Online Appendix to Monetary Policy and Asset Prices: A Markov-Switching DSGE Approach

Not for Publication

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This online appendix provides the details of the DSGE model including its steady state and log-linearized system, data used for the estimation, and additional results not included in the main text.

1. Model

1.1. **Households.** A representative household chooses sequences $\{c_t, \ell_t, d_t\}_{t=0}^{\infty}$ to maximize expected lifetime utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_{b,t} \left[\frac{c_t^{1-\sigma_c}}{1-\sigma_c} - \varphi \frac{\ell_t^2}{2} \right], \tag{1}$$

where β is the subjective discount factor, σ_c is the inverse of intertemporal elasticity of substitution, c_t is consumption of the final good, and ℓ_t is the labor input. The variable $\varepsilon_{b,t}$ is a general preference shock that follows

$$\varepsilon_{b,t} = \overline{\varepsilon}_b \left(\varepsilon_{b,t-1}/\overline{\varepsilon}_b \right)^{\rho_b} \exp\left(\sigma_b \epsilon_{b,t} \right), \quad \epsilon_{b,t} \sim \mathbb{N}(0,1),$$

where $\overline{\varepsilon}_b$ is steady-state preference.

The flow budget constraint in units of consumption goods for the household is given by

$$c_t + d_t \leq w_t \ell_t + \frac{r_{t-1} d_{t-1}}{\pi_t} + \tau_t + \Lambda_t,$$

where $\pi_t \equiv P_t / P_{t-1}$ is the gross inflation rate, w_t is the real wage of labor input, d_t and r_t denote real deposits in a financial intermediary and their gross nominal interest rate, respectively, τ_t is a real lump-sum transfer from the monetary authority, and Λ_t is real dividend payments from retailer firms. As in Christensen and Dib (2008), I assume that the rate of return from nominal deposits, r_t, is identical to that of government bonds. Then the representative household's optimality conditions imply

$$\lambda_t = \varepsilon_{b,t} (c_t)^{-\sigma_c}, \tag{2}$$

$$\lambda_t w_t = \varphi \varepsilon_{b,t} \ell_t, \tag{3}$$

$$\frac{\lambda_t}{r_t} = \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right), \tag{4}$$

where λ_t is the Lagrangian multiplier associated with the budget constraint.

1.2. **Production Sector.**

1.2.1. *Capital Goods Producers.* Capital goods are produced in a perfectly competitive environment by using a linear technology. Capital producers utilize a fraction of final goods purchased from retailers, i_t, as inputs and produce efficient investment goods, $\varepsilon_{i,t} i$, which are governed by an investment-specific shock $\varepsilon_{i,t}$. Then they produce new capital goods, k_{t+1}, by combining the efficient investment goods and the existing capital stock. Consequently, the law of motion of capital stock is given by

$$k_{t+1} = (1-\delta)k_t + \varepsilon_{i,t} i_t, \tag{5}$$

where δ is the depreciation rate of capital. The investment-specific shock process $\varepsilon_{i,t}$ follows

$$\varepsilon_{i,t} = \bar{\varepsilon}_i \left(\varepsilon_{i,t-1}/\bar{\varepsilon}_i\right)^{\rho_i} \exp\left(\sigma_i \epsilon_{i,t}\right), \quad \epsilon_{i,t} \sim \mathbb{N}(0,1),$$

where $\bar{\varepsilon}_i$ is the steady-state level of investment-specific shock process.

I further assume that capital producers pay an adjustment cost in transforming investment goods into new units of productive capital given by $\frac{\chi}{2}\left(\frac{i_t}{k_t} - \delta\right)^2 k_t$. Hence, the optimization problem for producers of capital goods becomes

$$\max_{i_t} \mathbb{E}_t \left[q_t \varepsilon_{i,t} i_t - i_t - \frac{\chi}{2}\left(\frac{i_t}{k_t} - \delta\right)^2 k_t \right], \tag{6}$$

where q_t is the real price of capital. Then their optimality condition is given by

$$\mathbb{E}_t \left[q_t \varepsilon_{i,t} - 1 - \chi\left(\frac{i_t}{k_t} - \delta\right)\right] = 0, \tag{7}$$

which states that the price of capital equals the marginal adjustment costs.

1.2.2. *Entrepreneurs.* Entrepreneurs purchase the capital stock, k_{t+1}, from capital goods producers at a given price, q_t, by using both their net worth (n_{t+1}) and external loans from a financial intermediary ($q_t k_{t+1} - n_{t+1}$).

Each entrepreneur produces final goods, y_t, according to the technology given as

$$y_t = k_t^{\alpha} \left(\varepsilon_{a,t} \ell_t\right)^{1-\alpha}, \tag{8}$$

where α is the share of capital in the production function. The variable $\varepsilon_{a,t}$ is a technology shock that follows

$$\varepsilon_{a,t} = \overline{\varepsilon}_a \left(\varepsilon_{a,t-1}/\overline{\varepsilon}_a\right)^{\rho_a} \exp\left(\sigma_a \epsilon_{a,t}\right), \quad \epsilon_{a,t} \sim \mathbb{N}(0,1),$$

where $\overline{\varepsilon}_a$ is steady-state technology.

Final goods are sold in a perfectly competitive market at a price that equals the entrepreneurs' nominal marginal cost. Entrepreneurs maximize their profits by choosing k_t and ℓ_t subject to the production function (8). Their optimality conditions imply

$$z_t = \alpha \mu_t \frac{y_t}{k_t}, \tag{9}$$

$$w_t = (1-\alpha)\mu_t \frac{y_t}{\ell_t}, \tag{10}$$

where μ_t denotes the Lagrangian multiplier associated with the production function and z_t is the real marginal productivity of capital.

The entrepreneurs' demand for capital is determined by the expected marginal return and the expected marginal external financing cost at $t+1$. Given the production technology in (8), the expected marginal return to capital, $\mathbb{E}_t f_{t+1}$, is given as

$$\mathbb{E}_t f_{t+1} = \mathbb{E}_t \left[\frac{z_{t+1} + (1-\delta) q_{t+1}}{q_t}\right],$$

which implies that the expected return to capital equals the opportunity cost of accumulating a unit of capital ($1/q_t$) multiplied by the sum of the expected rental rate of capital ($\mathbb{E}_t z_{t+1}$) plus the expected value of a unit of capital ($\mathbb{E}_t\left[(1-\delta)q_{t+1}\right]$).

Following the costly state verification framework of Bernanke et al. (1999), the cost of external finance is higher than the economy's nominal risk-free rate between t and $t+1$, r_t. The source of the premium is that financial intermediaries face the cost of monitoring the return on the market value of the entrepreneurs' capital stock, which is a random variable following a probability distribution. Bernanke et al. (1999) assume that the realization of the return is the entrepreneurs' private information, and financial intermediaries must pay an auditing cost if they wish to observe the entrepreneurs' realized returns. Financial intermediaries pass the monitoring cost to entrepreneurs in the form of the premium.[1]

Entrepreneurs maximize their net expected return by choosing k_{t+1} given $\mathbb{E}_t f_{t+1}$, q_t, n_{t+1} and r_t. Having solved the maximization problem, Bernanke et al. (1999) show

[1] See Bernanke et al. (1999) for more detailed derivations of the optimal contract between financial intermediaries and entrepreneurs under the presence of asymmetric information.

that entrepreneurs borrow up to the point at which the expected marginal external financing cost, $\mathbb{E}_t f_{t+1}$, equals an external finance premium over the real risk-free interest rate. Accordingly, the optimality condition is given by

$$\mathbb{E}_t f_{t+1} = \mathbb{E}_t \left[S\left(\frac{n_{t+1}}{q_t k_{t+1}}\right) \frac{r_t}{\pi_{t+1}} \right], \tag{11}$$

with $S'(\cdot) < 0$ and $S(1) = 1$.

The log-linearization of (11) yields the equation for the external funds rate as:

$$\mathbb{E}_t \hat{f}_{t+1} = (\hat{r}_t - \mathbb{E}_t \hat{\pi}_{t+1}) + \psi \mathbb{E}_t \left(\hat{q}_t + \hat{k}_{t+1} - \hat{n}_{t+1} \right), \tag{12}$$

where a hat ($\hat{}$) denotes log deviations from the deterministic steady state and ψ denotes the elasticity of the external finance premium with respect to a change in the leverage position of entrepreneurs. As in Bernanke et al. (1999), equation (12) implies that the external finance premium over the risk-free rate demanded by financial intermediaries falls as entrepreneurs' collateralized net worth increases.

Aggregate entrepreneurial net worth accumulates according to

$$n_{t+1} = \nu v_t + (1 - \nu) g_t, \tag{13}$$

where ν is the entrepreneurial survival rate such that its expected lifetime is $1/(1-\nu)$. v_t denotes the net worth of surviving entrepreneurs carried over from the previous period, and g_t is the transfer that newly established entrepreneurs receive from entrepreneurs who die in the previous period. The law of motion of v_t is given by

$$v_t = f_t q_{t-1} k_t - \mathbb{E}_{t-1} \left[f_t (q_{t-1} k_t - n_t) \right], \tag{14}$$

where f_t is the ex-post real return on capital held in t, and $\mathbb{E}_{t-1} f_t = \mathbb{E}_{t-1} \left[S(\cdot) r_{t-1}/\pi_t \right]$. As emphasized in Christensen and Dib (2008), entrepreneurs' debt contracts are written based upon a nominal interest rate. Thus an unanticipated rise in inflation induces a debt deflation effect that decreases the real cost of debt repayment. This, in turn, drives up the entrepreneurial net worth. An unanticipated fall in inflation has the opposite effects.

A combination of equations (11), (13), and (14), and its log-linearization yield the law of motion for entrepreneurial net worth as

$$\begin{aligned}\frac{1}{\nu f}\hat{n}_t =& \frac{kz}{nf}\hat{z}_t + \frac{k(1-\delta)}{nf}\hat{q}_{t-1} - \left(\frac{k}{n} - 1\right)(\hat{r}_{t-1} - \hat{\pi}_t) - \left[\frac{k}{n} + \psi\left(\frac{k}{n} - 1\right)\right]\hat{q}_{t-2} \\ &- \psi\left(\frac{k}{n} - 1\right)\hat{k}_t + \psi\left(\frac{k}{n} - 1\right)\hat{n}_{t-1} + \hat{\varepsilon}_{n,t},\end{aligned} \tag{15}$$

where $\varepsilon_{n,t}$ is a net worth shock that follows

$$\varepsilon_{n,t} = \overline{\varepsilon}_n \left(\varepsilon_{n,t-1}/\overline{\varepsilon}_n\right)^{\rho_n} \exp\left(\sigma_n \epsilon_{n,t}\right), \quad \epsilon_{n,t} \sim \mathbb{N}(0,1),$$

and $\overline{\varepsilon}_n$ denotes the steady-state level of net worth shock.

1.2.3. *Retailers.* The role of retailers is to introduce nominal rigidity into the economy. Retailers purchase the final goods produced by entrepreneurs and turn them into a continuum of differentiated goods at no cost. The differentiated retail goods then are sold in a monopolistically competitive environment. Following Calvo (1983) and Yun (1996), in each period a fraction of retail firms, ω, cannot update their prices. Thus, firms that are able to reset their price at t choose their optimal price, $p_t^*(j)$, to maximize the expected discounted present value of real profits,

$$\max_{\{p_t^*(j)\}} \mathbb{E}_0 \left[\sum_{s=0}^{\infty} (\beta\omega)^s \frac{\lambda_{t+s}\Omega_{t+s}(j)}{p_{t+l}}\right],$$

subject to the demand function

$$y_{t+s}(j) = \left[\frac{p_t^*(j)}{p_{t+s}}\right]^{-\theta} y_{t+s},$$

where θ is the retail goods elasticity of substitution and $\Omega_t(j)$ is the retailer j's nominal profit function given as

$$\Omega_{t+s}(j) = \left[\pi^s p_t^*(j) - p_{t+s}\zeta_{t+s}\right] y_{t+s}(j),$$

where ζ_t is the real marginal cost.

The optimality condition is given by

$$p_t^*(j) = \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\omega)^s \lambda_{t+s} y_{t+s}(j) \zeta_{t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\omega)^s \lambda_{t+s} y_{t+s}(j) \pi^s / p_{t+s}}, \tag{16}$$

so that the aggregate price is given as

$$p_t^{1-\theta} = \omega(\pi p_{t-1})^{1-\theta} + (1-\omega) p_t^{*1-\theta}, \tag{17}$$

where π is steady-state inflation.

The combination of (16) and (17), and its log-linearization yield the new Keynesian Philips curve:

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \frac{(1-\beta\omega)(1-\omega)}{\omega}\hat{\zeta}_t. \tag{18}$$

1.3. **Monetary Policy.** The monetary authority sets policy according to

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho_r} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{y_t}{y_{t-1}}\right)^{\phi_y} \left(\frac{n_t}{n_{t-1}}\right)^{\phi_n(\xi_t^P)}\right]^{1-\rho_r} \exp\left(\sigma_r \epsilon_{r,t}\right), \tag{19}$$

where ϕ_π, ϕ_y, and ϕ_n measure the policy responses to inflation, output growth, and net worth growth, respectively, r denotes the steady-state nominal interest rate, and $\epsilon_{r,t} \sim \mathbb{N}(0,1)$. ξ_t^P is an unobservable state variable which governs the structural parameter regime at time t. This specification posits that the central bank responds to inflation, output growth, and net worth growth, whereas any discretionary changes in the nominal interest rate are captured by the monetary policy shock, $\epsilon_{r,t}$.

1.4. **Steady State.** I assume that $q = 1$. Then the steady state level of the model variables are given as follows.

$$\begin{aligned}
\zeta =& \frac{\theta-1}{\theta} \\
r =& \pi/\beta \\
f =& sr/\pi \qquad \text{where } s \text{ is steady state gross external finance premium} \\
z =& f + \delta - 1 \\
\frac{k}{y} =& \frac{\alpha\zeta}{z} \\
\frac{c}{y} =& 1 - \delta\frac{k}{y} \\
w\ell\lambda =& \frac{(1-\alpha)\zeta}{c/y} \\
\ell =& (w\ell\lambda)^{1/2} \\
\frac{i}{y} =& 1 - \frac{c}{y}
\end{aligned}$$

1.5. **The Log-linearized System of the DSGE Model.** Conditional on the structural parameter regimes, the log-linearized system of the DSGE model is given as follows.

FOC consumption:

$$\hat{\lambda}_t = \hat{\varepsilon}_{b,t} - \sigma_c \hat{c}_t \tag{20}$$

FOC labor supply:

$$\hat{\lambda}_t = \hat{\ell}_t - \hat{w}_t + \hat{\varepsilon}_{b,t} \tag{21}$$

FOC nominal deposit:

$$\hat{\lambda}_t = \hat{r}_t - \mathbb{E}_t \hat{\pi}_{t+1} + \mathbb{E}_t \hat{\lambda}_{t+1} \tag{22}$$

Production function:

$$\hat{y}_t = (1-\alpha)(\hat{\ell}_t + \hat{\varepsilon}_{a,t}) + \alpha \hat{k}_t \tag{23}$$

Aggregate resource constraint:

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{i}_t \tag{24}$$

Real wage:

$$\hat{w}_t = \hat{y}_t + \hat{\zeta}_t - \hat{\ell}_t \tag{25}$$

Real marginal product of capital:

$$\hat{z}_t = \hat{y}_t + \hat{\zeta}_t - \hat{k}_t \tag{26}$$

Entrepreneurs' capital demand:

$$\hat{f}_t = \frac{z}{f}\hat{z}_t + \frac{1-\delta}{f}\hat{q}_t - \hat{q}_{t-1} \tag{27}$$

Real price of capital:

$$\hat{q}_t = \chi\left(\hat{i}_t - \hat{k}_t\right) - \hat{\varepsilon}_{i,t} \tag{28}$$

Phillips curve:

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \frac{(1-\beta\omega)(1-\omega)}{\omega}\hat{\zeta}_t \tag{29}$$

Capital law of motion:

$$\hat{k}_{t+1} = \delta\left(\hat{i}_t + \hat{\varepsilon}_{i,t}\right) + (1-\delta)\hat{k}_t \tag{30}$$

External funds rate:

$$\mathbb{E}_t\hat{f}_{t+1} = (\hat{r}_t - \mathbb{E}_t\hat{\pi}_{t+1}) + \psi\mathbb{E}_t\left(\hat{q}_t + \hat{k}_{t+1} - \hat{n}_{t+1}\right) \tag{31}$$

Entrepreneurial net worth law of motion:

$$\begin{aligned}\frac{1}{\nu f}\hat{n}_t =& \frac{kz}{nf}\hat{z}_t + \frac{k(1-\delta)}{nf}\hat{q}_{t-1} - \left(\frac{k}{n}-1\right)(\hat{r}_{t-1} - \hat{\pi}_t) - \left[\frac{k}{n} + \psi\left(\frac{k}{n}-1\right)\right]\hat{q}_{t-2} \\ &- \psi\left(\frac{k}{n}-1\right)\hat{k}_t + \psi\left(\frac{k}{n}-1\right)\hat{n}_{t-1} + \hat{\varepsilon}_{n,t}\end{aligned} \tag{32}$$

Technology shock process:

$$\hat{\varepsilon}_{a,t} = \rho_a\hat{\varepsilon}_{a,t-1} + \sigma_a\epsilon_{a,t} \tag{33}$$

Preference shock process:

$$\hat{\varepsilon}_{b,t} = \rho_b\hat{\varepsilon}_{b,t-1} + \sigma_b\epsilon_{b,t} \tag{34}$$

Investment-efficiency shock process:

$$\hat{\varepsilon}_{i,t} = \rho_i\hat{\varepsilon}_{i,t-1} + \sigma_i\epsilon_{i,t} \tag{35}$$

Net worth shock process:

$$\hat{\varepsilon}_{n,t} = \rho_n\hat{\varepsilon}_{n,t-1} + \sigma_n\epsilon_{n,t} \tag{36}$$

Monetary policy rule:

$$\hat{r}_t = \rho_r\hat{r}_{t-1} + (1-\rho_r)\left(\phi_\pi\pi_t + \phi_y\Delta\hat{y}_t\right) + \sigma_r\epsilon_{r,t}, \tag{37}$$

where $\Delta\hat{y}_t = \hat{y}_t - \hat{y}_{t-1}$.

For the model with the extended monetary policy rule, Equation (37) is replaced by

$$\hat{r}_t = \rho_r\hat{r}_{t-1} + (1-\rho_r)\left(\phi_\pi\pi_t + \phi_y\Delta\hat{y}_t + \phi_n\Delta\hat{n}_t\right) + \sigma_r\epsilon_{r,t}, \tag{38}$$

where $\Delta\hat{n}_t = \hat{n}_t - \hat{n}_{t-1}$.

2. Data

The model is estimated using U.S. quarterly data from 1984:Q1 to 2009:Q2. Detailed data descriptions are as follows.

$$\begin{aligned}
\text{Output Growth} &= \log\left[\text{Real GDP Per Cap. / Real GDP Per Cap.(-1)}\right] \times 100,\\
\text{Investment Growth} &= \log\left[\text{Real Invest. Per Cap. / Real Invest. Per Cap.(-1)}\right] \times 100,\\
\text{Inflation Rate} &= \log\left[\text{GDPDEF / GDPDEF(-1)}\right] \times 100,\\
\text{Nominal Interest Rate} &= \text{Federal Funds Rate}/4,\\
\text{Value of Stock Market Growth} &= \log\left[\text{Real Stock Index Per Cap. / Real Stock Index Per Cap.(-1)}\right] \times 100,
\end{aligned}$$

and then all the left-hand side variables are demeaned. Each per capita real variable is obtained as:

$$\begin{aligned}
\text{Real GDP Per Cap.} &= \text{Nominal GDP / (Population} \times \text{GDPDEF)},\\
\text{Real Invest. Per Cap.} &= \text{Nominal Investment / (Population} \times \text{GDPDEF)},\\
\text{Real Stock Index Per Cap.} &= \text{Value of Stock Market / (Population} \times \text{GDPDEF)}.
\end{aligned}$$

The sources of the original data are:

- Nominal GDP: Nominal Gross Domestic Product (U.S. Department of Commerce, Bureau of Economic Analysis, Table 1.1.5, Line 1)
- Nominal Investment: Nominal Gross Private Domestic Investment (U.S. Department of Commerce, Bureau of Economic Analysis, Table 1.1.5, Line 7)
- Population: Civilian Noninstitutional Population, Ages 16 Years and Over—Seasonally Adjusted (U.S. Department of Labor, Bureau of Labor Statistics, Series No. "LNS10000000"). Due to data availability, I use "LNU00000000Q" (seasonally unadjusted) for the population series prior to 1976
- GDPDEF: GDP Deflator—Index Numbers, 2009=100, Seasonally Adjusted (U.S. Department of Commerce, Bureau of Economic Analysis, Table 1.1.4, Line 1)
- Federal Funds Rate: Averages of Daily Figures—Percent (Board of Governors of the Federal Reserve System)
- Value of Stock Market: The Wilshire 5000 Total Market Full Cap Index—Index Numbers, Not Seasonally Adjusted (Federal Reserve Economic Data, Federal Reserve Bank of St. Louis, Series ID "WILL5000INDFC")

3. Estimation Results for the MS-DSGE Models

3.1. **Convergence Diagnostics.** Table 1 summarizes the p-values for Geweke's chi-squared test for two sets of MCMC sample draws of the posterior distributions. The Geweke's chi-squared test statistics are calculated by comparing the mean of the first 30% and the last 50% of the MCMC draws.

Parameter	Geweke Chi-square p-value
σ_c (Risk aversion)	0.90
χ (Capital adjustment cost)	0.69
ω (Degree of price stickiness)	0.75
ρ_a (Technology shock AR(1))	0.81
ρ_b (Preference shock AR(1))	0.74
ρ_i (Investment-efficiency shock AR(1))	0.10
ρ_n (Net worth shock AR(1))	0.00
k/n (Steady-state ratio of capital to net worth)	0.58
ν (Survival rate of entrepreneurs)	0.07
ψ (Elasticity of the external finance premium w.r.t. firm leverage)	0.73
s_{ss} (Gross steady-state risk premium)	0.20
ρ_r (MP rule AR(1))	0.12
ϕ_π (MP response to inflation)	0.93
ϕ_y (MP response to output growth)	0.64
ϕ_n (MP response to net worth growth, policy regime 1)	0.47
(MP response to net worth growth, policy regime 2)	Fixed at zero ($\phi_n = 0$)
σ_a (Technology shock std.)	0.80
σ_b (Preference shock std.)	0.81
σ_i (Investment-efficiency shock std.)	0.48
σ_r (Monetary policy shock std.)	0.54
σ_n (Net worth shock std.)	0.48
P_{11} (Prob. of policy regime 1)	0.57
P_{22} (Prob. of policy regime 2)	0.98

Table 1. P-values for Geweke's chi-squared test for the estimated parameters of the benchmark MS-DSGE model.

3.2. **Prior and Posterior Distributions.** Figure 1 displays the prior and posterior distributions of the estimated parameters for the benchmark specification.

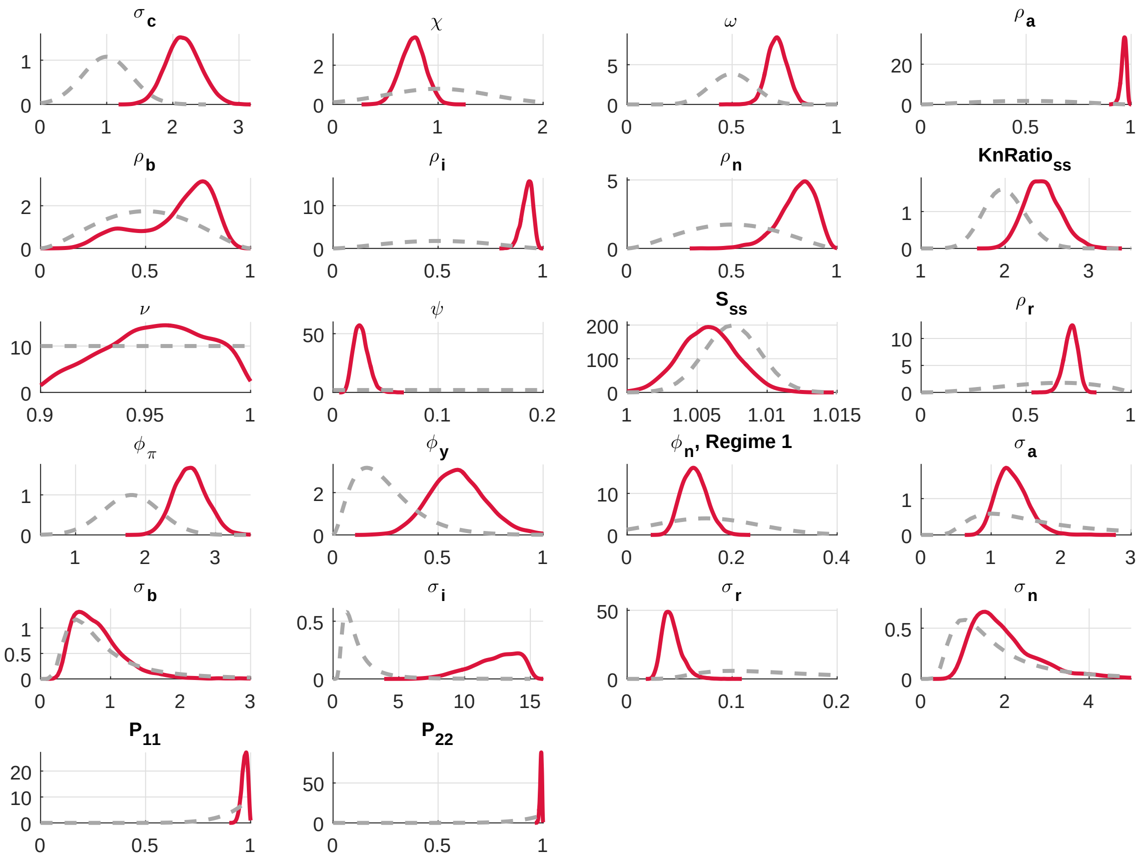

FIGURE 1. Prior (dashed lines) and posterior (solid lines) distributions of the estimated parameters.

3.3. **Regime Probabilities.** Figure 2 plots the smoothed probabilities of the monetary policy regime responding to stock prices (regime 1) for the benchmark specification.

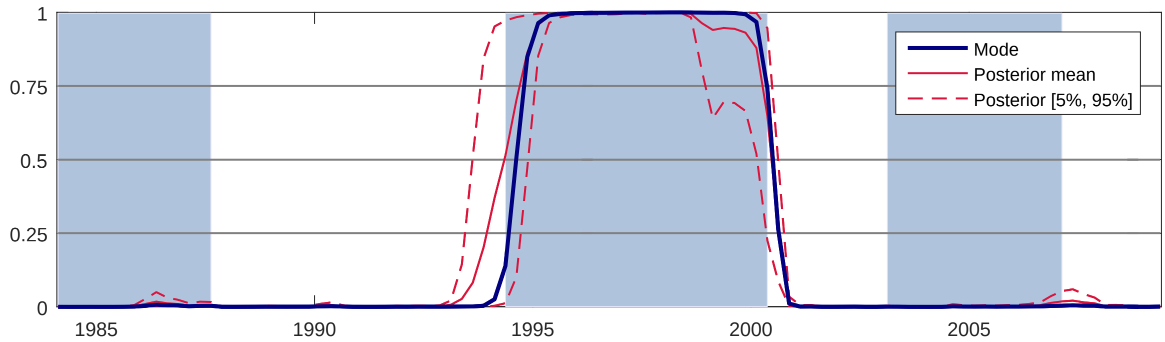

Figure 2. Smoothed probabilities of the monetary policy regime 1 (responsive regime). The probability estimates evaluated at the posterior mode (thick solid line) as well as the posterior mean (thin solid line) and 90% bands (thin dashed line) are reported. The shaded areas indicate the stock market boom dates in Christiano et al. (2010).

4. Impulse Responses

This section presents the details of the impulse responses, which are not reported in the main text. Figures 3 through 7 report the impulse responses to a 1% shock. Notice that the presented impulse responses are calculated by taking into account the possibility of regime changes in the structural parameter and conditioning on an initial regime.[2]

[2]See Bianchi (2016) for details of implementing this method.

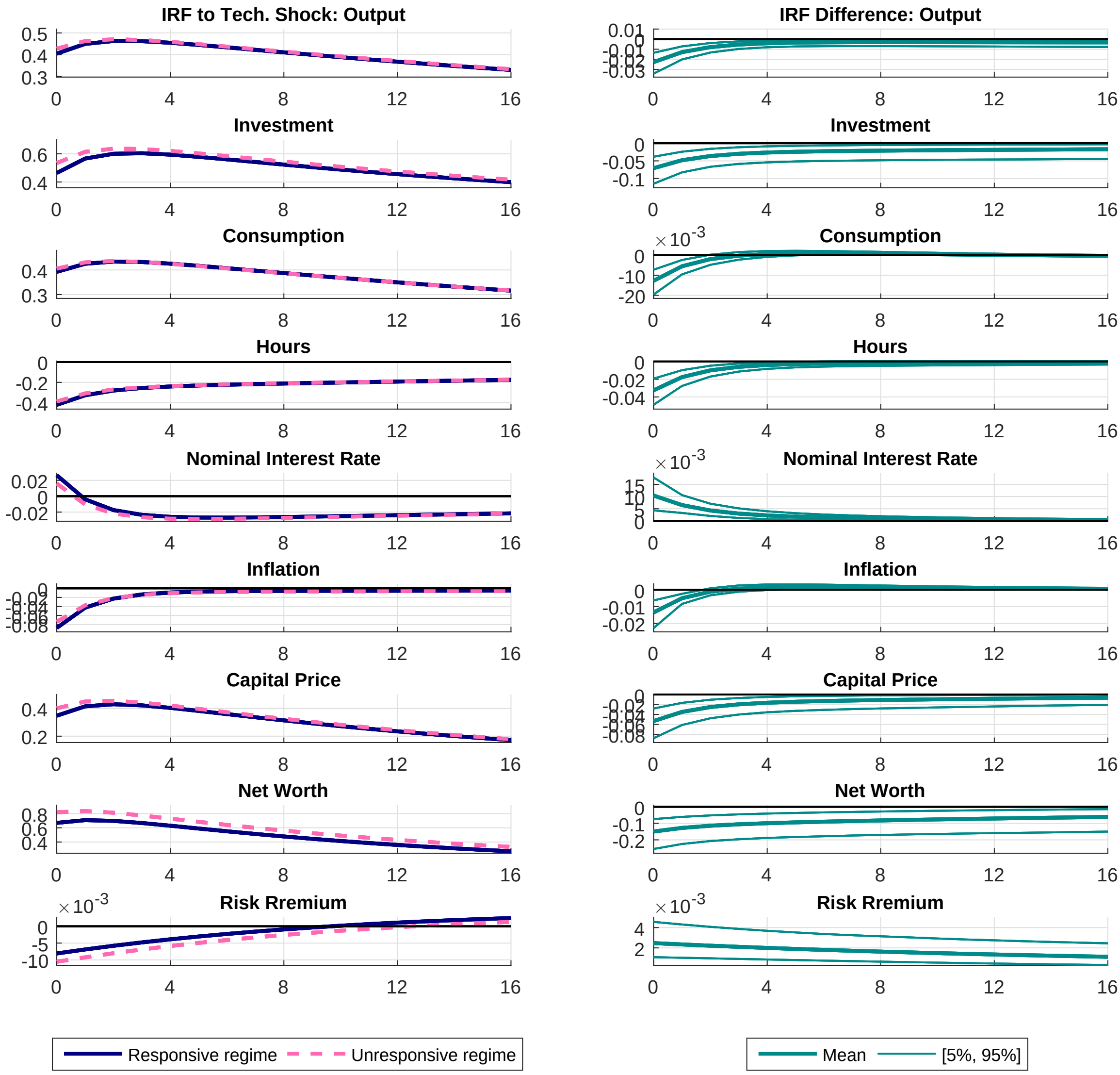

FIGURE 3. **[Left panels]** Mean impulse responses to 1% technology shocks, taking into account the possibility of regime changes. In each figure, impulse responses associated with the responsive regime as the initial regime (solid lines) and with the unresponsive regime as the initial regime (dashed lines) are reported. **[Right panels]** Mean (thick lines) and 90% error bands (thin lines) for the difference between the two responses. The x-axis measures quarters.

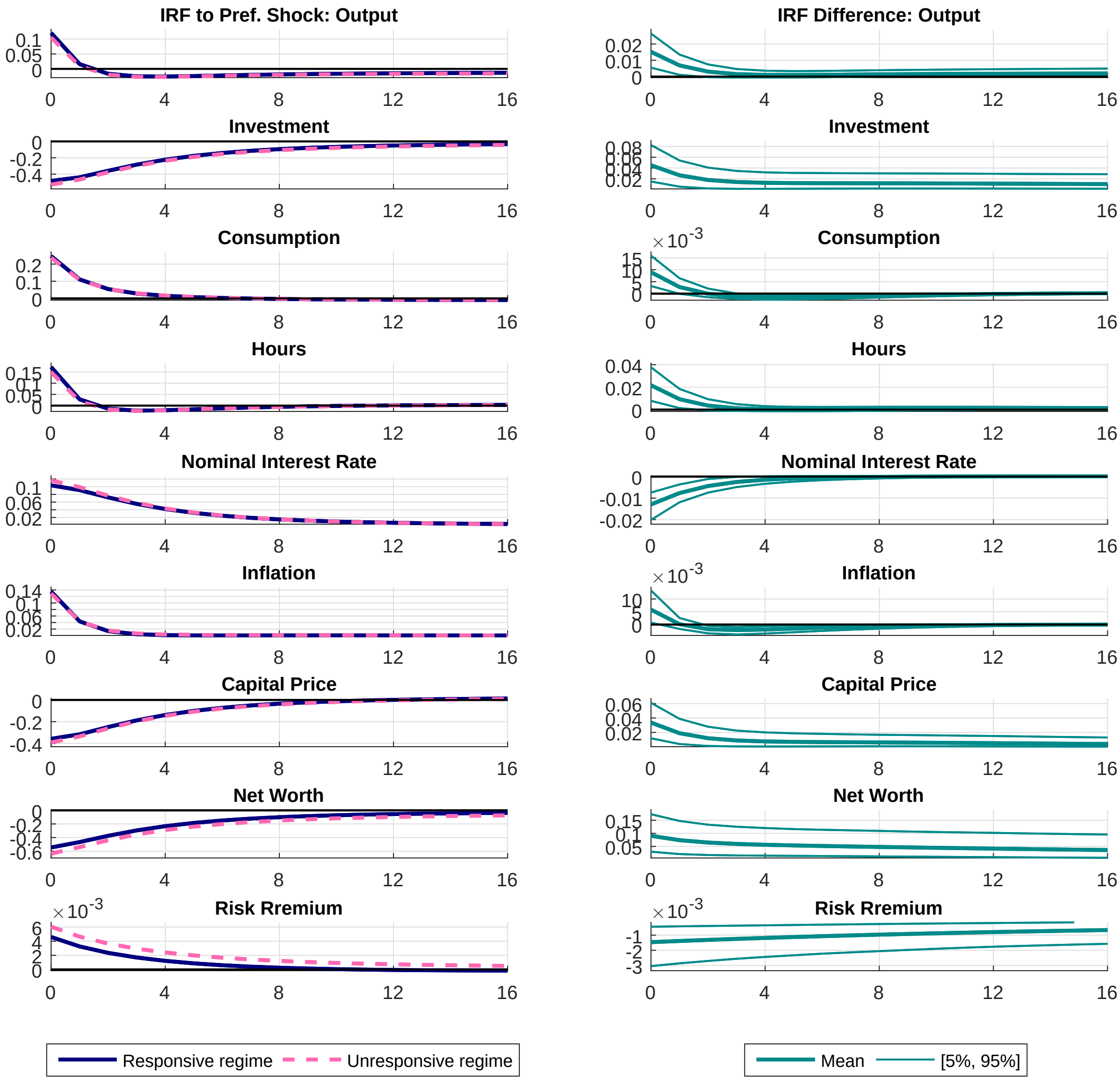

FIGURE 4. **[Left panels]** Mean impulse responses to 1% preference shocks, taking into account the possibility of regime changes. In each figure, impulse responses associated with the responsive regime as the initial regime (solid lines) and with the unresponsive regime as the initial regime (dashed lines) are reported. **[Right panels]** Mean (thick lines) and 90% error bands (thin lines) for the difference between the two responses. The x-axis measures quarters.

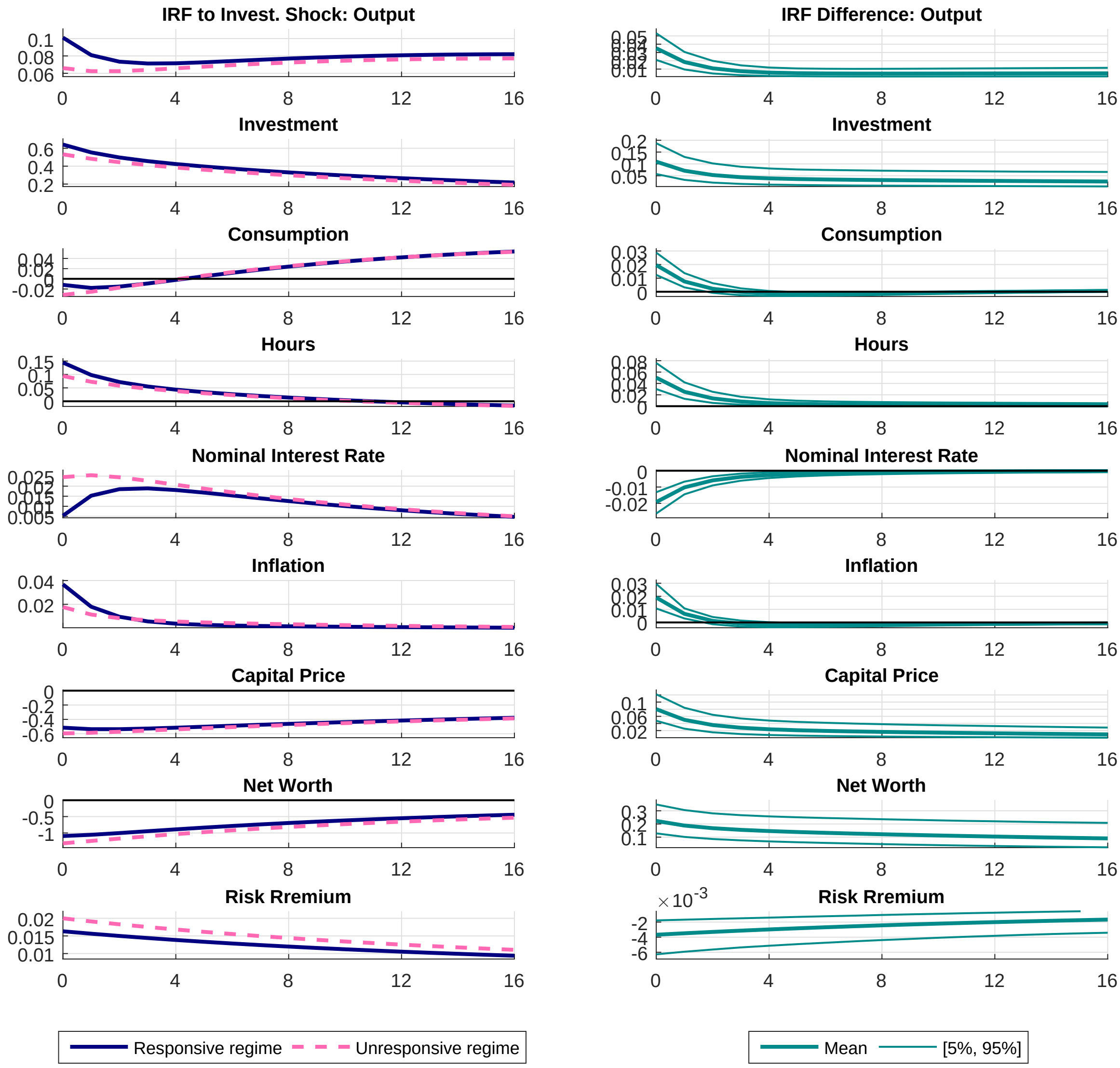

FIGURE 5. **[Left panels]** Mean impulse responses to 1% investment-efficiency shocks, taking into account the possibility of regime changes. In each figure, impulse responses associated with the responsive regime as the initial regime (solid lines) and with the unresponsive regime as the initial regime (dashed lines) are reported. **[Right panels]** Mean (thick lines) and 90% error bands (thin lines) for the difference between the two responses. The x-axis measures quarters.

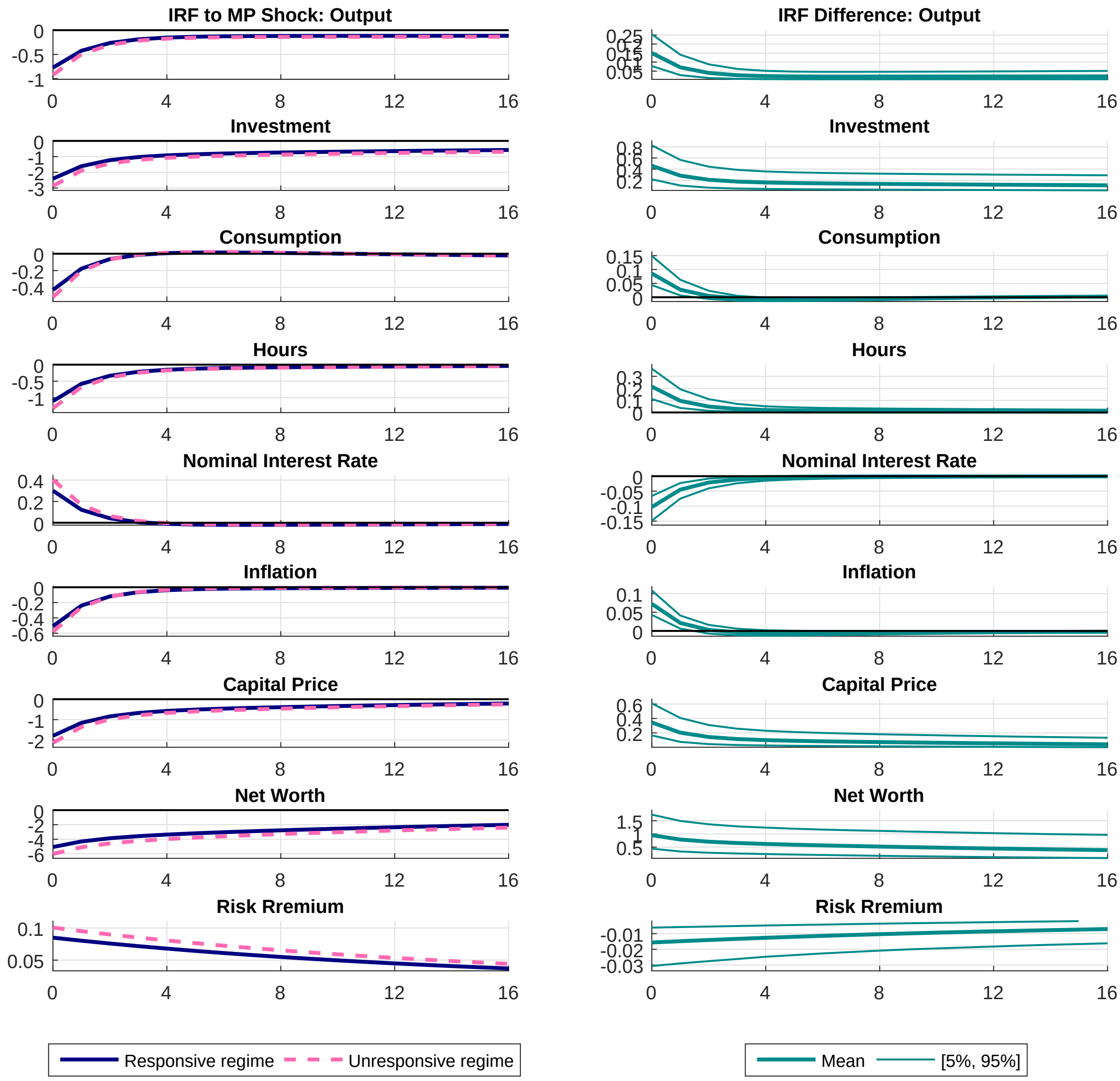

FIGURE 6. **[Left panels]** Mean impulse responses to 1% monetary policy shocks, taking into account the possibility of regime changes. In each figure, impulse responses associated with the responsive regime as the initial regime (solid lines) and with the unresponsive regime as the initial regime (dashed lines) are reported. **[Right panels]** Mean (thick lines) and 90% error bands (thin lines) for the difference between the two responses. The x-axis measures quarters.

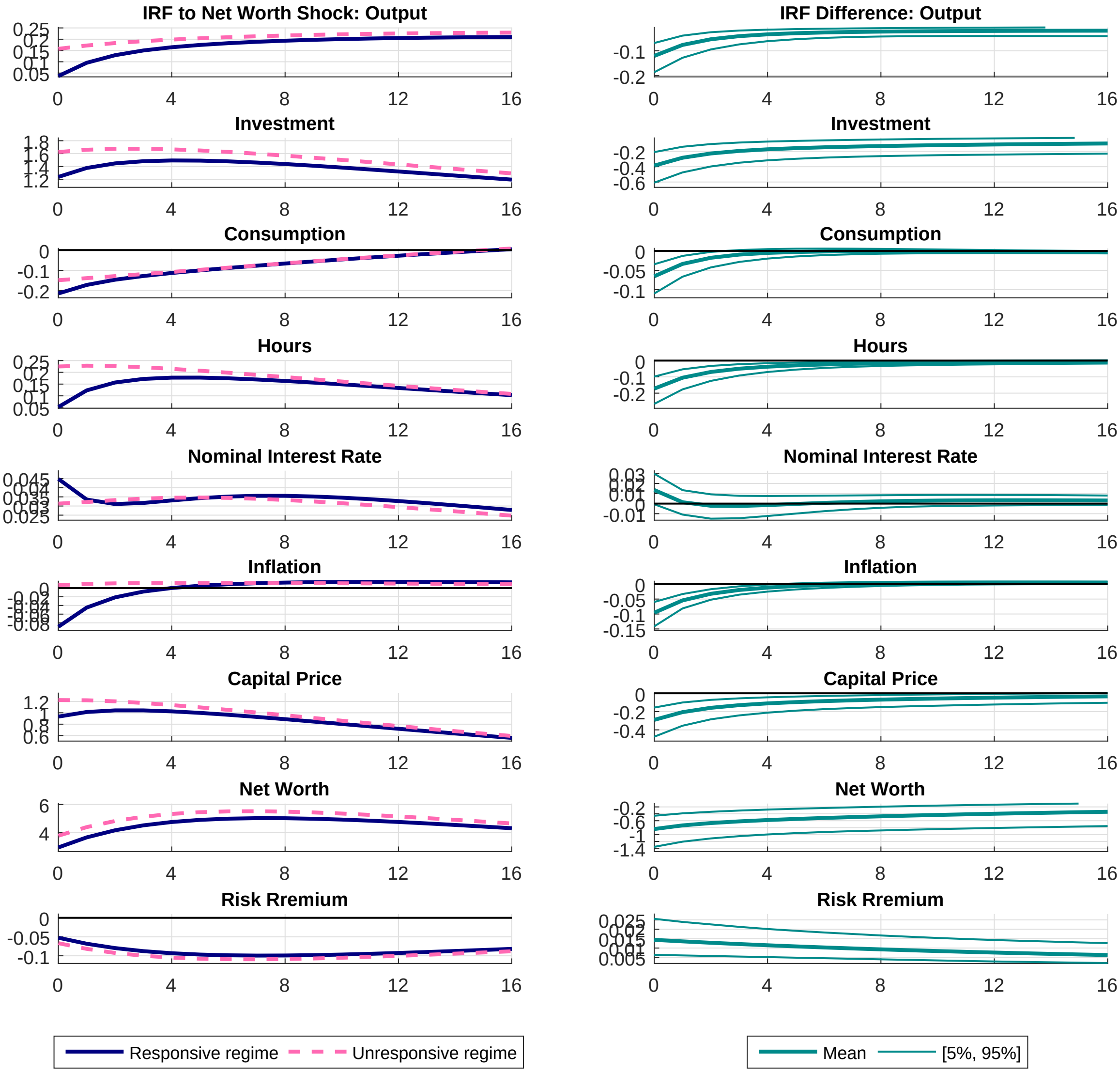

FIGURE 7. **[Left panels]** Mean impulse responses to 1% net worth shocks, taking into account the possibility of regime changes. In each figure, impulse responses associated with the responsive regime as the initial regime (solid lines) and with the unresponsive regime as the initial regime (dashed lines) are reported. **[Right panels]** Mean (thick lines) and 90% error bands (thin lines) for the difference between the two responses. The x-axis measures quarters.

5. Posterior Parameter Estimates for the Alternative Specifications

Table 2 reports the posterior parameter estimates for the three additional specifications.

Parameter	Fixed unresponsive MP		Fixed responsive MP		Full Markov-switching	
	Mean	[5%, 95%]	Mean	[5%, 95%]	Mean	[5%, 95%]
σ_c (Risk aversion)	2.15	[1.70, 2.59]	2.17	[1.73, 2.60]	2.01	[1.56, 2.46]
χ (Capital adjustment cost)	0.73	[0.53, 0.94]	0.73	[0.52, 0.93]	0.75	[0.54, 0.97]
ω (Degree of price stickiness)	0.72	[0.64, 0.80]	0.72	[0.64, 0.79]	0.75	[0.64, 0.84]
ρ_a (Technology shock AR(1))	0.97	[0.94, 0.98]	0.97	[0.94, 0.98]	0.96	[0.93, 0.98]
ρ_b (Preference shock AR(1))	0.60	[0.28, 0.87]	0.59	[0.28, 0.87]	0.74	[0.39, 0.91]
ρ_i (Investment-efficiency shock AR(1))	0.92	[0.87, 0.96]	0.91	[0.86, 0.96]	0.93	[0.89, 0.97]
ρ_n (Net worth shock AR(1))	0.70	[0.35, 0.88]	0.65	[0.27, 0.87]	0.80	[0.62, 0.93]
ν (Survival rate of entrepreneurs)	0.95	[0.91, 0.99]	0.95	[0.91, 0.99]	0.95	[0.91, 0.99]
ψ (Elasticity of external finance premium)	0.03	[0.02, 0.05]	0.03	[0.02, 0.05]	0.03	[0.02, 0.04]
k/n (Steady-state capital/net worth ratio)	2.28	[1.96, 2.63]	2.30	[1.98, 2.65]	2.43	[2.10, 2.80]
s_{ss} (Gross steady-state risk premium)	1.006	[1.002, 1.009]	1.006	[1.003, 1.009]	1.006	[1.003, 1.010]
ρ_r (MP rule AR(1))	0.72	[0.66, 0.77]	0.71	[0.66, 0.76]		
ϕ_π (MP response to inflation)	2.51	[2.15, 2.88]	2.50	[2.14, 2.88]		
ϕ_y (MP response to output growth)	0.64	[0.42, 0.90]	0.60	[0.36, 0.86]		
ϕ_n (MP response to net worth growth)			0.01	[-0.01, 0.03]		
σ_a (Technology shock std.)	1.31	[0.99, 1.71]	1.33	[1.00, 1.74]	1.34	[0.97, 1.86]
σ_b (Preference shock std.)	0.78	[0.39, 1.52]	0.78	[0.39, 1.56]	0.96	[0.44, 1.84]
σ_i (Investment-efficiency shock std.)	12.33	[8.83, 14.70]	12.12	[8.49, 14.67]	12.14	[8.63, 14.71]
σ_r (Monetary policy shock std.)	0.04	[0.03, 0.05]	0.04	[0.03, 0.06]	0.05	[0.03, 0.06]
σ_n (Net worth shock std.)	3.00	[1.22, 7.50]	3.41	[1.21, 10.43]	2.20	[1.07, 4.69]
ρ_r (MP rule AR(1), **regime 1**)					0.64	[0.50, 0.75]
ϕ_π MP response to inflation, **regime 1**)					2.54	[2.15, 2.98]
ϕ_y (MP response to output growth, **regime 1**)					0.53	[0.25, 0.84]
ϕ_n (MP response to net worth growth, **regime 1**)					0.12	[0.07, 0.17]
ρ_r (MP rule AR(1), **regime 2**)					0.70	[0.64, 0.76]
ϕ_π MP response to inflation, **regime 2**)					2.21	[1.69, 2.72]
ϕ_y (MP response to output growth, **regime 2**)					0.66	[0.36, 1.01]
ϕ_n (MP response to net worth growth, **regime 2**)					Fixed at zero ($\phi_n = 0$)	
P_{11} (Prob. of policy regime 1)					0.97	[0.93, 0.99]
P_{22} (Prob. of policy regime 2)					0.99	[0.98, 1.00]

TABLE 2. Posterior distributions of the estimated parameters for the three alternative specifications.

Figure 8 plots the ϕ_n estimates associated with the fixed responsive regime assumption. The dashed line represents the posterior distribution for ϕ_n accompanied by a fixed coefficient version of the model, in which the Fed is presumed to be responsive to stock prices over the entire sample period. The solid line with circles represents Rigobon and Sack's (2003) estimates for the degree of monetary policy responsiveness toward the stock market. Based on stock returns between 1985 and 1999, Rigobon and Sack (2003) establish a VAR framework that identifies the policy response to the stock market by controlling for the endogenous responses of stock prices to monetary policy disturbances. They document a significant reaction of monetary policy to fluctuations in stock prices as displayed in the figure.

The estimation result under the fixed-coefficient setup shows that, if a homogeneous regime for ϕ_n is assumed over the entire sample, the magnitude of stock price targeting becomes much weaker. The estimated response of the interest rate to stock prices is centered at 0.01 with the 90% interval of [-0.01, 0.03]. Compared to Rigobon and Sack (2003), the posterior distribution of ϕ_n associated with the fixed-coefficient specification encompasses their estimates, with a slightly lower mean value. In spite of the difference in methodology and sample span, it is notable that the fixed-regime estimates in this article characterize a degree of monetary policy responsiveness toward stock prices comparable to the previous study.

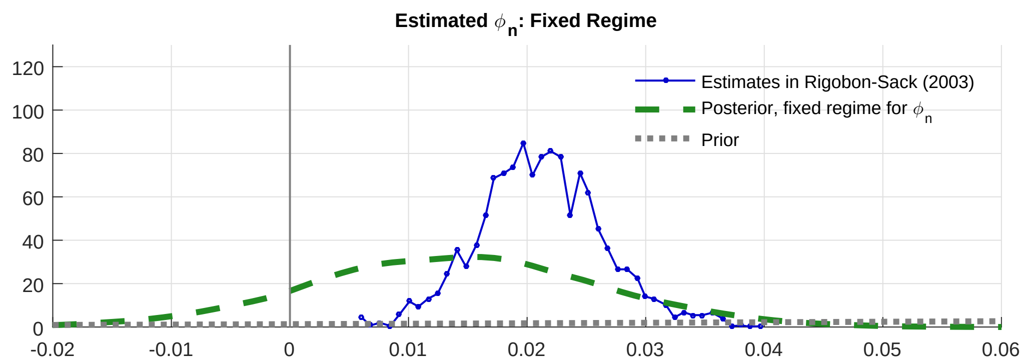

Figure 8. Posterior of the fixed-coefficient specification in which the Fed always responds to stock prices (dashed line), and the estimates in Rigobon and Sack (2003, solid line with circles).

6. Robustness

6.1. Regime Shifts in the Whole Monetary Policy Rule Coefficients. The benchmark specification assumes that the three coefficients in the Taylor rule—the autoregressive, inflation response, and output response parameters—are constant throughout the sample. A potential concern with this assumption is whether the estimation is subject to a bias in the degree of time variation in the coefficient on stock prices, which is allowed to vary over time.

Before addressing this issue, Figure 9 plots the posterior estimates of ϕ_π and ϕ_y against ϕ_n to check whether there is a systematic relationship between them. The left panel shows that the inflation coefficient is positively linked to the stock price coefficient, as their correlation is 0.26. An explanation for the positive correlation can be gleaned from one of the findings in Section 6.2 of the main text. The significantly positive ϕ_n estimates associated with the responsive regime raise the model-implied inflation volatility, and thus the estimation requires a higher degree of the interest rate reaction to inflation, ϕ_π, in order to match the inflation variability in the data. In contrast, the right panel of the figure suggests that there is no systematic pattern between ϕ_y and ϕ_n.

Table 3 summarizes the estimation results associated with the specification allowing for time variation in all the monetary policy rule coefficients. In doing so, the alternative specification maintains the empirical strategy of the benchmark: the stock price coefficient is estimated for one regime, while it is fixed at zero for the other one. As

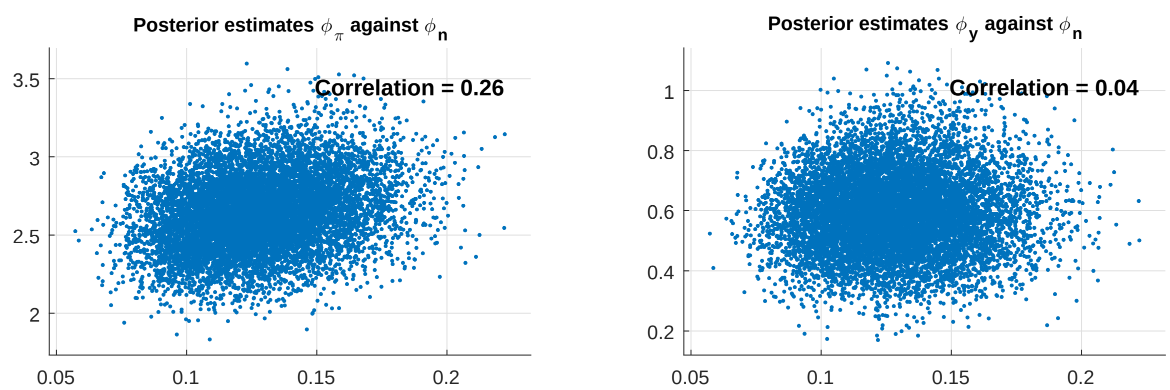

Figure 9. Scatter plots for the posterior estimates of ϕ_π (left panel) and ϕ_y (right panel) against those of ϕ_n associated with the responsive regime.

Parameter	Benchmark Regime 1	Benchmark Regime 2	Full Markov-switching Regime 1	Full Markov-switching Regime 2
ρ_r	0.71; [0.66, 0.76]		0.64; [0.50, 0.75]	0.70; [0.64, 0.76]
ϕ_π	2.64; [2.26, 3.04]		2.54; [2.15, 2.98]	2.21; [1.69, 2.72]
ϕ_y	0.59; [0.38, 0.82]		0.53; [0.25; 0.84]	0.66; [0.36; 1.01]
ϕ_n	0.13; [0.09, 0.17]	fixed at zero	0.12; [0.07, 0.17]	fixed at zero
P_{11}	0.97; [0.95, 0.99]		0.97; [0.93, 0.99]	
P_{22}		0.99; [0.98, 1.00]		0.99; [0.98, 1.00]
Average Log Marginal Density	-650.7		-652.2	
DIC	1221.6		1226.6	
BPIC	1231.7		1238.8	

Table 3. Posterior distributions of the estimated monetary policy rule parameters as well as the model fit, associated with the benchmark specification and the one imposing a full-fledged Markov-switching structure in the policy coefficients. This table reports the mean and associated 90% error bands (in brackets).

in the last two columns in the table, the posterior estimates characterize two policy regimes which display the central bank's distinct responses toward stock prices. The responsiveness to the stock market under the regime 1 is estimated to be away from zero, with a mean of 0.12 and 90% interval of [0.07, 0.17]. These estimates are slightly lower than the ones associated with the benchmark specification.

It turns out that the estimates for the rest of the policy coefficients vary across the regimes. Regime 1 is accompanied by a lower degree of interest rate smoothing and output response, but more aggressive inflation targeting than the other regime. More importantly, however, the posterior intervals for these parameters overlap considerably between the regimes, suggesting that regimes 1 and 2 are distinguished mainly by the policy stance toward the stock market. The model fit provided in the last three rows in Table 3 confirms this point. Regardless of the criteria, the alternative specification is less preferred than the benchmark. This indicates that the additional time variation in ρ_r, ϕ_π, and ϕ_y has almost no role in fitting the data, which is rationalized by the significant overlap of these parameters across the regimes. Regarding the probabilities of moving across the two monetary policy regimes, they are not substantially altered by the alternative specification, as the persistence of regime 1 decreases only mildly compared to the benchmark.

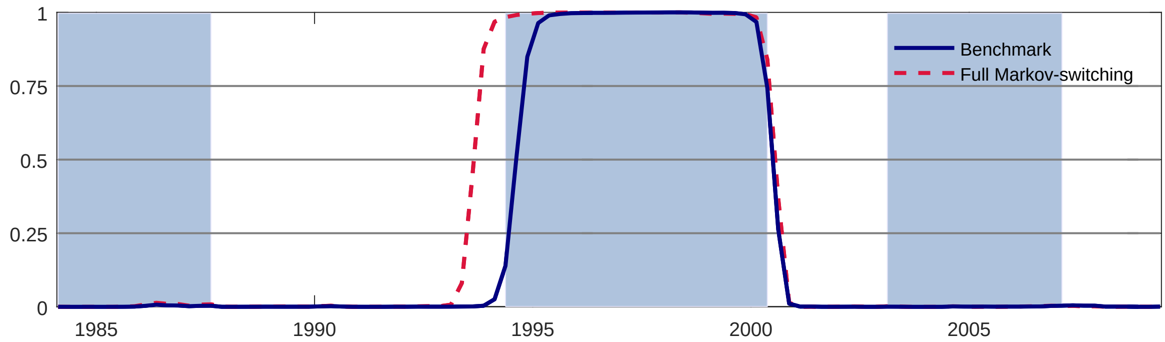

FIGURE 10. Smoothed probabilities of monetary policy regime 1 for the benchmark specification (solid line) and for the one imposing a full-fledged Markov-switching structure in the policy coefficients (dashed line), evaluated at the posterior mode. The shaded areas indicate the stock market boom dates in Christiano et al. (2010).

Finally, Figure 10 shows the smoothed probability estimates of regime 1 evaluated at the posterior mode, both for the benchmark and alternative specifications. The full-fledged Markov-switching structure extends the timing of the responsive regime, as the probability of regime 1 starts rising before the stock market boom period in the 1990s. Nevertheless, allowing for time variation in all the monetary policy parameters does not alter the conclusion that monetary policy was responsive to equity prices during the stock market boom in the 1990s.

6.2. **Alternative Priors for the Regime-switching Probability.** Another issue with the estimation results is the highly persistent monetary policy regimes. The estimated mean probabilities of staying in the responsive and unresponsive regimes are 0.97 and 0.99, respectively. A natural question is to what extent the regime probability estimates are affected by the informative prior. To assess the sensitivity of results to this dimension, I re-estimate the benchmark model under a less informative prior distribution for P_{11} and P_{22}: a beta distribution of mean 0.9 and standard deviation 0.1. Notice that the alternative prior is quite uninformative as its standard deviation is more than 10 times larger than that of the benchmark specification. Also, the prior distribution has the same mean as that of Davig and Doh (2014), but with a *twice enlarged* standard deviation.

As summarized in Table 4, the posterior estimates concerning the monetary policy behavior are almost unaltered by the prior assumption on the regime-switching probability. This finding indicates that the posterior regime probability estimates are

Parameter	Benchmark Regime 1	Benchmark Regime 2	Alternative Prior for P_{ii} Regime 1	Alternative Prior for P_{ii} Regime 2
ρ_r	0.71; [0.66, 0.76]		0.71; [0.66, 0.76]	
ϕ_π	2.64; [2.26, 3.04]		2.66; [2.28, 3.06]	
ϕ_y	0.59; [0.38, 0.82]		0.59; [0.38, 0.81]	
ϕ_n	0.13; [0.09, 0.17]	fixed at zero	0.13; [0.09, 0.17]	fixed at zero
P_{11}	0.97; [0.95, 0.99]		0.97; [0.95, 0.99]	
P_{22}		0.99; [0.98, 1.00]		0.99; [0.98, 1.00]

Table 4. Posterior distributions of the estimated monetary policy rule parameters, associated with the benchmark specification and the one with alternative prior distribution for the regime-switching probability. This table reports the mean and associated 90% error bands (in brackets).

unlikely to be driven by a specific choice of the prior distributions. Rather, the data tend to be informative in identifying the parameters.

7. Model Validation

Macro DSGE models are known to have poor asset pricing implications. The model in this paper may be subject to this caveat due to the inclusion of the stock market index as an observable. In the present section, I evaluate the empirical plausibility of incorporating stock prices by comparing the key characteristics of the model-implied variables to those of the actual time series.

7.1. **Net Worth and Stock Prices.** The underlying DSGE model assumes that the dynamics of net worth are assumed to be driven by exogenous shocks, instead of being determined endogenously as equilibrium outcomes. Then the estimation uses the stock price index as the proxy for fluctuations in entrepreneurial net worth. This modeling choice may raise a question as to how reasonable the approximation is in practice. In order to address the issue, I compute the key posterior moments for the model-implied net worth growth, and make a comparison of them with those of the actual stock price growth. The model-implied moments are calculated by Monte Carlo simulations evaluated at each posterior draw.

Figure 11 reports the standard deviation, autocorrelation, and cross-correlations with investment growth and with the nominal interest rate for the actual stock price

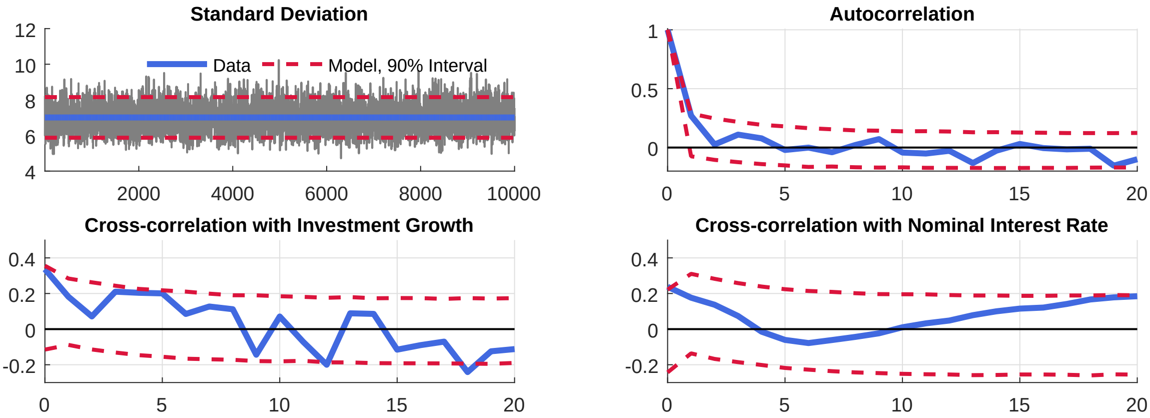

FIGURE 11. Standard deviation, autocorrelations, and cross-correlations with investment growth and with nominal interest rate for the actual stock price growth (solid lines) and the model-implied net worth growth (dashed lines—90% posterior intervals). In the first figure, the thin solid line represents the standard deviation evaluated at each posterior draw, and the x-axis is the number of posterior draws. In the latter three figures, the x-axis is in quarters.

growth (solid lines) and the model-implied net worth growth (dashed lines, 90% posterior interval). Overall, most of the actual data moments are nested into the 90% intervals for the model-implied moments, indicating that the model-implied net worth series replicate fairly well the key moments of the actual stock series. As in the first panel of the figure, the 90% interval for the model-implied standard deviation of net worth growth is [5.88, 8.11], which includes its data counterpart of 7.01. Concerning the autocorrelation in the second panel, the data moments are within the 90% posterior intervals up to the 19 lags. The model is also able, with rare exceptions, to generate the data-consistent cross-correlations of stock prices with investment. Most of the empirical cross-correlations fall within the 90% posterior intervals. Finally, a similar finding is observed for the model-implied cross-correlations between net worth growth and the policy rate, as shown in the last panel. The 90% posterior intervals encompass the actual data moments up to the 19 lags.

7.2. **External Finance Premium.** In the model, the external finance premium, $q_t k_{t+1}/n_{t+1}$, is the channel through which the stock market affects the real economy. Thus, another important dimension of the model validation is to inspect its ability in producing the data-consistent premium. Here I consider two readily available series, which are widely used as proxies for the financing cost of firms. The first of them is the corporate bond spread, which is Baa-rated corporate bond yield less the Aaa-rated corporate bond yield. The second one is the spread between the Baa-rated corporate bond yield and the 10-year US government bond yield, employed in Christiano et al. (2010).

Figure 12 compares the mean estimates for the model-implied external finance premium (solid lines with circles) to the two actual spreads (solid lines). Overall, the model-implied series closely mimics the high frequency movements in the actual data, as it tracks well the ups and downs of the two actual spreads. One of the consequences is the countercyclicality of the model-implied finance premium, which is also observed in the data. Notice that this property contrasts that of De Graeve (2008), who documents a cyclical pattern of the model-implied finance premium by estimating a DSGE model with the financial accelerator mechanism. A primary difference of this paper from De Graeve (2008) is the inclusion of the financial market information in estimating the model. The observable variables in De Graeve (2008) consist only of macroeconomic aggregates, whereas this article uses the stock market index to impose restrictions on the behavior of endogenous net worth movements. Thus, this finding suggests that incorporating additional information from the financial market can be important for generating the data-consistent countercyclical property of the model-implied external finance premium.

Table 5 reports the correlation between the model-implied finance premium and the two actual spreads, associated with the lags from -4 to 4 quarters.[3] The correlation is much higher with the Baa−Aaa spread than the Baa−government bond spread, as the contemporaneous correlations are as much as 0.59 and 0.23, respectively. This suggests that the model-implied external finance premium seems to display a substantial degree of comovement with the Baa−Aaa spread. Regarding the statistics across the leads and lags, the correlations associated with the positive lags exceed those associated with the negative ones. This implies that the model-implied series tends to lead the actual spreads, indicating the model's predictive power for the financial indicators.

[3]Positive lags indicate that the model-implied series leads the actual spread, and vice versa for negative lags.

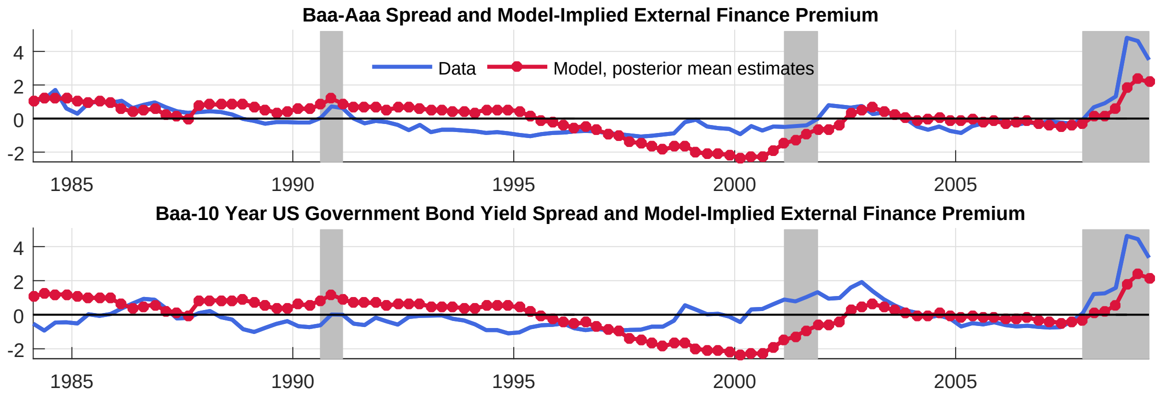

FIGURE 12. The proxy variables for the external finance premium (solid lines) and model-implied external finance premium (solid lines with circles). The proxy variables use the difference between the Baa- and Aaa-rated corporate bond yields (upper panel) and difference between the BAA-rated corporate bond yield and the 10-year US government bond yield (lower panel), respectively. The model-implied series are the posterior mean estimates. To ease comparison, all the series are standardized by subtracting the mean and dividing by the standard deviation. Shaded areas indicate NBER recession dates.

Lag	-4	-3	-2	-1	0	1	2	3	4
Baa−Aaa	0.14	0.22	0.34	0.49	0.59	0.61	0.61	0.62	0.59
Baa−10-year Govt. Bond Yield	-0.24	-0.17	-0.05	0.10	0.23	0.25	0.23	0.20	0.20

TABLE 5. Correlation between the proxy variables for the external finance premium and mean estimates for model-implied external finance premium. Positive lags indicate the model-implied series leading the actual spread, and vice versa for negative lags.

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