required for the derivation.

(10 marks)

(b) Define the p-value of the t-statistic for $\hat{\beta}_2$ when the null hypothesis H_0: $\beta_2 = 0$ is tested against each of the following three alternative hypotheses:

(1) H_1: $\beta_2 \neq 0$;

(2) H_1: $\beta_2 > 0$;

(3) H_1: $\beta_2 < 0$.

(30 marks)

2. Consider the multiple linear regression model for which the population regression equation can be written in conventional notation as:

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \qquad \textbf{(1)}$$

where Y_i, X_{2i} and X_{3i} are observable variables; β_1, β_2 and β_3 are unknown (constant) regression coefficients; and u_i is an unobservable random error term. The Ordinary Least Squares (OLS) sample regression equation corresponding to regression equation (1) is

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i} + \hat{u}_i \qquad (i = 1, ..., N) \qquad \textbf{(2)}$$

where $\hat{\beta}_1$ is the OLS estimator of the intercept coefficient β_1, $\hat{\beta}_2$ is the OLS estimator of the slope coefficient β_2, $\hat{\beta}_3$ is the OLS estimator of the slope coefficient β_3, $\hat{u}_i$ is the OLS residual for the i-th sample observation, and N is sample size (the number of observations in the estimation sample).

(20 marks)

(a) State the Ordinary Least Squares (OLS) estimation criterion. Derive the OLS normal equations for regression equation (1) from the OLS estimation criterion.

(10 marks)

(b) You are asked to change the specification of regression equation (1) to allow the marginal effect of X_{2i} on Y_i to vary with the values of both X_{2i} and X_{3i}. Write the re-specified regression equation you would estimate to incorporate these changes. Write the expression for the marginal effect of X_{2i} on Y_i and the expression for the marginal effect of X_{3i} on Y_i implied by your re-specified model.

PART B (180 marks)

Instructions: Answer ***all*** **parts of questions 3, 4 and 5** in this part. Question 3 is worth a total of 80 marks. Question 4 is worth a total of 50 marks. Question 5 is worth a total of 40 marks. Marks for each part are given in parentheses. **Show explicitly all formulas and calculations**.

(80 marks)

3. You are conducting an empirical investigation into the selling prices of houses sold in an urban housing market in the years 1978 and 1981. The sample contains data on 321 houses, 179 of which were sold in 1978 and 142 of which were sold in 1981. The sample data consist of observations on the following observable variables for each of these 321 houses:

$price_i$ = the selling price of the i-th house, in *thousands* of dollars;

$size_i$ = the house size of the i-th house, in *hundreds* of square feet;

lot_i = the lot size of the i-th house, in *hundreds* of square feet;

$rooms_i$ = the number of rooms in the i-th house;

$baths_i$ = the number of bathrooms in the i-th house;

age_i = the age of the i-th house, in years;

$d81_i$ = an indicator variable defined such that $d81_i = 1$ if the i-th house was sold in 1981, and $d81_i = 0$ if the i-th house was sold in 1978;

$d81_i baths_i$ = $d81_i$ multiplied by $baths_i$.

Your research assistant estimates the following model of house prices on the sample of N = 321 observations. The OLS estimation results for the model are given below.

Regression Model

$$price_i = \beta_1 + \beta_2 size_i + \beta_3 lot_i + \beta_4 rooms_i + \beta_5 baths_i + \beta_6 age_i + \beta_7 d81_i + \beta_8 d81_i baths_i + u_i \qquad \ldots (1)$$

where the β_j (j = 1, 2, …, 8) are regression coefficients, and u_i is a random error term.

OLS Estimates of Equation (1) , Question 3: (*standard errors* in parentheses)

$\hat{\beta}_1 = -12.35$ (10.41) $\hat{\beta}_2 = 1.975$ (0.2734) $\hat{\beta}_3 = 0.01156$ (0.003429) $\hat{\beta}_4 = 4.578$ (1.894)

$\hat{\beta}_5 = 8.032$ (3.180) $\hat{\beta}_6 = -0.1954$ (0.04614) $\hat{\beta}_7 = 5.822$ (8.398) $\hat{\beta}_8 = 13.54$ (3.440)

3. (continued)

Summary Statistics from OLS Estimation of Equation (1) , Question 3:

$$\text{RSS} = \sum_{i=1}^{N} \hat{u}_i^2 = 169{,}628.42; \qquad \text{TSS} = \sum_{i=1}^{N} \left(\text{price}_i - \overline{\text{price}}\right)^2 = 597{,}853.03; \qquad N = 321$$

RSS is the Residual Sum-of-Squares and TSS is the Total Sum-of-Squares from OLS estimation of regression equation (1).

(10 marks)

(a) Interpret each of the slope coefficient estimates $\hat{\beta}_2$ for size_i and $\hat{\beta}_6$ for age_i in regression equation (1); that is, explain in words what the numerical values of the slope coefficient estimates $\hat{\beta}_2$ and $\hat{\beta}_6$ mean.

(10 marks)

(b) Use the estimation results for regression equation (1) to test the *individual* significance of each of the slope coefficient estimates $\hat{\beta}_2$ for size_i and $\hat{\beta}_6$ for age_i. For each test, state the null and alternative hypotheses, show how you calculate the required test statistic, and state its null distribution. Which of these two slope coefficient estimates are individually significant at the 5 percent significance level? Which of these two slope coefficient estimates are individually significant at the 1 percent significance level?

(10 marks)

(c) Use the estimation results for regression equation (1) to test the *joint* significance of all the slope coefficient estimates at the 1 percent significance level (i.e., for significance level α = 0.01). State the null and alternative hypotheses, show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test.

(10 marks)

(d) Use the estimation results for regression equation (1) to construct a two-sided 95 percent confidence interval for the slope coefficient β_4 of rooms_i. Explain how you would use the two-sided 95 percent confidence interval you have computed for β_4 to perform a two-tail test of the hypothesis that $\beta_4 = 0$ at the 5 percent significance level (i.e., for significance level α = 0.05).

3. (continued)

(10 marks)

(e) Use the estimation results for regression equation (1) to test the proposition that lot_i (lot size) is positively related to price_i. Perform the test at the 1 percent significance level. State the null and alternative hypotheses. Show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test.

(10 marks)

(f) Use the estimation results for regression equation (1) to test the proposition that $\beta_5 = 5$. Explain in words what this proposition means. Perform the test at the 5 percent significance level. State the null and alternative hypotheses. Show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test. Would your inference be the same at the 10 percent significance level?

(10 marks)

(g) Use the estimation results for regression equation (1) to test the proposition that the marginal effect on price_i of baths_i, the number of bathrooms in a house, is the same for houses sold in 1981 and houses sold in 1978. Perform the test at the 1 percent significance level. State the null hypothesis H_0 and the alternative hypothesis H_1. Write the *restricted* regression equation implied by the null hypothesis H_0. Calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test.

(10 marks)

(h) Use the estimation results for regression equation (1) to test the proposition that the average price of houses sold in 1981 equals the average price of similar houses sold in 1978. Perform the test at the 1 percent significance level. State the null hypothesis H_0 and the alternative hypothesis H_1. Write the *restricted* regression equation implied by the null hypothesis H_0. OLS estimation of this *restricted* regression equation yields a Residual Sum-of-Squares value of **RSS = 278,410.15**. Use this information, together with the results from OLS estimation of equation (1), to calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test.

(50 marks)

4. You are conducting an econometric investigation into the wage rates of employees. The sample data consist of observations for 935 employees on the following variables:

W_i = the monthly earnings of the i-th employee, in dollars per month;

ED_i = years of formal education completed by the i-th employee, in years;

AGE_i = age of the i-th employee, in years;

TEN_i = firm tenure of the i-th employee, in years;

$BLACK_i$ = an indicator variable defined such that $BLACK_i = 1$ if the i-th employee is black, and $BLACK_i = 0$ if the i-th employee is non-black.

The regression model you propose to use is the log-lin (semi-log) regression equation

$$\ln W_i = \beta_1 + \beta_2 ED_i + \beta_3 AGE_i + \beta_4 TEN_i + \beta_5 ED_i^2 + \beta_6 TEN_i^2 + \beta_7 ED_i AGE_i + \beta_8 BLACK_i + u_i \qquad \textbf{... (1)}$$

where the β_j (j = 1, 2, …, 8) are regression coefficients, $\ln W_i$ denotes the natural logarithm of the variable W_i, and u_i is a random error term.

Using the sample data described above, your research assistant computes OLS estimates of regression equation (1) and of three restricted versions of equation (1). For each of the sample regression equations estimated on the N = 935 observations, the following table contains the OLS coefficient estimates (with estimated *standard errors* in parentheses below the coefficient estimates) and the summary statistics RSS (residual sum-of-squares), TSS (total sum-of-squares), and N (number of sample observations).

4. (continued)

<u>Question 4</u>: OLS Sample Regression Equations for $\ln W_i$
(*standard errors* in parentheses)

Regressors		(1)	(2)	(3)	(4)
Intercept	$\hat{\beta}_1$	5.632 (1.006)	6.035 (0.1511)	5.360 (0.1625)	5.370 (0.5249)
ED_i	$\hat{\beta}_2$	0.1067 (0.09666)	----	0.05379 (0.005872)	0.1376 (0.07556)
AGE_i	$\hat{\beta}_3$	–0.02311 (0.02606)	0.01905 (0.004479)	0.01790 (0.004294)	----
TEN_i	$\hat{\beta}_4$	0.02951 (0.008572)	0.03864 (0.008865)	0.02818 (0.008571)	0.02379 (0.008575)
ED_i^2	$\hat{\beta}_5$	–0.005732 (0.002709)	----	----	–0.002952 (0.002676)
TEN_i^2	$\hat{\beta}_6$	–0.001035 (0.0005018)	–0.001668 (0.0005172)	–0.0009579 (0.0005016)	–0.0005074 (0.0004934)
$ED_i AGE_i$	$\hat{\beta}_7$	0.003220 (0.001935)	----	----	----
$BLACK_i$	$\hat{\beta}_8$	–0.2052 (0.03814)	–0.2744 (0.03917)	–0.2091 (0.03820)	–0.2085 (.03857)
	RSS =	135.018	148.289	136.003	138.365
	TSS =	165.656	165.656	165.656	165.656
	N =	935	935	935	935

<u>Note</u>: The symbol "----" means that the corresponding regressor was omitted from the estimated sample regression equation. The figures in parentheses below the coefficient estimates are the estimated *standard errors*. RSS is the Residual Sum-of-Squares, TSS is the Total Sum-of-Squares, and N is the number of sample observations.

(10 marks)

(a) Compare the goodness-of-fit to the sample data of the four sample regression equations (1), (2), (3) and (4) in the table. Calculate the value of an appropriate goodness-of-fit measure for each of the sample regression equations (1), (2), (3) and (4) in the table. Which of the four sample regression equations provides the best fit to the sample data? Which of the four sample regression equations provides the worst fit to the sample data?

4. (continued)

(10 marks)

(b) Use the estimation results for regression equation (1) in the above table to perform a test of the proposition that black employees of any given education, age and firm tenure have lower average log-wages than non-black employees of the same education, age and firm tenure. Perform the test at the 5 percent significance level (i.e., for significance level α = 0.05). State the null hypothesis H_0 and the alternative hypothesis H_1. Show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test.

(10 marks)

(c) State the coefficient restrictions that regression equation (2) in the table imposes on regression equation (1). Explain in words what the restrictions mean. Use the estimation results given in the table to perform a test of these coefficient restrictions at the 5 percent significance level (i.e., for significance level α = 0.05). State the null and alternative hypotheses, show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test. Would your inference be the same at the 1 percent significance level (i.e., for significance level α = 0.01)? Based on the outcome of the test, which of the two regression equations would you choose, equation (1) or equation (2)?

(10 marks)

(d) State the coefficient restrictions that regression equation (3) in the table imposes on regression equation (1). Explain in words what the restrictions mean. Use the estimation results given in the table to perform a test of these coefficient restrictions at the 5 percent significance level (i.e., for significance level α = 0.05). State the null and alternative hypotheses, show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test. Would your inference be the same at the 1 percent significance level (i.e., for significance level α = 0.01)? Based on the outcome of the test, which of the two regression equations would you choose, equation (1) or equation (3)?

(10 marks)

(e) State the coefficient restrictions that regression equation (4) in the table imposes on regression equation (1). Explain in words what the restrictions mean. Use the estimation results given in the table to perform a test of these coefficient restrictions at the 5 percent significance level (i.e., for significance level α = 0.05). State the null and alternative hypotheses, show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test. Would your inference be the same at the 1 percent significance level (i.e., for significance level α = 0.01)? Based on the outcome of the test, which of the two regression equations would you choose, equation (1) or equation (4)?

(40 marks)

5. You are conducting an econometric investigation into the college entrance exam scores of male and female high school graduates. Your particular interest is in comparing the determinants of college entrance exam scores for female and male students. The sample data consist of a random sample of observations on 3,829 high school students, 1,717 of whom are females and 2,112 of whom are males. The sample data provide observations on the following variables:

$SCORE_i$ = the college entrance exam score of student i, measured in points;
$\ln SCORE_i$ = the natural logarithm of $SCORE_i$;
$HSIZE_i$ = the size of student i's high school graduating class, in hundreds;
$(HSIZE_i)^2$ = $HSIZE_i$-squared;
F_i = a female indicator variable, defined such that $F_i = 1$ if student i is female and $F_i = 0$ if student i is male.

Your astute client proposes that the following pooled regression equation be estimated on the full sample of 3,829 observations for male and female students:

$$\ln SCORE_i = \beta_1 + \beta_2 F_i + \beta_3 HSIZE_i + \beta_4 (HSIZE_i)^2 + \beta_5 F_i\, HSIZE_i + \beta_6 F_i (HSIZE_i)^2 + u_i \qquad \textbf{(1)}$$

where the β_j (j = 1, 2, …, 6) are regression coefficients, $\ln SCORE_i$ denotes the natural logarithm of the variable $SCORE_i$, and u_i is a random error term.

Using the sample data described above, your research assistant computes OLS estimates of regression equation (1) and OLS estimates of several restricted versions of equation (1) on the full sample of N = 3,829 observations for male and female students. The following are the OLS coefficient estimates of regression equation (1) with estimated *standard errors* in parentheses below the coefficient estimates. RSS is the residual sum-of-squares, TSS is the total sum-of-squares, and N is the number of sample observations.

OLS Estimates of Equation (1), Question 5: (standard errors in parentheses)

$\hat{\beta}_1 = 6.918$ (0.008039)　　$\hat{\beta}_4 = -0.002360$ (0.0007019)

$\hat{\beta}_2 = -0.02314$ (0.01202)　　$\hat{\beta}_5 = -0.009803$ (0.007691)

$\hat{\beta}_3 = 0.02287$ (0.005130)　　$\hat{\beta}_6 = 0.0007840$ (0.001052)

$$RSS = \sum_{i=1}^{N} \hat{u}_i^2 = 63.146; \qquad TSS = \sum_{i=1}^{N} \left(\ln SCORE_i - \overline{\ln SCORE}\right)^2 = 65.379; \qquad N = 3{,}829$$

5. (continued)

(10 marks)

(a) State the OLS estimates of the slope coefficients of the regressors HSIZE_i and $(\text{HSIZE}_i)^2$ for *male* students implied by the estimation results for regression equation (1). State the OLS estimates of the slope coefficients of the regressors HSIZE_i and $(\text{HSIZE}_i)^2$ for *female* students implied by the estimation results for regression equation (1).

(15 marks)

(b) Write the expression (or formula) for the marginal effect of HSIZE_i on $\ln\text{SCORE}_i$ for *male* students implied by regression equation (1). Write the expression (or formula) for the marginal effect of HSIZE_i on $\ln\text{SCORE}_i$ for *female* students implied by regression equation (1). Compute a test of the null hypothesis that the marginal effect of HSIZE_i on $\ln\text{SCORE}_i$ for *male* students is equal to the marginal effect of HSIZE_i on $\ln\text{SCORE}_i$ for *female* students for all values of HSIZE_i. State the null hypothesis H_0 and the alternative hypothesis H_1. Write the *restricted* regression equation implied by the null hypothesis H_0. OLS estimation of this *restricted* regression equation yields a Residual Sum-of-Squares value of **RSS = 63.210**. Use this information, together with the results from OLS estimation of equation (1), to calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test. Choose an appropriate significance level for the test.

(15 marks)

(c) Compute a test of the proposition that the mean log-score of *female* students with any given value of HSIZE_i equals the mean log-score of *male* students with the same value of HSIZE_i. State the coefficient restrictions on regression equation (1) implied by this proposition; that is, state the null hypothesis H_0 and the alternative hypothesis H_1. Write the *restricted* regression equation implied by the null hypothesis H_0. OLS estimation of this *restricted* regression equation yields a Residual Sum-of-Squares value of **RSS = 64.895**. Use this information, together with the results from OLS estimation of equation (1), to calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test. Choose an appropriate significance level for the test.

Selected Formulas

For the Simple (Two-Variable) Linear Regression Model

$$Y_i = \beta_1 + \beta_2 X_i + u_i \qquad (i = 1, \ldots, N)$$

θ Deviations from sample means are defined as:

$$y_i \equiv Y_i - \overline{Y}; \qquad x_i \equiv X_i - \overline{X};$$

where

$\overline{Y} = \Sigma_i Y_i / N = \dfrac{\Sigma_i Y_i}{N}$ is the sample mean of the Y_i values;

$\overline{X} = \Sigma_i X_i / N = \dfrac{\Sigma_i X_i}{N}$ is the sample mean of the X_i values.

θ Formulas for the variance of the OLS intercept coefficient estimator $\hat{\beta}_1$ and the covariance of the OLS coefficient estimators $\hat{\beta}_1$ and $\hat{\beta}_2$ in the two-variable linear regression model:

$$\mathrm{Var}\left(\hat{\beta}_1\right) = \frac{\sigma^2 \Sigma_i X_i^2}{N\, \Sigma_i \left(X_i - \overline{X}\right)^2} = \frac{\sigma^2 \Sigma_i X_i^2}{N\, \Sigma_i x_i^2};$$

$$\mathrm{Cov}\left(\hat{\beta}_1, \hat{\beta}_2\right) = -\overline{X}\left(\frac{\sigma^2}{\Sigma_i x_i^2}\right).$$

θ Formulas for the variance of the conditional predictor $\hat{Y}_0 = \hat{\beta}_1 + \hat{\beta}_2 X_0$:

- When $\hat{Y}_0$ is used as a mean predictor of $E\left(Y_0 \mid X_0\right) = \beta_1 + \beta_2 X_0$,

$$\mathrm{Var}\left(\hat{Y}_0^m\right) = \sigma^2\left[\frac{1}{N} + \frac{\left(X_0 - \overline{X}\right)^2}{\Sigma_i x_i^2}\right].$$

- When $\hat{Y}_0$ is used as an individual predictor of $Y_0 \mid X_0 = \beta_1 + \beta_2 X_0 + u_0$,

$$\mathrm{Var}\left(\hat{Y}_0\right) = \sigma^2 + \sigma^2\left[\frac{1}{N} + \frac{\left(X_0 - \overline{X}\right)^2}{\Sigma_i x_i^2}\right].$$

Selected Formulas (continued)

<u>For the Multiple (Three-Variable) Linear Regression Model</u>

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \qquad (i = 1, \ldots, N)$$

θ Deviations from sample means are defined as:

$$y_i \equiv Y_i - \overline{Y}; \qquad x_{2i} \equiv X_{2i} - \overline{X}_2; \qquad x_{3i} \equiv X_{3i} - \overline{X}_3;$$

where

$\overline{Y} = \Sigma_i Y_i / N = \dfrac{\Sigma_i Y_i}{N}$ is the sample mean of the Y_i values;

$\overline{X}_{2i} = \Sigma_i X_{2i} / N = \dfrac{\Sigma_i X_{2i}}{N}$ is the sample mean of the X_{2i} values;

$\overline{X}_{3i} = \Sigma_i X_{3i} / N = \dfrac{\Sigma_i X_{3i}}{N}$ is the sample mean of the X_{3i} values.

θ The OLS slope coefficient estimators $\hat{\beta}_2$ and $\hat{\beta}_3$ in deviation-from-means form are:

$$\hat{\beta}_2 = \frac{\left(\Sigma_i x_{3i}^2\right)\left(\Sigma_i x_{2i} y_i\right) - \left(\Sigma_i x_{2i} x_{3i}\right)\left(\Sigma_i x_{3i} y_i\right)}{\left(\Sigma_i x_{2i}^2\right)\left(\Sigma_i x_{3i}^2\right) - \left(\Sigma_i x_{2i} x_{3i}\right)^2};$$

$$\hat{\beta}_3 = \frac{\left(\Sigma_i x_{2i}^2\right)\left(\Sigma_i x_{3i} y_i\right) - \left(\Sigma_i x_{2i} x_{3i}\right)\left(\Sigma_i x_{2i} y_i\right)}{\left(\Sigma_i x_{2i}^2\right)\left(\Sigma_i x_{3i}^2\right) - \left(\Sigma_i x_{2i} x_{3i}\right)^2}.$$

θ Formulas for the variances and covariances of the slope coefficient estimators $\hat{\beta}_2$ and $\hat{\beta}_3$:

$$\mathrm{Var}\left(\hat{\beta}_2\right) = \frac{\sigma^2 \Sigma_i x_{3i}^2}{\left(\Sigma_i x_{2i}^2\right)\left(\Sigma_i x_{3i}^2\right) - \left(\Sigma_i x_{2i} x_{3i}\right)^2};$$

$$\mathrm{Var}\left(\hat{\beta}_3\right) = \frac{\sigma^2 \Sigma_i x_{2i}^2}{\left(\Sigma_i x_{2i}^2\right)\left(\Sigma_i x_{3i}^2\right) - \left(\Sigma_i x_{2i} x_{3i}\right)^2};$$

$$\mathrm{Cov}\left(\hat{\beta}_2, \hat{\beta}_3\right) = \frac{\sigma^2 \Sigma_i x_{2i} x_{3i}}{\left(\Sigma_i x_{2i}^2\right)\left(\Sigma_i x_{3i}^2\right) - \left(\Sigma_i x_{2i} x_{3i}\right)^2}.$$

Percentage Points of the t-Distribution

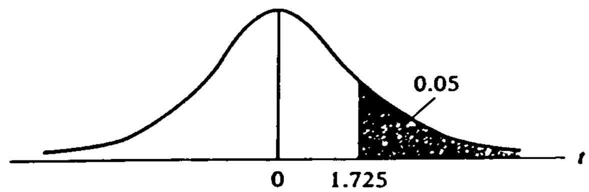
TABLE D.2
Percentage points of the *t* distribution

Example

$\Pr(t > 2.086) = 0.025$

$\Pr(t > 1.725) = 0.05$ for df = 20

$\Pr(|t| > 1.725) = 0.10$



Pr / df	0.25 0.50	0.10 0.20	0.05 0.10	0.025 0.05	0.01 0.02	0.005 0.010	0.001 0.002
1	1.000	3.078	6.314	12.706	31.821	63.657	318.31
2	0.816	1.886	2.920	4.303	6.965	9.925	22.327
3	0.765	1.638	2.353	3.182	4.541	5.841	10.214
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090

Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 12, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

Source: Damodar N. Gujarati, *Basic Econometrics*, Third Edition. New York: McGraw-Hill, 1995, p. 809.

Selected Upper Percentage Points of the F-Distribution

TABLE D.3
Upper percentage points of the *F* distribution (*continued*)

df for denominator N_2	Pr	df for numerator N_1: 1	2	3	4	5	6	7	8	9	10	11	12
22	.25	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
	.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12
24	.25	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37	1.36
	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21	2.18
	.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03
26	.25	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.36	1.35
	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.84	1.81
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96
28	.25	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79
	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12
	.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90
30	.25	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
	.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84
40	.25	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31
	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.73	1.71
	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
	.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66
60	.25	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.29
	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50
120	.25	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26
	.10	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.62	1.60
	.05	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.87	1.83
	.01	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34
200	.25	1.33	1.39	1.38	1.36	1.34	1.32	1.31	1.29	1.28	1.27	1.26	1.25
	.10	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66	1.63	1.60	1.57
	.05	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80
	.01	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.34	2.27
∞	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.24
	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75
	.01	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18

Source: Damodar N. Gujarati, *Basic Econometrics,* Third Edition. New York: McGraw-Hill, 1995, p. 814.