ECON 351* Final Exam – Winter Term 2009

Coverage of exam **Notes 1-9, 11-23 (Winter Term 2009)**

Part I, Sections 1-5, 7; Part II, Sections 8-12; Part III, Sections 13-14.

Format of questions

- **Definitions, Proofs, Derivations, and Explanations**

- **Numerical Answer Questions**
  - computation
  - interpretation of results
  - statistical inference: hypothesis tests and confidence interval estimation

Proofs and Derivations to Know

For the Simple Linear Regression Model $Y_i = \beta_0 + \beta_1 X_i + u_i$

- Derivation of OLS normal equations, the first-order conditions for the OLS coefficient estimators $\hat{\beta}_0$ and $\hat{\beta}_1$ from the RSS function
- Solution of OLS normal equations to obtain formulas for the OLS coefficient estimators $\hat{\beta}_0$ and $\hat{\beta}_1$
- Proof of linearity and unbiasedness of $\hat{\beta}_1$, i.e., proof that $E(\hat{\beta}_1) = \beta_1$
- Derivation of expression (formula) for $Var(\hat{\beta}_1)$
- Derivation of OLS decomposition equation from OLS SRE $Y_i = \hat{Y}_i + \hat{u}_i$
- Derivations of the t-statistic and F-statistic for $\hat{\beta}_1$
- Derivation of two-sided $100(1-\alpha)$ percent confidence interval for β_1 or β_0
- Basic concepts of hypothesis testing

For the Multiple Linear Regression Model $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$

- Derivation of OLS normal equations, the first-order conditions for the OLS coefficient estimators $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\beta}_2$
- Derivation of OLS decomposition equation

Important Things to Know

- Assumptions A1-A8 of the Classical Linear Regression Model
- Definition and meaning of the following statistical properties of estimators:
  (1) unbiasedness; (2) minimum variance; and (3) efficiency.
- Statistical properties of the OLS coefficient estimators $\hat{\beta}_j$
- Computational properties of the OLS sample regression equation
- How to compute, interpret, and use the coefficient of determination R^2 and the adjusted R-squared ($\bar{R}^2$)
- The normality assumption A9 and its implications for the distribution of the Y_i values and for the sampling distributions of the OLS coefficient estimators $\hat{\beta}_j$
- How to compute and interpret two-sided confidence intervals for β_j
- How to perform two-tail and one-tail hypothesis tests for β_j
- How to perform t-tests and F-tests of single linear coefficient restrictions
- How to perform F-tests of two or more linear coefficient restrictions
- How to use interaction terms to allow for nonconstant marginal effects of explanatory variables in regression models
- How to use dummy variables as regressors in regression models, including derivations in Note 21
- Properties of restricted and unrestricted OLS coefficient estimators ($\tilde{\beta}_j$ and $\hat{\beta}_j$)
- How to test for coefficient differences between two regression functions, both with and without dummy variables
- How to formulate and interpret lin-lin, log-log, and log-lin regression models

Important Test Statistics to Know

$$t(\hat{\beta}_j) = \frac{\hat{\beta}_j - \beta_j}{\hat{se}(\hat{\beta}_j)} \sim t[N-K]; \quad F(\hat{\beta}_j) = \frac{\left(\hat{\beta}_j - \beta_j\right)^2}{\hat{Var}(\hat{\beta}_j)} \sim F[1, N-K]; \qquad \left[t(\hat{\beta}_j)\right]^2 = F(\hat{\beta}_j).$$

$$F = \frac{(RSS_0 - RSS_1)/(df_0 - df_1)}{RSS_1/df_1} = \frac{(R_U^2 - R_R^2)/(df_0 - df_1)}{(1 - R_U^2)/df_1} \sim F[df_0 - df_1, df_1]$$

$$t(c_j\hat{\beta}_j + c_h\hat{\beta}_h) = \frac{(c_j\hat{\beta}_j + c_h\hat{\beta}_h) - (c_j\beta_j + c_h\beta_h)}{\hat{se}(c_j\hat{\beta}_j + c_h\hat{\beta}_h)} \sim t[N-K]$$

$$F(c_j\hat{\beta}_j + c_h\hat{\beta}_h) = \frac{\left[(c_j\hat{\beta}_j + c_h\hat{\beta}_h) - (c_j\beta_j + c_h\beta_h)\right]^2}{\hat{Var}(c_j\hat{\beta}_j + c_h\hat{\beta}_h)} \sim F[1, N-K]$$