The Early Millennium Slowdown: Replicating the Peersman (2005) Results: Online Appendix

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A Priors

This section provides the details of the priors used for both models. The priors for the models are chosen with a methodology similar to that in Primiceri (2005).

Constant Coefficients VAR

For the constant coefficients VAR, the following independent prior distributions are assumed: $\boldsymbol{\beta} \sim \mathcal{N}(\boldsymbol{\beta}_0, \mathbf{V}_{\boldsymbol{\beta}})$, $\boldsymbol{\Sigma} \sim \mathcal{IW}(\nu_{\boldsymbol{\Sigma}}, \mathbf{S}_{\boldsymbol{\Sigma}})$, where $\mathcal{IW}(\cdot, \cdot)$ denotes the inverse-Wishart distribution. Specifically, $\boldsymbol{\beta}_0 = \widehat{\boldsymbol{\beta}}_{\text{OLS}}$, $\mathbf{V}_{\boldsymbol{\beta}} = 4 \times \mathbf{V}(\widehat{\boldsymbol{\beta}}_{\text{OLS}})$, $\nu_{\boldsymbol{\Sigma}} = n + 2$, and $\mathbf{S}_{\boldsymbol{\Sigma}} = \widehat{\boldsymbol{\Sigma}}_{\text{OLS}}$, where $\widehat{\boldsymbol{\beta}}_{\text{OLS}}$ is a $q \times 1$ vector of ordinary least squares (OLS) point estimates, $\mathbf{V}(\widehat{\boldsymbol{\beta}}_{\text{OLS}})$ is a $q \times q$ diagonal matrix of the variances of the OLS estimates and $\widehat{\boldsymbol{\Sigma}}_{\text{OLS}}$ is an $n \times n$ diagonal matrix of the estimated variances of the error terms.

The OLS estimates are calculated with a constant coefficients VAR over a sample that consists of 10 years of data from 1961Q1 to 1970Q4. For this sample, all series are sourced as outlined in Section 3 of the replication paper, except for the oil price. The IMF oil price series is not available quarterly from 1961Q1 to 1970Q4, so this sample is constructed by splicing on the changes in the fuel oil and other fuels subcomponent series in the US consumer price index, which is highly correlated with changes in the IMF oil price series.

Time-Varying Parameter VAR

For the TVP-VAR, recall that the error variance-covariance matrix of the state equation $\boldsymbol{\Omega}_{\boldsymbol{\beta}} = \text{diag}(\omega^2_{\beta,1}, \ldots, \omega^2_{\beta,q})$ is diagonal, and its diagonal elements are assumed to be independently distributed as $\omega^2_{\beta,i} \sim \mathcal{IG}(\nu_{\beta,i}, S_{\beta,i}), i = 1, ..., q$, where $\mathcal{IG}(\cdot,\cdot)$ denotes the inverse-gamma distribution. Specifically, $\nu_{\beta,i} = 40$ and $S_{\beta,i} = k_Q^2 \times 40 \times v_i^2$, where $k_Q = 0.0001$ and v_i^2 is the i-th diagonal element of $\mathbf{V}(\widehat{\boldsymbol{\beta}}_{\text{OLS}})$. The state equation is initialized with $\boldsymbol{\beta}_1 \sim \mathcal{N}(\boldsymbol{\beta}_0, \mathbf{V}_{\boldsymbol{\beta}})$, where $\boldsymbol{\beta}_0 = \widehat{\boldsymbol{\beta}}_{\text{OLS}}$ and $\mathbf{V}_{\boldsymbol{\beta}} = 4 \times \mathbf{V}(\widehat{\boldsymbol{\beta}}_{\text{OLS}})$, consistent with the priors of the constant coefficients VAR. For the error variance-covariance matrix in the measurement equation, $\boldsymbol{\Sigma}$, its prior is assumed to be the same as in the constant coefficients VAR, i.e., $\boldsymbol{\Sigma} \sim \mathcal{IW}(\nu_{\boldsymbol{\Sigma}}, \mathbf{S}_{\boldsymbol{\Sigma}})$ with $\nu_{\boldsymbol{\Sigma}} = n + 2$ and $\mathbf{S}_{\boldsymbol{\Sigma}} = \widehat{\boldsymbol{\Sigma}}_{\text{OLS}}$.

B Gibbs Samplers

This section provides the estimation details of the two models. Gibbs samplers are used to simulate from the posterior distributions of all models. The full conditional distributions are provided below.[1]

Constant Coefficients VAR

1. Draw from $(\boldsymbol{\beta} \,|\, \mathbf{y}, \boldsymbol{\Sigma}) \sim \mathcal{N}(\widehat{\boldsymbol{\beta}}, \mathbf{D}_{\boldsymbol{\beta}})$, where

$$\mathbf{D}_{\boldsymbol{\beta}}^{-1} = \mathbf{X}'(\mathbf{I}_T \otimes \boldsymbol{\Sigma}^{-1})\mathbf{X} + \mathbf{V}_{\boldsymbol{\beta}}^{-1}, \quad \widehat{\boldsymbol{\beta}} = \mathbf{D}_{\boldsymbol{\beta}} \left(\mathbf{X}'(\mathbf{I}_T \otimes \boldsymbol{\Sigma}^{-1})\mathbf{y} + \mathbf{V}_{\boldsymbol{\beta}}^{-1}\boldsymbol{\beta}_0 \right).$$

2. Draw from $(\boldsymbol{\Sigma} \,|\, \mathbf{y}, \boldsymbol{\beta}) \sim \mathcal{IW}\left(T + \nu_{\boldsymbol{\Sigma}}, \mathbf{S}_{\boldsymbol{\Sigma}} + \sum_{t=1}^{T} (\mathbf{y}_t - \mathbf{X}_t\boldsymbol{\beta})(\mathbf{y}_t - \mathbf{X}_t\boldsymbol{\beta})' \right)$.

3. Repeat steps (1)-(2) N times.

Time-Varying Parameter VAR

The first step is to stack the measurement equation (??) over t:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \tag{1}$$

where $\mathbf{y} = (\mathbf{y}_1', \ldots, \mathbf{y}_T')'$, $\boldsymbol{\beta} = (\boldsymbol{\beta}_1', \ldots, \boldsymbol{\beta}_T')'$, $\boldsymbol{\varepsilon} = (\boldsymbol{\varepsilon}_1', \ldots, \boldsymbol{\varepsilon}_T')'$, $\boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_T \otimes \boldsymbol{\Sigma})$, and

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_2 & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{X}_T \end{pmatrix}.$$

[1] The Matlab code for the reduced-form constant coefficients VAR and TVP-VAR is based on a template provided by Joshua Chan, while the sign restrictions code is based on a template provided by Renée Fry-McKibbin.

Next, let $\mathbf{H}$ denote the first difference matrix, i.e.,

$$\mathbf{H} = \begin{pmatrix} \mathbf{I}_q & \mathbf{0} & \cdots & \mathbf{0} \\ -\mathbf{I}_q & \mathbf{I}_q & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & -\mathbf{I}_q & \mathbf{I}_q \end{pmatrix}.$$

Then, (??) can be rewritten as

$$\mathbf{H}\boldsymbol{\beta} = \widetilde{\boldsymbol{\alpha}}_{\boldsymbol{\beta}} + \boldsymbol{\zeta},$$

where $\widetilde{\boldsymbol{\alpha}}_{\boldsymbol{\beta}} = (\boldsymbol{\beta}_0', \mathbf{0}, \ldots, \mathbf{0})'$, $\boldsymbol{\zeta} = ({\zeta_1}', \ldots, {\zeta_T}')' \sim \mathcal{N}(\mathbf{0}, \mathbf{S}_{\boldsymbol{\beta}})$ and $\mathbf{S}_{\boldsymbol{\beta}} = \mathrm{diag}(\mathbf{V}_{\boldsymbol{\beta}}, \boldsymbol{\Omega}_{\boldsymbol{\beta}}, \ldots, \boldsymbol{\Omega}_{\boldsymbol{\beta}})$. Since the determinant of $\mathbf{H}$ is unity, it is invertible and $(\boldsymbol{\beta} \,|\, \boldsymbol{\Omega}_{\boldsymbol{\beta}}) \sim \mathcal{N}(\boldsymbol{\alpha}_{\boldsymbol{\beta}}, (\mathbf{H}'\mathbf{S}_{\boldsymbol{\beta}}^{-1}\mathbf{H})^{-1})$ where $\boldsymbol{\alpha}_{\boldsymbol{\beta}} = \mathbf{H}^{-1}\widetilde{\boldsymbol{\alpha}}_{\boldsymbol{\beta}}$.

The full conditional distributions are as follows.

1. Draw from $(\boldsymbol{\beta} \,|\, \mathbf{y}, \boldsymbol{\Sigma}, \boldsymbol{\Omega}_{\boldsymbol{\beta}}) \sim \mathcal{N}(\widehat{\boldsymbol{\beta}}, \mathbf{D}_{\boldsymbol{\beta}})$, using the precision-based algorithm in Chan and Jeliazkov (2009), where

$$\mathbf{D}_{\boldsymbol{\beta}}^{-1} = \mathbf{X}'(\mathbf{I}_T \otimes \boldsymbol{\Sigma}^{-1})\mathbf{X} + \mathbf{H}'\mathbf{S}_{\boldsymbol{\beta}}^{-1}\mathbf{H}, \quad \widehat{\boldsymbol{\beta}} = \mathbf{D}_{\boldsymbol{\beta}}\left(\mathbf{X}'(\mathbf{I}_T \otimes \boldsymbol{\Sigma}^{-1})\mathbf{y} + \mathbf{H}'\mathbf{S}_{\boldsymbol{\beta}}^{-1}\mathbf{H}\boldsymbol{\alpha}_{\boldsymbol{\beta}}\right).$$

2. Draw from $(\boldsymbol{\Sigma} \,|\, \mathbf{y}, \boldsymbol{\beta}, \boldsymbol{\Omega}_{\boldsymbol{\beta}}) \sim \mathcal{IW}\left(T + \nu_{\boldsymbol{\Sigma}}, \mathbf{S}_{\boldsymbol{\Sigma}} + \sum_{t=1}^{T}(\mathbf{y}_t - \mathbf{X}_t\boldsymbol{\beta}_t)(\mathbf{y}_t - \mathbf{X}_t\boldsymbol{\beta}_t)'\right)$.

3. Draw each $\omega_{\beta,i}^2$ for $i = 1, \ldots, q$ from

$$(\omega_{\beta,i}^2 \,|\, \mathbf{y}, \boldsymbol{\beta}, \boldsymbol{\Sigma}) \sim \mathcal{IG}\left(\nu_{\beta,i} + \frac{T-1}{2}, S_{\beta,i} + \frac{1}{2}\sum_{t=2}^{T}(\boldsymbol{\beta}_{i,t} - \boldsymbol{\beta}_{i,t-1})^2\right).$$

4. Repeat steps (1)-(3) N times.

C Results for Constant Coefficients VAR

This section reports the impulse response functions, structural shocks and historical decompositions based on the constant coefficients VAR for both the 1980Q1 to 2002Q2 and 1980Q1 to 2014Q2 sample periods.

Impulse Response Functions

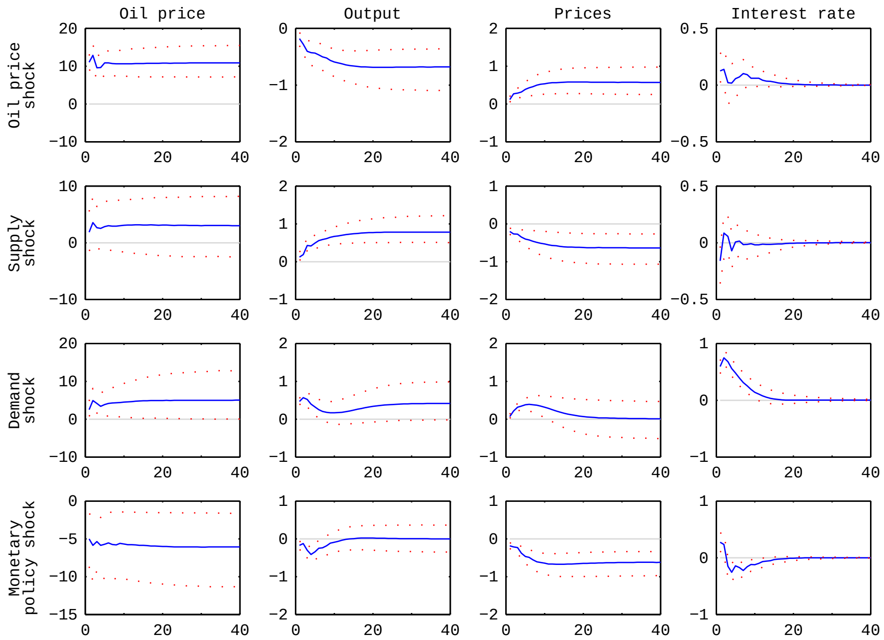


Figure C.1: Impulse response functions, constant coefficients VAR, 1980Q1 to 2002Q2.

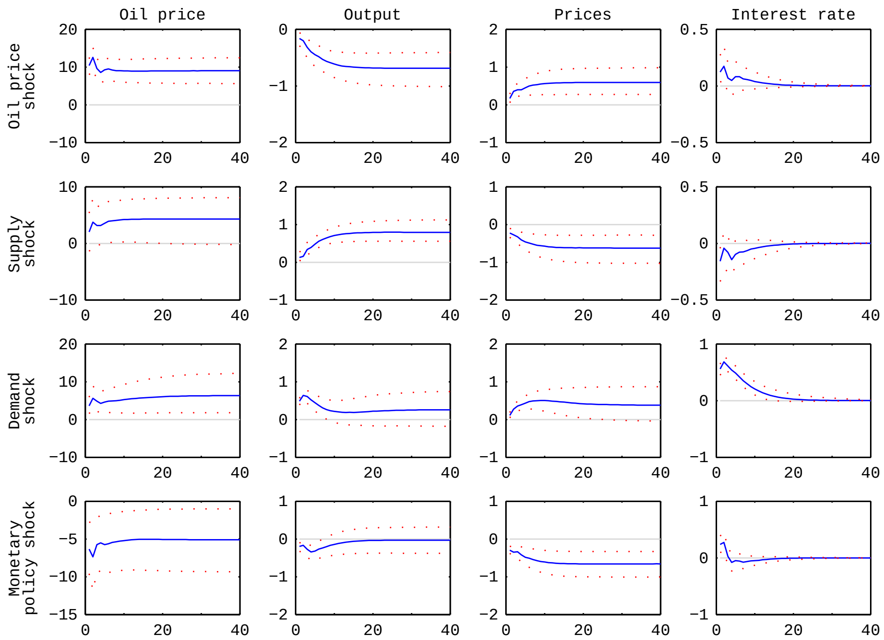
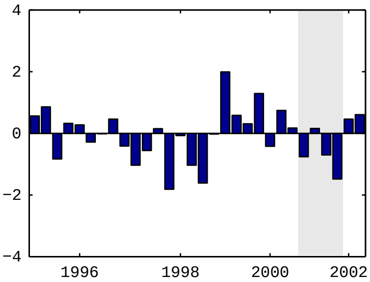
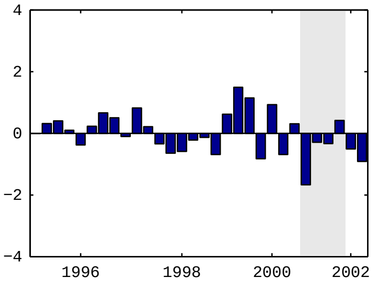


Figure C.2: Impulse response functions, constant coefficients VAR, 1980Q1 to 2014Q2.

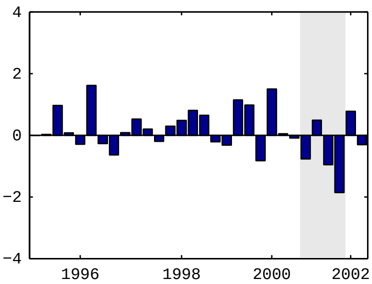
Structural Shocks



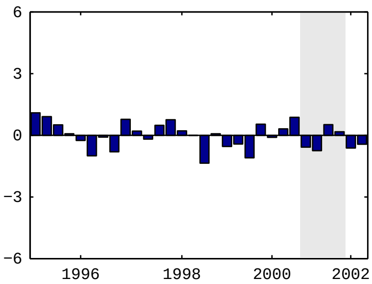
(a) Oil price shock.



(b) Supply shock.

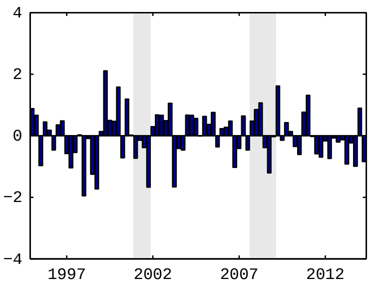


(c) Demand shock.

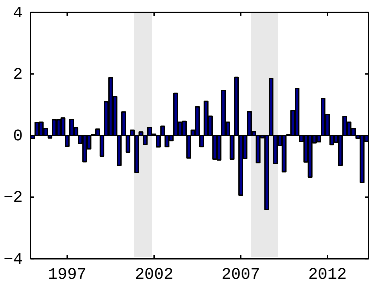


(d) Monetary policy shock.

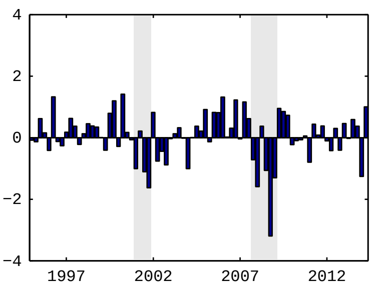
Figure C.3: Structural shocks, constant coefficients VAR, 1980Q1 to 2002Q2.



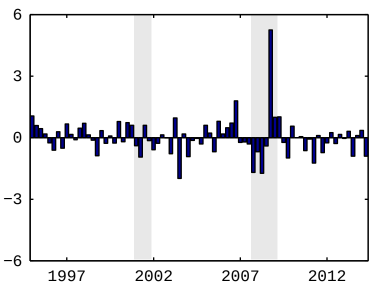
(a) Oil price shock.



(b) Supply shock.



(c) Demand shock.



(d) Monetary policy shock.

Figure C.4: Structural shocks, constant coefficients VAR, 1980Q1 to 2014Q2.

Historical Decompositions

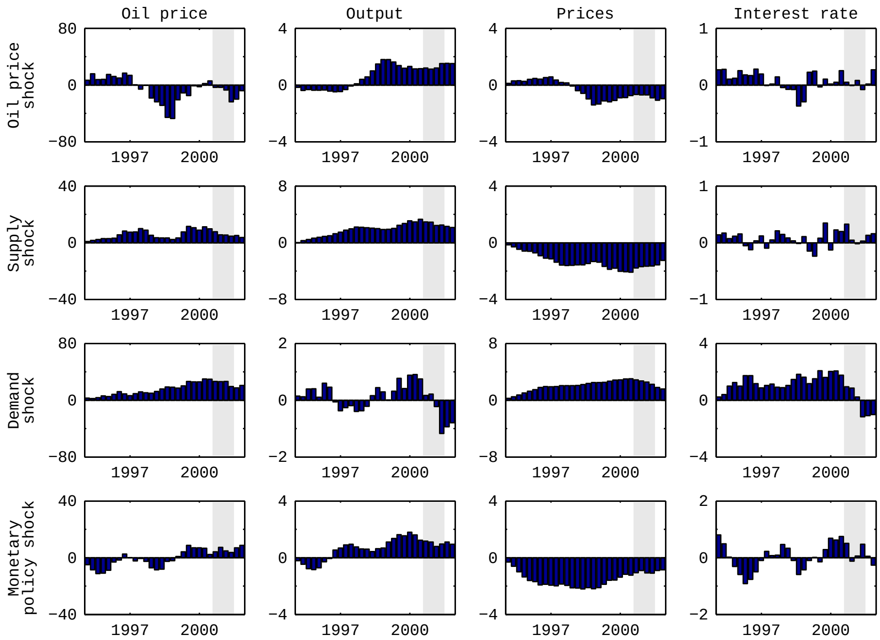


Figure C.5: Historical decomposition, constant coefficients VAR, 1980Q1 to 2002Q2.

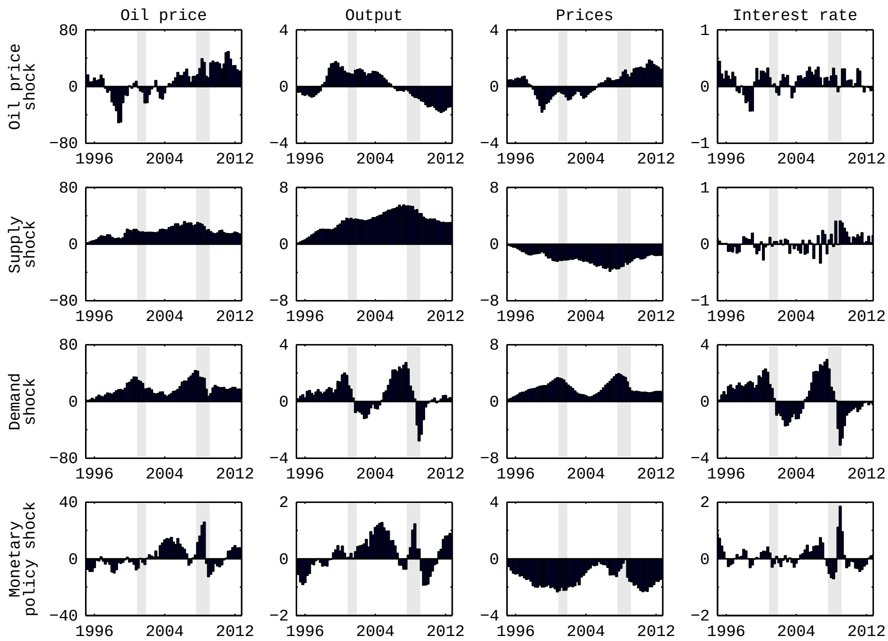
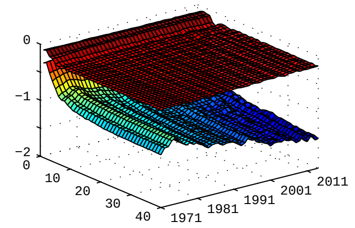


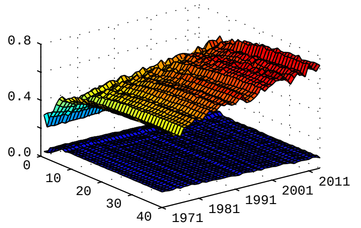
Figure C.6: Historical decomposition, constant coefficients VAR, 1980Q1 to 2014Q2.

D Error Bands for TVP-VAR

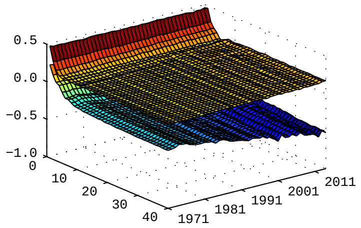
This section contains the error bands for the impulse response functions constructed with the TVP-VAR.



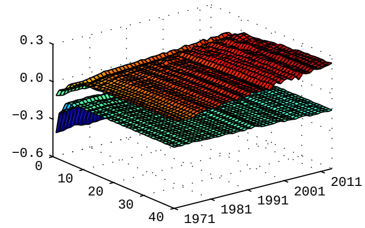
(a) Oil price shock.



(b) Supply shock.

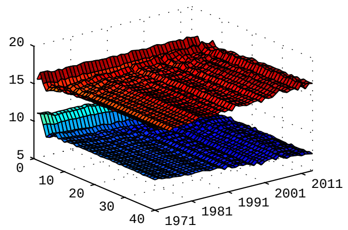


(c) Demand shock.

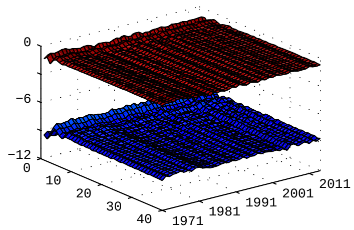


(d) Monetary policy shock.

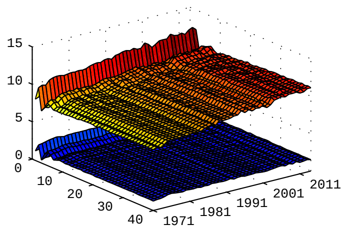
Figure D.1: Error bands, impulse response functions for output, TVP-VAR, 1971Q1 to 2014Q2.



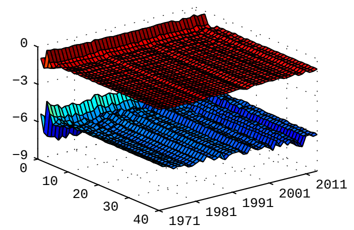
(a) Oil price shock.



(b) Supply shock.

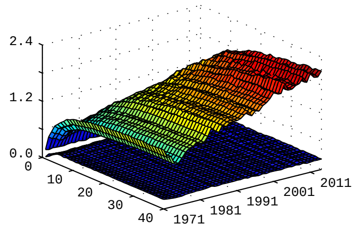


(c) Demand shock.

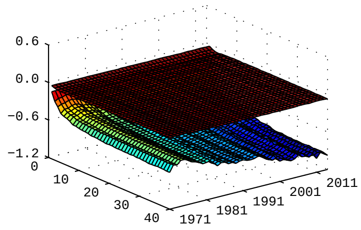


(d) Monetary policy shock.

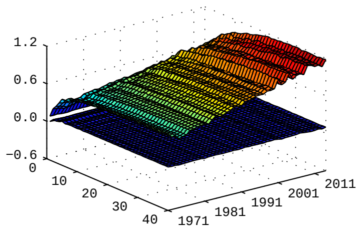
Figure D.2: Error bands, impulse response functions for the oil price, TVP-VAR, 1971Q1 to 2014Q2.



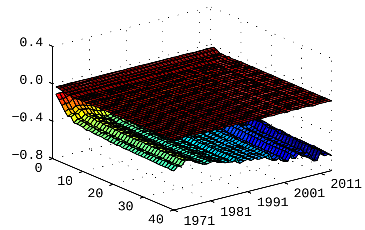
(a) Oil price shock.



(b) Supply shock.

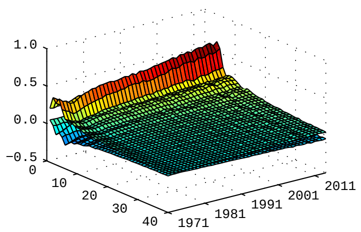


(c) Demand shock.

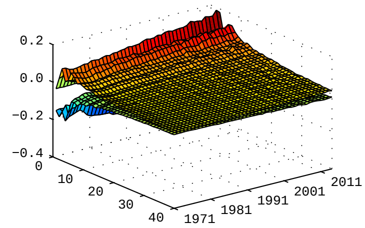


(d) Monetary policy shock.

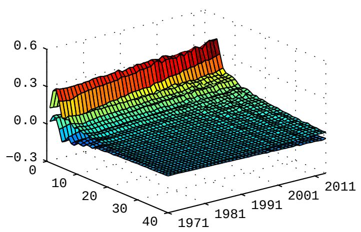
Figure D.3: Error bands, impulse response functions for prices, TVP-VAR, 1971Q1 to 2014Q2.



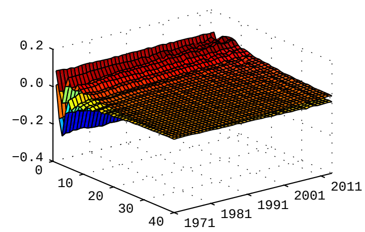
(a) Oil price shock.



(b) Supply shock.



(c) Demand shock.

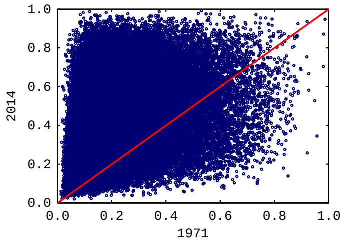


(d) Monetary policy shock.

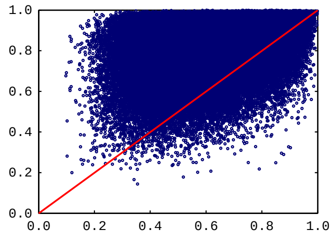
Figure D.4: Error bands, impulse response functions for the interest rate, TVP-VAR, 1971Q1 to 2014Q2.

E R^2 Statistic of Cogley et al. (2010)

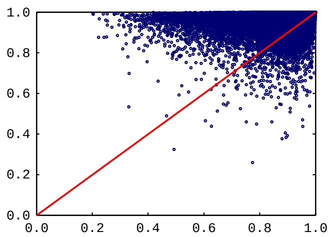
This section reports the R^2 statistic for the one-step-ahead forecast for each of the variables in the model using the methodology of Cogley et al. (2010).



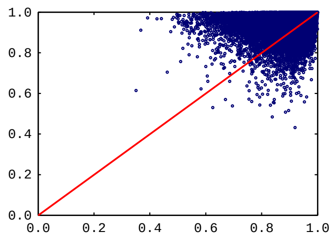
(a) Oil price.



(b) Output.



(c) Prices.



(d) Interest Rate.

Figure E.1: R^2 statistic for one-step-ahead forecasts, TVP-VAR, 1971Q1 to 2014Q1.