Another important consequence of the definition of a conditional expectation is the so-called **law of iterated expectations**, which can be stated as follows:

$$E\big(E(y \,|\, \boldsymbol{z})\big) = E(y).$$

The proof of this is an immediate consequence of using the whole of $\mathbb{R}^k$ as the set G in (4.29).

The definitions which follow are rather technical, as are the statements of the laws of large numbers that make use of them. Some readers may therefore wish to skip over them and the discussion of central limit theorems to the definitions of the two sets of regularity conditions, which we call WULLN and CLT, presented at the end of this section. Such readers may return to this point when some reference to it is made later in the book.

Definition 4.10.

The sequence $\{y_t\}$ is said to be **stationary** if for all finite k the joint distribution of the linked set $\{y_t, y_{t+1}, \ldots, y_{t+k}\}$ is independent of the index t.

Definition 4.11.

The stationary sequence $\{y_t\}$ is said to be **ergodic** if, for any two bounded mappings $Y\colon \mathbb{R}^{k+1} \to \mathbb{R}$ and $Z\colon \mathbb{R}^{l+1} \to \mathbb{R}$,

$$\begin{aligned} &\lim_{n\to\infty} \big|E\big(Y(y_i, \ldots, y_{i+k})Z(y_{i+n}, \ldots, y_{i+n+l})\big)\big| \\ &= \big|E\big(Y(y_i, \ldots, y_{i+k})\big)\big|\,\big|E\big(Z(y_i, \ldots, y_{i+l})\big)\big|. \end{aligned}$$

Definition 4.12.

The sequence $\{y_t\}$ is said to be **uniformly mixing**, or **$\boldsymbol{\phi}$–mixing**, if there is a sequence of positive numbers $\{\phi_n\}$, convergent to zero, such that, for any two bounded mappings $Y\colon \mathbb{R}^{k+1} \to \mathbb{R}$ and $Z\colon \mathbb{R}^{l+1} \to \mathbb{R}$,

$$\big|E\big(Y(y_t, \ldots, y_{t+k}) \,|\, Z(y_{t+n}, \ldots, y_{t+n+l})\big) - E\big(Y(y_t, \ldots, y_{t+k})\big)\big| < \phi_n.$$

The symbol $E(\cdot \,|\, \cdot)$ denotes a conditional expectation, as defined above.

Definition 4.13.

The sequence $\{y_t\}$ is said to be **$\boldsymbol{\alpha}$–mixing** if there is a sequence of positive numbers $\{\alpha_n\}$, convergent to zero, such that, if Y and Z are as in the preceding definition, then

$$\big|E\big(Y(y_t, \ldots, y_{t+k})Z(y_{t+n}, \ldots, y_{t+n+l})\big) - E\big(Y(\cdot)\big)E\big(Z(\cdot)\big)\big| < \alpha_n.$$

The last three definitions can be thought of as defining various forms of **asymptotic independence**. According to them, random variables y_t and y_s are more nearly independent (in some sense) the farther apart are the indices t