Appendix 4: Derivation of the asymptotic standard errors using the Delta Method

Let the kurtosis coefficient of student t is $Kurt = 3\left(\frac{\nu - 2}{\nu - 4}\right)$, we have the variance of kurtosis coefficient by delta approach based on t distribution is $Var(Kurt) = \left(\frac{\partial Kurt}{\partial \nu}\right)^2 Var(\nu)$, where υ is the estimated degree of freedom from (E)GARCH-t model.

The skewness coefficient of EGB2 distribution is given by

$$skew - egb2 = g(p,q) = \frac{\psi''(p) - \psi''(q)}{\left[\psi'(p) + \psi'(q)\right]^{1.5}}$$

The variance of skewness coefficient by delta approach based on EGB2 distribution is

$$var(skew - egb2) = \left[g'_p(p,q)\right]^2 var(p) + \left[g'_q(p,q)\right]^2 var(q) + 2g'_p(p,q)g'_q(p,q)cov(p,q)$$

where

$$g'_p(p,q) = \frac{\psi'''(p)\left[\psi'(p) + \psi'(p)\right]^{1.5} - 1.5\psi''(p)\left[\psi'(p) + \psi'(p)\right]^{0.5}\left[\psi''(p) - \psi''(q)\right]}{\left[\psi'(p) + \psi'(p)\right]^3}$$

$$g'_q(p,q) = \frac{\psi'''(q)\left[\psi'(p) + \psi'(q)\right]^{1.5} + 1.5\psi''(q)\left[\psi'(p) + \psi'(q)\right]^{0.5}\left[\psi''(p) - \psi''(q)\right]}{\left[\psi'(p) + \psi'(q)\right]^3}$$

The kurtosis coefficient of EGB2 distribution is given by

$$kurt\_egb2 = h(p,q) = \frac{\psi'''(p) + \psi'''(q)}{\left[\psi'(p) + \psi'(q)\right]^2} + 3$$

The variance of kurtosis coefficient by delta approach based on EGB2 distribution is

$$\mathrm{var}(kurt - egb2) = \left[h'_p(p,q)\right]^2 \mathrm{var}(p) + \left[h'_q(p,q)\right]^2 \mathrm{var}(q) + 2h'_p(p,q)h'_q(p,q)\mathrm{cov}(p,q)$$

where

$$h'_p(p,q) = \frac{\psi''''(p)\left[\psi'(p) + \psi'(q)\right]^2 - 2\psi''(p)\left[\psi'(p) + \psi'(q)\right]\left[\psi'''(p) + \psi'''(q)\right]}{\left[\psi'(p) + \psi'(q)\right]^4}$$

and

$$h'_q(p,q) = \frac{\psi''''(q)\left[\psi'(p) + \psi'(q)\right]^2 - 2\psi''(q)\left[\psi'(p) + \psi'(q)\right]\left[\psi'''(p) + \psi'''(q)\right]}{\left[\psi'(p) + \psi'(q)\right]^4}$$