The difference between z_t and w_t is a rolling estimate of the amount by which the value of y_t for the current quarter tends to differ from its average value over the year. Thus one way to define a seasonally adjusted series would be

$$\begin{aligned} y_t^* &\equiv y_t - z_t + w_t \\ &= .0909y_{t-5} - .2424y_{t-4} + .0909y_{t-3} + .0909y_{t-2} \\ &\quad + .0909y_{t-1} + .7576y_t + .0909y_{t+1} + .0909y_{t+2} \\ &\quad + .0909y_{t+3} - .2424y_{t+4} + .0909y_{t+5}. \end{aligned} \tag{19.37}$$

This example corresponds to a linear filter in which the p^{th} row of $\boldsymbol{\Phi}$ (for $5 < p < n - 5$) would consist first of $p - 6$ zeros, followed by the eleven coefficients that appear in (19.37), followed by $n - p - 5$ more zeros.

This example was deliberately made too simple, but the basic approach that it illustrates may be found, in various modified forms, in almost all official seasonal adjustment procedures. The latter generally do not actually employ linear filters, but do employ a number of moving averages in a way similar to the example. These moving averages tend to be longer than the ones in the example; z_t generally consists of at least 5 terms and w_t consists of at least 25 terms with quarterly data. They also tend to give progressively less weight to observations farther from t. The weight given to y_t by these procedures is generally between 0.75 and 0.9, but it is always well below 1. For more on the relationship between official procedures and ones based on linear filters, see Wallis (1974), Burridge and Wallis (1984), and Ghysels and Perron (1993).

We have asserted that official seasonal adjustment procedures in most cases have much the same properties as linear filters applied to either the levels or the logarithms of the raw data. This assertion can be checked empirically. If it is true, regressing a seasonally adjusted series y_t^* on enough leads and lags of the corresponding seasonally unadjusted series y_t should yield an extremely good fit. The coefficient on y_t should be large and positive, but less than 1, and the coefficients on y_{t+j} should be negative whenever j is an integer multiple of 4 or 12, for quarterly and monthly data, respectively.

As an illustration, we regressed the logarithm of the seasonally adjusted housing start series for Canada that corresponds to the unadjusted series in Figure 19.1 on a constant and the current value and 13 leads and lags of the unadjusted series, for the period 1957:1 to 1986:4. The R^2 was .992 and the coefficient on the current period value was 0.80. We also regressed the logarithm of real personal consumption expenditure, seasonally adjusted at annual rates, on a constant, the current value and 13 leads and lags of the corresponding unadjusted series, for 1953:1 to 1984:4.[5] This time, the R^2

[5] All data were taken from the CANSIM database of Statistics Canada. The adjusted and unadjusted housing start series are numbers D2717 and D4945. The adjusted and unadjusted expenditure series are D20131 and D10131.