If $\boldsymbol{u}$ were asymptotically uncorrelated with $\boldsymbol{X}$, this quantity would just be σ_0^2. Instead, it is *smaller* than σ_0^2. Thus using least squares makes the model fit too well. Because least squares minimizes the distance between $\boldsymbol{y}$ and $\mathcal{S}(\boldsymbol{X})$, part of the variation in $\boldsymbol{y}$ that is really due to variation in the error terms $\boldsymbol{u}$ has incorrectly been attributed to variation in the regressors.

Unfortunately, there are many situations in econometrics in which the error terms cannot be expected to be orthogonal to the $\boldsymbol{X}$ matrix. We will discuss two of them, the cases of errors in variables and simultaneous equations bias, in Sections 7.2 and 7.3. The most general technique for handling such situations is the method of **instrumental variables**, or **IV** for short. This technique, proposed originally by Reiersøl (1941) and further developed by Durbin (1954) and Sargan (1958), among many others, is very powerful and very general. Numerous variants of it appear in many branches of econometrics. These include **two-stage least squares** (Section 7.5), **three-stage least squares** (Chapter 18), and the **generalized method of moments** (Chapter 17).

The plan of the chapter is as follows. In the next section, we discuss the very common problem of errors in variables, for which the method of instrumental variables was originally proposed as a solution. Then, in Section 7.3, we provide an introduction to the linear simultaneous equations model and show that OLS is biased when applied to one equation of such a model. In Section 7.4, we introduce the method of instrumental variables in the context of a linear regression equation and discuss many of its properties. In the following section, we discuss two-stage least squares, which is really just another name for the IV estimator of the parameters of a linear regression model. In Section 7.6, we show how the IV method may be used to estimate nonlinear regression models. In Section 7.7, we generalize the Gauss-Newton regression to the IV case and discuss how to test hypotheses about the coefficients of regression models when they have been estimated by IV. In Section 7.8, we discuss the issue of identification in regression models estimated by IV. Finally, in Section 7.9, we consider a class of tests called Durbin-Wu-Hausman tests, which may be used to decide whether or not it is necessary to employ instrumental variables.

7.2 Errors in Variables

Almost all economic variables are measured with error. This is true to a greater or lesser extent of all macroeconomic time series and is especially true of survey data and many other cross-section data sets. Unfortunately, the statistical consequences of errors in explanatory variables are severe, since explanatory variables that are measured with error are necessarily correlated with the error terms. When this occurs, the problem is said to be one of **errors in variables**. We will illustrate the problem of errors in variables with a simple example.