Economics 813: Assignment 1

January 20, 2010.
Due: January 27, 2010.

Question 1. Suppose that $\lambda, \mu, \lambda', \mu', \nu$ are five distributions on three outcomes $Y = \{y_1, y_2, y_3\}$. Suppose that $\lambda \sim \mu \succ \lambda' \sim \mu' \succ \nu$. Assuming the independence axiom holds, show that the indifference curve through λ and μ is parallel to the indifference curve through λ' and μ'.

Question 2.
i. In the weighted utility model, for $i = 1, 2, 3$, let $u(x_i) = i$ and $w(x_i) = 2i$. Plot some indifference curves and find the point of intersection of the indifference curves.
ii. For the weighted utility model, prove that the indifference curves intersect outside the simplex.

Question 3. For the rank dependent utility model, suppose that there are three outcomes x_1, x_2 and x_3. Let $u(x_i) = i$. Plot indifference curves for the cases where $g(\alpha) = \alpha^2$ and $g(\alpha) = \sqrt{\alpha}$.

Question 4. Given a set of states, S, and a set of consequences, C. The theory of Savage provides a set of axioms, including the "sure thing principle" which lead to a representation of preferences in terms of expected utility. An act is a function f, $f : S \to C$, so the set of acts is C^S. A preference ordering, $\succeq$, on acts is given. Let f, g, f', g' be four acts. The sure thing principle says that if on a set of states $Q \subseteq S$, $f = g$, and $f' = g'$ then if $f = f'$ and $g = g'$ and on Q^c, $f \succeq g$ if and only if $f' \succeq g'$. (Put differently, let f and g be any two acts that agree on some states Q. Then two new acts f' and g' which agree on Q and equal f and g respectively on Q^c must be ranked the same way as f, g: $f \succeq g \Leftrightarrow f' \succeq g'$.)

Suppose that an act, f, has utility $U(f) = \sum_s v(f(s), s)$. (A special case is that where $v(f(s), s) = u(f(s), s)\pi(s)$ and $\pi(s)$ is the probability of state s.) Let $f \succeq g$ if and only if $U(f) \geq U(g)$. Show that $\succeq$ satisfies the sure thing principle.

Question 5. In the example below, develop a Gilboa-Schmeidler model consistent with the behavior described there.

An urn contains 60 balls: 20 red balls, 40 green and blue balls. However, the specific number of blue and green balls is unknown. In one experiment, the individual announces a color, either red or green. A ball is drawn from the urn and if it matches the announced color, the individual receives \$100, and nothing otherwise. Announcing red gives the individual a $\frac{20}{60}$ chance of winning \$100, whereas announcing green gives a $\frac{x}{60}$ chance of winning \$100 — where $\frac{x}{60}$ is the subjective probability of green and $\frac{y}{60}$ is the subjective probability of blue determined by the Savage theory ($x + y = 40$). In this experiment, most people choose red — so it must be that $\frac{20}{60} > \frac{x}{60}$. In a second experiment the individual must select a color pair — either red and blue ($r - b$) or green and blue ($g - b$). A ball is then drawn. If the person chose $r - b$ and either a red or blue ball was drawn, they receive a \$100 and 0 otherwise. If the person choose $g - b$ and the ball drawn is either green or blue the person receives \$100 and 0 otherwise. In this experiment, most people choose $g - b$ which has a probability of $\frac{40}{60}$ whereas the $r - b$ choice has probability $\frac{20+y}{60}$. Thus $\frac{40}{60} > \frac{20+y}{60}$ or $\frac{20}{60} > \frac{y}{60}$. So the first experiment implies $x < 20$ and the second implies that $y < 20$. These are inconsistent with $x + y = 40$. (Note that in both cases, the individual selects the choice which has a known probability. This is taken as indicative of a dislike for the "ambiguity" associated with the unknown probabilities of green and blue.)