QUEEN'S UNIVERSITY AT KINGSTON
Department of Economics

CONFIDENTIAL
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question paper

ECONOMICS 351* - Winter Term 2005

Introductory Econometrics

Winter Term 2005 **FINAL EXAMINATION** M.G. Abbott

DATE: **Friday April 15, 2005**

TIME: **Three (3) hours (180 minutes); 2:00 p.m. - 5 p.m.**

INSTRUCTIONS: The examination is divided into two parts.

PART A contains two questions; students are required to **answer ONE** of the two questions 1 and 2 in Part A.

PART B contains two questions; students are required to **answer BOTH** questions 3 and 4 in Part B.

- Answer all questions in the exam booklets provided. Be sure **your *student number*** is printed clearly and legibly on the front page of all exam booklets used.
- **Do not write answers to questions on the front page of the first exam booklet**.
- **Please label clearly** each of your answers in the exam booklets with the appropriate number and letter.
- **Please write legibly.** GOOD LUCK!

The candidate is urged to submit with the answer paper a clear statement of any assumptions made if doubt exists as to the interpretation of any question that requires a written answer.

MARKING: The marks for each question are indicated in parentheses immediately above each question. **Total marks** for the exam **equal 200**.

PART A: Questions 1 and 2 (30 marks for each question) .... **30 marks**
Answer ***either one of*** **Questions 1 and 2**.

PART B: Questions 3 (100 marks) and 4 (70 marks) .... **170 marks**
Answer ***all parts of*** **Questions 3 *and* 4**.

TOTAL MARKS .... **200 marks**

PART A (30 marks)

Instructions: Answer **EITHER ONE (1) of questions 1 and 2** in this part. Total marks for each question equal 30; marks for each part are given in parentheses.

(30 marks)

1. Consider the simple (two-variable) linear regression model for which the population regression equation can be written in conventional notation as:

$$Y_i = \beta_0 + \beta_1 X_i + u_i \qquad \textbf{(1)}$$

where Y_i and X_i are observable variables, β_0 and β_1 are unknown (constant) regression coefficients, and u_i is an unobservable random error term. The Ordinary Least Squares (OLS) sample regression equation corresponding to regression equation (1) is

$$Y_i = \hat{\beta}_0 + \hat{\beta}_1 X_i + \hat{u}_i \qquad (i = 1, ..., N) \qquad \textbf{(2)}$$

where $\hat{\beta}_0$ is the OLS estimator of the intercept coefficient β_0, $\hat{\beta}_1$ is the OLS estimator of the slope coefficient β_1, $\hat{u}_i$ is the OLS residual for the i-th sample observation, and N is sample size (the number of observations in the sample).

(20 marks)

(a) Give a general definition of the F-distribution. Starting from this definition, derive the F-statistic for the OLS slope coefficient estimator $\hat{\beta}_1$. State all assumptions required for the derivation.

(5 marks)

(b) The significance level of a statistical test of a null hypothesis H_0 against an alternative hypothesis H_1 is defined as:

(1) the probability of rejecting H_0 when H_0 is false;
(2) the probability of not rejecting H_0 when H_0 is false;
(3) the probability of rejecting H_0 when H_0 is true;
(4) the probability of not rejecting H_0 when H_0 is true.

(5 marks)

(c) The power of a statistical test of a null hypothesis H_0 against an alternative hypothesis H_1 is defined as:

(1) the probability of rejecting H_0 when H_0 is false;
(2) the probability of not rejecting H_0 when H_0 is false;
(3) the probability of rejecting H_0 when H_0 is true;
(4) the probability of not rejecting H_0 when H_0 is true.

(30 marks)

2. Consider the multiple linear regression model for which the population regression equation can be written in conventional notation as:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i \qquad \textbf{(1)}$$

where Y_i, X_{1i} and X_{2i} are observable variables; β_0, β_1 and β_2 are unknown (constant) regression coefficients; and u_i is an unobservable random error term. The Ordinary Least Squares (OLS) sample regression equation corresponding to regression equation (1) is

$$Y_i = \hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 X_{2i} + \hat{u}_i \qquad (i = 1, ..., N) \qquad \textbf{(2)}$$

where $\hat{\beta}_0$ is the OLS estimator of the intercept coefficient β_0, $\hat{\beta}_1$ is the OLS estimator of the slope coefficient β_1, $\hat{\beta}_2$ is the OLS estimator of the slope coefficient β_2, $\hat{u}_i$ is the OLS residual for the i-th sample observation, and N is sample size (the number of observations in the estimation sample).

(20 marks)

(a) State the Ordinary Least Squares (OLS) estimation criterion. Derive the OLS normal equations for regression equation (1) from the OLS estimation criterion.

(10 marks)

(b) Write an interpretive formula for $\text{Var}(\hat{\beta}_2)$, the variance of the OLS slope coefficient estimator $\hat{\beta}_2$ in sample regression equation (2). Define all terms in the formula you give. Which of the following factors makes $\text{Var}(\hat{\beta}_2)$ *larger*?

(1) a larger estimation sample;

(2) more linear dependence between the sample values X_{1i} and X_{2i};

(3) less sample variation of the X_{2i} values;

(4) a larger error variance;

(5) more sample variation of the X_{2i} values;

(6) less linear dependence between the sample values X_{1i} and X_{2i};

(7) a smaller estimation sample;

(8) a smaller error variance.

PART B (170 marks)

Instructions: Answer ***all*** **parts of questions 3 and 4** in this part. Question 3 is worth a total of 100 marks. Question 4 is worth a total of 70 marks. Marks for each part are given in parentheses. **Show explicitly all formulas and calculations**.

(100 marks)

3. You are investigating the relationship between the birth weights of newborn babies and four of their determinants: mother's average daily cigarette consumption during pregnancy; the number of prenatal visits made by the mother to a physician or medical facility during pregnancy; the mother's age; and the mother's race. You have sample data for 1656 babies born during a given year on the following variables:

$bwght_i$ = birth weight of the baby born to the i-th mother, measured in *hundreds* of *grams*;

$cigs_i$ = average number of cigarettes per day smoked by the i-th mother during pregnancy, measured in *cigarettes per day*;

$npvis_i$ = number of prenatal visits to a doctor or medical facility made by the i-th mother during pregnancy;

age_i = age of the i-th mother, measured in *years*;

$white_i$ = an indicator variable defined such that $white_i = 1$ if the i-th mother is white, and $white_i = 0$ if the i-th mother is non-white.

Using the given sample data on 1656 newborn babies, your trusty research assistant has estimated regression equation (1) and obtained the following estimation results (with estimated *standard errors* given in parentheses below the coefficient estimates):

$$bwght_i = \beta_0 + \beta_1 cigs_i + \beta_2 npvis_i + \beta_3 npvis_i^2 + \beta_4 age_i + \beta_5 age_i^2 + \beta_6 white_i + \beta_7 white_i age_i + u_i \qquad \ldots \textbf{(1)}$$

$\hat{\beta}_0 = 16.076$ (4.618) $\quad \hat{\beta}_1 = -0.1010$ (0.03338) $\quad \hat{\beta}_2 = 0.2971$ (0.1061) $\quad \hat{\beta}_3 = -0.006045$ (0.003429)

$\hat{\beta}_4 = 0.8083$ (0.2825) $\quad \hat{\beta}_5 = -0.01010$ (0.004534) $\quad \hat{\beta}_6 = 6.8850$ (2.587) $\quad \hat{\beta}_7 = -0.2039$ (0.08734)

$\hat{Cov}(\hat{\beta}_4, \hat{\beta}_5) = -0.0012263$ $\quad \hat{Cov}(\hat{\beta}_4, \hat{\beta}_6) = 0.20304$ $\quad \hat{Cov}(\hat{\beta}_4, \hat{\beta}_7) = -0.0071391$

$\hat{Cov}(\hat{\beta}_5, \hat{\beta}_6) = -0.0001694$ $\quad \hat{Cov}(\hat{\beta}_5, \hat{\beta}_7) = 8.666e\text{-}06$ $\quad \hat{Cov}(\hat{\beta}_6, \hat{\beta}_7) = -0.22269$

RSS = 53099.886 $\quad$ TSS = 54516.777 $\quad$ N = 1656

3. (continued)

RSS is the Residual Sum-of-Squares and TSS is the Total Sum-of-Squares for sample regression equation (1). Sample size N = 1656. $\hat{\text{Cov}}(\hat{\beta}_j, \hat{\beta}_h)$ is the estimated covariance between coefficient estimates $\hat{\beta}_j$ and $\hat{\beta}_h$. Estimated *standard errors* are given in parentheses below the coefficient estimates $\hat{\beta}_j$ (j = 0, 1, …, 7).

(10 marks)

(a) Use the estimation results for regression equation (1) to test the *joint* significance of the slope coefficient estimates $\hat{\beta}_j$ (j = 1, …, 7) in regression equation (1). Perform the test at the 1 percent significance level (i.e., for significance level $\alpha = 0.01$). State the null and alternative hypotheses, show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test at the 1 percent significance level.

(5 marks)

(b) Interpret the slope coefficient estimate $\hat{\beta}_1$ in sample regression equation (1). That is, explain in words what the numerical value of the slope coefficient estimate $\hat{\beta}_1$ means.

(15 marks)

(c) Use the estimation results for regression equation (1) to compute the two-sided 95 percent confidence interval for the slope coefficient β_1. Use the computed confidence interval for β_1 to perform a test of the proposition that mothers' cigarette consumption during pregnancy ($cigs_i$) has no effect on the birth weight of their babies. Perform the test at the 5 percent significance level (i.e., for significance level $\alpha = 0.05$). State the null hypothesis H_0 and the alternative hypothesis H_1. State the decision rule you use, and the inference you would draw from the test at the 5 percent significance level.

(10 marks)

(d) Compute a test of the proposition that the number of prenatal visits by the mother to a doctor or medical facility has no effect on the birth weight of newborn babies. Perform the test at the 5 percent significance level (i.e., for significance level $\alpha = 0.05$). State the coefficient restrictions on regression equation (1) implied by this proposition; that is, state the null hypothesis H_0 and the alternative hypothesis H_1. Write the *restricted* regression equation implied by the null hypothesis H_0. OLS estimation of this *restricted* regression equation yields a Residual Sum-of-Squares value of **RSS = 53542.009**. Use this information, together with the results from OLS estimation of equation (1), to calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test at the 5 percent significance level.

3. (continued)

(10 marks)

(e) Use the estimation results for regression equation (1) to test the proposition that the marginal effect of mother's age (age_i) on the birth weight of newborn babies is negatively related to age_i. Perform the test at the 5 percent significance level (i.e., for significance level $\alpha = 0.05$). State the null hypothesis H_0 and the alternative hypothesis H_1 implied by this proposition. Show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test at the 5 percent significance level.

(10 marks)

(f) Compute a test of the proposition that the marginal effect of mother's age (age_i) on birth weight is zero for babies born to *non-white* mothers. State the coefficient restrictions on regression equation (1) implied by this proposition; that is, state the null hypothesis H_0 and the alternative hypothesis H_1. Write the *restricted* regression equation implied by the null hypothesis H_0. OLS estimation of this *restricted* regression equation yields an R-squared value of $\mathbf{R^2 = 0.0193}$. Use this information, together with the results from OLS estimation of equation (1), to calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test at the 1 percent significance level (i.e., for significance level $\alpha = 0.01$).

(10 marks)

(g) Compute a test of the proposition that the marginal effect of mother's age (age_i) on birth weight is zero for babies born to *white* mothers. State the coefficient restrictions on regression equation (1) implied by this proposition; that is, state the null hypothesis H_0 and the alternative hypothesis H_1. Write the *restricted* regression equation implied by the null hypothesis H_0. OLS estimation of this *restricted* regression equation yields a Residual Sum-of-Squares value of **RSS = 53260.992**. Use this information, together with the results from OLS estimation of equation (1), to calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test at the 1 percent significance level (i.e., for significance level $\alpha = 0.01$).

(10 marks)

(h) Use the estimation results for regression equation (1) to test the proposition that the marginal effect of mother's age (age_i) on birth weight is zero for non-white mothers who are 30 years of age. State the null hypothesis H_0 and the alternative hypothesis H_1 implied by this proposition. Show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test at the 1 percent significance level (i.e., for significance level $\alpha = 0.01$).

3. (continued)

(10 marks)

(i) Compute a test of the proposition that the mean birth weight of babies born to white mothers equals the mean birth weight of babies born to non-white mothers. State the coefficient restrictions on regression equation (1) implied by this proposition; that is, state the null hypothesis H_0 and the alternative hypothesis H_1. Write the *restricted* regression equation implied by the null hypothesis H_0. OLS estimation of this *restricted* regression equation yields a Residual Sum-of-Squares value of **RSS = 53420.710**. Use this information, together with the results from OLS estimation of equation (1), to calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test. Choose an appropriate significance level for the test.

(10 marks)

(j) Use the estimation results for regression equation (1) to test the proposition that, for any given values of cigs_i and npvis_i, the mean birth weight of babies born to 25-year-old white mothers is greater than the mean birth weight of babies born to 25-year-old non-white mothers. Perform the test at the 1 percent significance level (i.e., for significance level α = 0.01). State the null hypothesis H_0 and the alternative hypothesis H_1 implied by this proposition. Show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test at the 1 percent significance level (i.e., for significance level $\alpha = 0.01$).

(70 marks)

4. You are conducting an econometric investigation into the selling prices of houses in a single urban area in the years 1993 and 2003. You have available for this purpose a random sample of houses that were sold in the years 1993 and 2003. The sample data consist of observations for these houses on the following variables:

$PRICE_i$ = the selling price of the i-th house, in *thousands of dollars*;

$HSIZE_i$ = the living area of the i-th house, in *hundreds of square feet*;

LOT_i = lot size of the i-th house, in *hundreds of square feet*;

AGE_i = the age of the i-th house at the time it was sold, *in years*;

$VIEW_i$ = an indicator variable defined such that $VIEW_i = 1$ if the i-th house has a view, and $VIEW_i = 0$ if the i-th house does not have a view;

$D03_i$ = an indicator variable defined such that $D03_i = 1$ if the i-th house was sold in 2003, and $D03_i = 0$ if the i-th house was sold in 1993.

The regression model you are asked to estimate is given by the population regression equation

$$PRICE_i = \beta_0 + \beta_1 HSIZE_i + \beta_2 HSIZE_i^2 + \beta_3 LOT_i + \beta_4 AGE_i + \beta_5 HSIZE_i AGE_i + \beta_6 VIEW_i$$
$$+ \beta_7 D03_i + \beta_8 D03_i HSIZE_i + \beta_9 D03_i HSIZE_i^2 + \beta_{10} D03_i LOT_i$$
$$+ \beta_{11} D03_i AGE_i + \beta_{12} D03_i HSIZE_i AGE_i + \beta_{13} D03_i VIEW_i + u_i \qquad \textbf{... (1)}$$

where the β_j (j = 0, 1, 2, …, 13) are regression coefficients and u_i is a random error term.

State the null hypothesis H_0 and alternative hypothesis H_1 of the statistical test that you would perform on regression equation (1) to assess the evidence for each of the following empirical propositions. In addition, for each hypothesis test, state which of the following tests you would use: (1) a two-tail t-test; (2) a left-tail t-test; (3) a right-tail t-test; or (4) an F-test.

(7 marks)
(a) The marginal effect of house size on price was zero in 1993.

(7 marks)
(b) The relationship of house size to price exhibited increasing marginal returns in 2003.

(7 marks)
(c) The marginal effect of house age on price was the same in 2003 as it was in 1993.

4. (continued)

(7 marks)

(d) Holding constant house size, lot size and view status, house prices in 2003 were unrelated to house age.

(7 marks)

(e) House size and house age were substitutable for one another in determining house prices in 2003.

(7 marks)

(f) The marginal effect of house size on price was constant in 2003.

(7 marks)

(g) The marginal effect of lot size on price was zero in both 1993 and 2003.

(7 marks)

(h) The mean price of houses with a view was equal to the mean price of houses without a view in both 1993 and 2003.

(7 marks)

(i) The mean price of houses in 1993 was equal to the mean price of similar houses in 2003.

(7 marks)

(j) For houses with 2500 square feet of living area and lots of 6000 square feet that were 10 years old and had a view, mean price in 2003 was greater than mean price in 1993.

Selected Formulas

For the Simple (Two-Variable) Linear Regression Model

$$Y_i = \beta_0 + \beta_1 X_i + u_i \qquad (i = 1, \ldots, N)$$

- Deviations from sample means are defined as:

$$y_i \equiv Y_i - \overline{Y}; \qquad x_i \equiv X_i - \overline{X};$$

where

$\overline{Y} = \Sigma_i Y_i / N = \dfrac{\Sigma_i Y_i}{N}$ is the sample mean of the Y_i values;

$\overline{X} = \Sigma_i X_i / N = \dfrac{\Sigma_i X_i}{N}$ is the sample mean of the X_i values.

- Formulas for the variance of the OLS intercept coefficient estimator $\hat{\beta}_0$ and the covariance of the OLS coefficient estimators $\hat{\beta}_0$ and $\hat{\beta}_1$ in the two-variable linear regression model:

$$\mathrm{Var}(\hat{\beta}_0) = \frac{\sigma^2 \Sigma_i X_i^2}{N \Sigma_i (X_i - \overline{X})^2} = \frac{\sigma^2 \Sigma_i X_i^2}{N \Sigma_i x_i^2};$$

$$\mathrm{Cov}(\hat{\beta}_0, \hat{\beta}_1) = -\overline{X}\left(\frac{\sigma^2}{\Sigma_i x_i^2}\right).$$

- Formulas for the variance of the conditional predictor $\hat{Y}_0 = \hat{\beta}_0 + \hat{\beta}_1 X_0$:

  - When $\hat{Y}_0$ is used as a mean predictor of $E(Y_0 | X_0) = \beta_0 + \beta_1 X_0$,

$$\mathrm{Var}(\hat{Y}_0^m) = \sigma^2 \left[\frac{1}{N} + \frac{(X_0 - \overline{X})^2}{\Sigma_i x_i^2}\right].$$

  - When $\hat{Y}_0$ is used as an individual predictor of $Y_0 | X_0 = \beta_0 + \beta_1 X_0 + u_i$,

$$\mathrm{Var}(\hat{Y}_0) = \sigma^2 + \sigma^2 \left[\frac{1}{N} + \frac{(X_0 - \overline{X})^2}{\Sigma_i x_i^2}\right].$$

Selected Formulas (continued)

For the Multiple (Three-Variable) Linear Regression Model

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i \qquad (i = 1, \ldots, N)$$

- Deviations from sample means are defined as:

$$y_i \equiv Y_i - \overline{Y}; \qquad x_{1i} \equiv X_{1i} - \overline{X}_1; \qquad x_{2i} \equiv X_{2i} - \overline{X}_2;$$

where

$\overline{Y} = \Sigma_i Y_i / N = \dfrac{\Sigma_i Y_i}{N}$ is the sample mean of the Y_i values;

$\overline{X}_{1i} = \Sigma_i X_{1i} / N = \dfrac{\Sigma_i X_{1i}}{N}$ is the sample mean of the X_{1i} values;

$\overline{X}_{2i} = \Sigma_i X_{2i} / N = \dfrac{\Sigma_i X_{2i}}{N}$ is the sample mean of the X_{2i} values.

- The OLS slope coefficient estimators $\hat{\beta}_1$ and $\hat{\beta}_2$ and in deviation-from-means form are:

$$\hat{\beta}_1 = \frac{\left(\Sigma_i x_{2i}^2\right)\left(\Sigma_i x_{1i} y_i\right) - \left(\Sigma_i x_{1i} x_{2i}\right)\left(\Sigma_i x_{2i} y_i\right)}{\left(\Sigma_i x_{1i}^2\right)\left(\Sigma_i x_{2i}^2\right) - \left(\Sigma_i x_{1i} x_{2i}\right)^2};$$

$$\hat{\beta}_2 = \frac{\left(\Sigma_i x_{1i}^2\right)\left(\Sigma_i x_{2i} y_i\right) - \left(\Sigma_i x_{1i} x_{2i}\right)\left(\Sigma_i x_{1i} y_i\right)}{\left(\Sigma_i x_{1i}^2\right)\left(\Sigma_i x_{2i}^2\right) - \left(\Sigma_i x_{1i} x_{2i}\right)^2}.$$

- Formulas for the variances and covariances of the slope coefficient estimators $\hat{\beta}_1$ and $\hat{\beta}_2$:

$$\operatorname{Var}\left(\hat{\beta}_1\right) = \frac{\sigma^2 \Sigma_i x_{2i}^2}{\left(\Sigma_i x_{1i}^2\right)\left(\Sigma_i x_{2i}^2\right) - \left(\Sigma_i x_{1i} x_{2i}\right)^2};$$

$$\operatorname{Var}\left(\hat{\beta}_2\right) = \frac{\sigma^2 \Sigma_i x_{1i}^2}{\left(\Sigma_i x_{1i}^2\right)\left(\Sigma_i x_{2i}^2\right) - \left(\Sigma_i x_{1i} x_{2i}\right)^2};$$

$$\operatorname{Cov}\left(\hat{\beta}_1, \hat{\beta}_2\right) = \frac{\sigma^2 \Sigma_i x_{1i} x_{2i}}{\left(\Sigma_i x_{1i}^2\right)\left(\Sigma_i x_{2i}^2\right) - \left(\Sigma_i x_{1i} x_{2i}\right)^2}.$$

Percentage Points of the t-Distribution

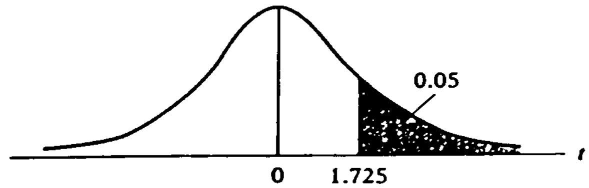
TABLE D.2
Percentage points of the t distribution

Example

$\Pr(t > 2.086) = 0.025$

$\Pr(t > 1.725) = 0.05$ for df = 20

$\Pr(|t| > 1.725) = 0.10$



df \ Pr	0.25 0.50	0.10 0.20	0.05 0.10	0.025 0.05	0.01 0.02	0.005 0.010	0.001 0.002
1	1.000	3.078	6.314	12.706	31.821	63.657	318.31
2	0.816	1.886	2.920	4.303	6.965	9.925	22.327
3	0.765	1.638	2.353	3.182	4.541	5.841	10.214
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090

Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 12, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

Source: Damodar N. Gujarati, *Basic Econometrics*, Third Edition. New York: McGraw-Hill, 1995, p. 809.

Selected Upper Percentage Points of the F-Distribution

TABLE D.3
Upper percentage points of the *F* distribution (*continued*)

df for denominator N_2	df for numerator N_1												
	Pr	1	2	3	4	5	6	7	8	9	10	11	12
	.25	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
22	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
	.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12
	.25	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37	1.36
24	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21	2.18
	.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03
	.25	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.36	1.35
26	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.84	1.81
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96
	.25	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
28	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79
	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12
	.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90
	.25	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
30	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
	.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84
	.25	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31
40	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.73	1.71
	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
	.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66
	.25	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.29
60	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50
	.25	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26
120	.10	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.62	1.60
	.05	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.87	1.83
	.01	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34
	.25	1.33	1.39	1.38	1.36	1.34	1.32	1.31	1.29	1.28	1.27	1.26	1.25
200	.10	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66	1.63	1.60	1.57
	.05	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80
	.01	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.34	2.27
	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.24
∞	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75
	.01	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18

Source: Damodar N. Gujarati, *Basic Econometrics*, Third Edition. New York: McGraw-Hill, 1995, p. 814.