Online Appendix to "Estimating Fiscal Limits: The Case of Greece"

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This appendix includes details of nonlinear numerical solutions, estimation diagnostics and additional results from the robustness analysis not included in the paper.

1. Solving the Nonlinear Model

The state vector at period t, ψ_t, includes the post-default debt level, b_t^d, the consumption from the previous period, c_{t-1}, and exogenous state variables that follow $AR(1)$ processes, $(A_t, u_t^g, z_t, u_t^\tau)$. c_{t-1} is included in the state vector because the household's utility for consumption is relative to a habit stock. Also, the post-default debt, b_t^d, instead of the pre-default debt, b_{t-1}, is included in the state space, because b_{t-1} always appears jointly with the default variable, Δ_t, in the model, making b_t^d the effective state variable.

To be specific, other than the end-of-period government debt b_t, all other variables are either exogenous or can be computed in terms of the current state, $\psi_t = (b_t^d, c_{t-1}, A_t, u_t^g, z_t, u_t^\tau)$. For instance, the tax rate and the government spending are determined by the fiscal rules,

$$\tau_t = u_t^\tau + \gamma^\tau \Big(b_t^d - b \Big) \tag{1}$$

$$g_t = u_t^g + \gamma^g \Big(b_t^d - b \Big). \tag{2}$$

Given the logarithm utility function, consumption is determined by the household optimization equation for labor supply and the aggregate resource constraint,

$$c_t = \frac{\phi h c_{t-1} + (A_t - g_t)(1 - \tau_t)}{1 + \phi - \tau_t}. \tag{3}$$

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The core equilibrium conditions for the model are,

$$q_t = \frac{b_t^d + z_t + g_t - \tau_t(c_t + g_t)}{b_t} \tag{4}$$

$$q_t = \beta(c_t - hc_{t-1})E_t\frac{1-\Delta_{t+1}}{c_{t+1} - hc_t}. \tag{5}$$

The government bond demand equation is derived from the government budget constraint, while the bond supply equation is from the household's first-order condition. In terms of computation, the most time-consuming part is the loop iterations of the numerical integration.

$$\begin{aligned} E_t\frac{1-\Delta_{t+1}}{c_{t+1} - hc_t} &= (1-p_t)\int_{\varepsilon^A_{t+1}}\int_{\varepsilon^g_{t+1}}\int_{\varepsilon^\tau_{t+1}}\frac{1}{c_{t+1} - hc_t}|_{\text{no default}} \\ &+ p_t\int_{\varepsilon^A_{t+1}}\int_{\varepsilon^g_{t+1}}\int_{\varepsilon^\tau_{t+1}}\frac{1-\delta}{c_{t+1} - hc_t}|_{\text{default}} \end{aligned}$$

where the default probability at the beginning of next period, p_t, is determined by,

$$p_t = \frac{\exp(\eta_1 + \eta_2 s_t)}{1 + \exp(\eta_1 + \eta_2 s_t)} \tag{6}$$

with s_t being the debt-GDP ratio. The integration can be re-written as,

$$\int_{\varepsilon^A_{t+1}}\int_{\varepsilon^g_{t+1}}\int_{\varepsilon^\tau_{t+1}}\frac{1}{c_{t+1} - hc_t} = \int_{\varepsilon^A_{t+1}}\int_{\varepsilon^g_{t+1}}\int_{\varepsilon^\tau_{t+1}}\frac{1+\phi-\tau_{t+1}}{(1-\tau_{t+1})(A_{t+1} - g_{t+1} - hc_t)} \tag{7}$$

$$= \int_{\varepsilon^\tau_{t+1}}\frac{1+\phi-\tau_{t+1}}{1-\tau_{t+1}}\int_{\varepsilon^A_{t+1}}\int_{\varepsilon^g_{t+1}}\frac{1}{A_{t+1} - g_{t+1} - hc_t} \tag{8}$$

The logarithm utility function helps to reduce the 4-dimension integration into 1- and 2-dimension integrations.

After obtaining the decision rule for government bond, $f^b(\psi_t)$, the pricing rule for the government bond, $q_t = f^q(\psi_t)$, can be computed using the government budget constraint.

$$f^q(\psi_t) = \frac{b_t^d + z_t + g_t - \tau_t(c_t + g_t)}{f^b(\psi_t)}$$

The pricing rule is smooth over the state space.[1] Figure 1 illustrates the response of net interest rate, $r_t = 1/f^q(\psi_t) - 1$, to the current government liability under different productivity state, while keeping other state variables at their steady states. It shows that the interest

[1] In the simulation, the path for government debt may jump if default ever occurs. Section 2. provides more details on the simulations.

rate rises with the current government liability in a nonlinear way, as the household starts to demand a risk premium on the government bond when the default probability rises above zero. Also, the lower the productivity level, the higher the interest rate.

2. Dynamic Euler-equation Accuracy Test

We evaluate the accuracy of numerical solutions using the dynamic Euler-equation test proposed by Den Haan (2010). The idea is to compare a time series for consumption, c_t, that is constructed using the decision rule directly, with an alternative series, $\tilde{c}_t$, that is implied by the Euler equation and the budget constraint.

2.1 Constructing c_t

1. Start the simulation with a given initial state $(b_0, c_0, A_0, g_0, z_0, \tau_0)$.

2. Draw shocks on productivity, government spending, tax rate, transfers, and default probability for T periods: $\varepsilon_t^A \sim \mathcal{N}(0, \sigma_A^2)$, $\varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2)$, $\varepsilon_t^\tau \sim \mathcal{N}(0, \sigma_\tau^2)$, $\varepsilon_t^z \sim \mathcal{N}(0, \sigma_z^2)$, and $u_t^p \sim \mathcal{U}(0, 1)$ with $t = 1...T$.

3. Construct the current state $\psi_t = (b_t^d, c_{t-1}, A_t, u_t^g, z_t, u_t^\tau)$ using the variables from the previous period, $(b_{t-1}, c_{t-1}, A_{t-1}, g_{t-1}, z_{t-1}, \tau_{t-1})$:

   (a) Compute the debt-GDP ratio at the end of the previous period, s_{t-1}, and also the default probability, p_{t-1}:

$$
\begin{aligned}
s_{t-1} &= \frac{b_{t-1}}{c_{t-1} + g_{t-1}} \\
p_{t-1} &= \frac{\exp\left(\eta_1 + \eta_2 s_{t-1}\right)}{1 + \exp\left(\eta_1 + \eta_2 s_{t-1}\right)}
\end{aligned}
$$

   If the default probability, p_{t-1}, is higher than the random shock drawn from the uniform distribution, u_t^p, then the government defaults and $\Delta_t = \delta$; otherwise the government pays the debt in full amount and $\Delta_t = 0$.

(b) Construct the current state, ψ_t:

$$\begin{aligned}
b_t^d &= b_{t-1}(1-\Delta_t) \\
u_t^\tau &= (1-\rho^\tau)\tau + \rho^\tau \tau_{t-1} + \varepsilon_t^\tau \\
u_t^g &= (1-\rho^g)g + \rho^g g_{t-1} + \varepsilon_t^g \\
z_t &= (1-\rho^z)z + \rho^z z_{t-1} + \varepsilon_t^z \\
A_t &= (1-\rho^A)A + \rho^A A_{t-1} + \varepsilon_t^A
\end{aligned}$$

4. Update the tax rate and the government spending at time t:

$$\begin{aligned}
\tau_t &= u_t^\tau + \gamma^\tau \left(b_t^d - b\right) \\
g_t &= u_t^g + \gamma^g \left(b_t^d - b\right)
\end{aligned}$$

5. Compute the consumption at time t:

$$c_t = \frac{\phi h c_{t-1} + (A_t - g_t)(1-\tau_t)}{1+\phi-\tau_t}$$

6. Use the decision rule to update the end-of-period debt, $b_t = f^b(\psi_t)$.

7. With $(b_t, c_t, A_t, g_t, z_t, \tau_t)$, repeat step 3 to 6 until $t = T$.

2.2 Constructing $\tilde{c}_t$

The procedure of constructing $\tilde{c}_t$ is the same as above, except that the end-of-period debt, $\tilde{b}_t$, is updated using the household Euler equation and the government budget constraint, instead of directly using the decision rule $f^b(.)$. The idea is, if the solution is accurate, then the error from evaluating the expectation term in the Euler equation should be small enough so that the paths of $\tilde{b}_t$ and $\tilde{c}_t$ are close to those of b_t and c_t .

1. Start the chain with the same initial state as in section 2.1, $\tilde{b}_0 = b_0, \tilde{c}_0 = c_0, \tilde{A}_0 = A_0, \tilde{g}_0 = g_0, \tilde{z}_0 = z_0, \tilde{\tau}_0 = \tau_0$.

2. Use the same shocks as in section 2.1, $\left(\varepsilon_t^A, \varepsilon_t^g, \varepsilon_t^\tau, \varepsilon_t^z, u_t^p\right)$ $(t = 1, ..., T)$.

3. Construct the current state $\tilde{\psi}_t = (\tilde{b^d}_t, \tilde{c}_{t-1}, \tilde{A}_t, \tilde{u^g}_t, \tilde{z}_t, \tilde{u^\tau}_t)$ in the same way as step 3 in section 2.1.

(a) Compute the default probability, $\tilde{p}_{t-1}$. If it is higher than the random shock drawn from the uniform distribution, u_t^p, then the government defaults and $\tilde{\Delta}_t = \delta$; otherwise the government pays the debt in full amount and $\tilde{\Delta}_t = 0$.

(b) Construct the current state, $\tilde{\psi}_t$.

4. Update the tax rate, $\tilde{\tau}_t$, and the government spending, $\tilde{g}_t$, in the same way as step 4 in section 2.1.

5. Compute the consumption, $\tilde{c}_t$, in the same way as step 5 in section 2.1.

6. Update the end-of-period government debt, $\tilde{b}_t$, using the household Euler equation and the government budget constraint:

(a) Use the decision rule to construct an intermediate variable, $\hat{b}_t = f^b(\tilde{\psi}_t)$;

(b) For all exogenous shocks on the grid, $\left(\hat{\varepsilon}^A_{t+1}, \hat{\varepsilon}^g_{t+1}, \hat{\varepsilon}^\tau_{t+1}, \hat{\varepsilon}^z_{t+1}, \hat{u^p}_{t+1}\right)$, use the intermediate debt variable, $\hat{b}_t$, to construct the state for next period, $\hat{\psi}_{t+1}$, and the intermediate default rate, $\hat{\Delta}_{t+1}$. The procedure is similar to step 3.

(c) Compute the intermediate consumption, $\hat{c}_{t+1}$, in a similar way as step 5.

(d) Compute the intermediate bond price, $\hat{q}_t$, by evaluating the expectation term in the household Euler equation,

$$\begin{aligned} \hat{q}_t &= E_t \left(1 - \hat{\Delta}_{t+1}\right) \frac{\tilde{c}_t - h\tilde{c}_{t-1}}{\hat{c}_{t+1} - h\tilde{c}_t} && (9) \\ &= \int_{\hat{\varepsilon}^A_{t+1}} \int_{\hat{\varepsilon}^g_{t+1}} \int_{\hat{\varepsilon}^\tau_{t+1}} \int_{\hat{u}^p_{t+1}} \left(1 - \hat{\Delta}_{t+1}\right) \frac{\tilde{c}_t - h\tilde{c}_{t-1}}{\hat{c}_{t+1} - h\tilde{c}_t} && (10) \end{aligned}$$

(e) Compute the end-of-period debt, $\tilde{b}_t$, using the government budget constraint,

$$\tilde{b}_t = \frac{\tilde{b}^d_t + \tilde{z}_t + \tilde{g}_t - \tilde{\tau}_t(\tilde{c}_t + \tilde{g}_t)}{\hat{q}_t} \tag{11}$$

7. With $\left(\tilde{b}_t, \tilde{c}_t, \tilde{A}_t, \tilde{g}_t, \tilde{z}_t, \tilde{\tau}_t\right)$, repeat step 3 to 6 until $t = T$.

2.3 Dynamic Euler-Equation Error

The dynamic Euler-equation error is then given by,

$$100 \left| \frac{c_t - \tilde{c}_t}{c_t} \right|. \tag{12}$$

We solve the model by setting the estimate parameters to their posterior median in the baseline case (i.e. the default rate δ is 0.3, and the measurement error is 20% of the standard deviations of the corresponding observable variables), and simulate the time series c_t and $\tilde{c}_t$ for 500 periods. The errors are low, with the mean error being 0.08%, which implies that households make a 8 cent mistake for each $100 dollars spent.[2]

3. Estimation Results

Tables 1 and 2 report estimation details for the two nonlinear model specifications in the main text, namely calibrations with $\delta^A = 0.3$ and $\delta^A = 0.2$. The tables list the estimated posterior means, medians, 90% credible intervals, and the Geweke Separated Partial Means (GSPM) test p-values.[3] Figures 4 and 5 plot priors against posterior distributions. The plots suggest that the data contain information for identifying most parameters, with the posteriors of $\sigma_{z,p}$ and $\gamma^{g,L}$ being most similar to the priors. Trace plots are given in figures 11 and 12.

Table 4 reports estimation details for the log-linearized model. Figure 7 plots priors against the posterior distributions while figure 14 gives trace plots.

4. Sensitivity Analysis

We investigate the robustness of our parameter estimates under several alternative model specifications. The results of these robustness checks are summarized in Table 5.[4] For all cases, unless otherwise noted, the calibration, priors, and estimation procedure is the same as that for the model specification with $\delta^A = 0.30$.

4.1 Higher default rate

To determine how sensitive our estimates and inferences are to a higher default rate calibration, we estimate the model for an alternative calibration with $\delta^A = 0.45$. The results are shown in the '$\delta^A = 0.45$' columns of Table 5. As previously noted, increasing the default

[2] A longer simulation is more likely to generate larger errors. Therefore, the errors generated by a simulation of 500 periods sets an upper bound for our model that uses a short data sample of 40 periods.

[3] The GSPM test determines whether the mean from the first 20% of the MCMC draws is identical to the mean of the final 50% of the draws. A Z-test of the hypothesis of equality of the two means is carried out and the corresponding chi-squared marginal significance is calculated (see Geweke (2005), pages 149-150, for more details). The reported p-values are based on assuming 4% tapered autocorrelation.

[4] For these estimations, we sample 500,000 draws from the posterior distribution and discard the first 250,000 draws. Posterior analysis is conducted using every 25th draw from the chains. The acceptance rates for these specifications varies from 22-49%.

rate implies higher estimates of $\tilde{s}$. The posterior median is raised to 1.61 from 1.56 with $\delta^A = 0.30$. Other estimated parameters are trivially affected by the alternative calibration. More detailed analysis of the posterior estimates are reported in table 3. Figure 6 plots priors against the posterior distributions while figure 13 gives trace plots.

Although the estimates imply the debt-to-GDP ratio associated with any given probability of default is higher, the implied probability of default by 2010Q4 is between 3-5% and by 2011Q4 is between 45-70%, as seen in figure 2. Figure 3 plots the data versus fitted and forecast values for the Greek real interest rate. The forecast bands from the $\delta^A = 0.45$ calibration are dashed magenta lines whereas the bands from the $\delta^A = 0.30$ calibration are solid blue lines. The surge in the real interest rate in Greece in 2011 is still well within forecast bands of the estimated model with a higher default rate calibration.

4.2 Varying Measurement Error Calibration

For the baseline estimation, the standard deviation of each measurement error was fixed to be 20% of the standard deviation of the respective observable variable. To determine how sensitive our estimates and inferences are to measurement error, we estimate the model for two alternative cases where each measurement error was fixed to be 25 and 30% of the standard deviation of the respective observable variable. The results are shown in the 'Meas. Err. 30%' and 'Meas. Err. 25%' columns of table 5. Most of the 90% intervals of the estimated parameters are similar to the benchmark case (the '$\delta^A = 0.3$' column). Figures 8 and 9 plot priors against the posterior distributions while figures 15 and 16 give trace plots.

In addition, the 'Meas. Err. 30%' and 'Meas. Err. 25%' rows of table 6 lists the mean absolute values of measurement errors and the standard deviations of the estimated measurement errors relative to the standard deviation of observables (i.e. relative standard deviation). For the 25% relative standard deviation case, the actual estimated relative standard deviations are still less than 20% and comparable to the benchmark 20% case. The actual estimated relative standard deviations are slightly larger for the 30% relative standard deviation case, with both the actual estimated relative standard deviations of the measurement errors for the tax revenue-to-GDP ratio and the real interest rate being above 20%.

4.3 Varying $\gamma^{\tau,L}$, $\gamma^{g,L}$ priors

We check whether our results are sensitive to the prior specifications by estimating a version of the model where both $\gamma^{\tau,L}$ and $\gamma^{g,L}$ have normal priors centered at 0.5 with a standard deviation of 0.18. The priors were chosen to allow a positive probability mass for negative parameter values, which allows the posterior estimates to potentially encompass zero, while still maintaining a large prior probability mass lying in the determinacy region of the parameter space. We focus on these two parameters as they are essential for the debt dynamics in the model, and as seen in figure 4 the posteriors are similar to the priors, particularly for $\gamma^{g,L}$. The results are shown in the 'Alt. $\gamma^{\tau,L}$, $\gamma^{g,L}$ priors' column of Table 5. The estimates for $\gamma^{\tau,L}$ are similar to the benchmark specification, although the 90% credible interval is more diffuse. The posterior for $\gamma^{g,L}$ varies considerably from the benchmark specification, although the posterior median of $\gamma^{g,L}$ is still quantitatively larger than the estimate of $\gamma^{\tau,L}$. The results confirm $\gamma^{g,L}$ is not well identified from the data. Figure 10 plots priors against posterior distributions while figure 17 provides trace plots.

Table 1: Posterior Estimates $\delta^A = 0.3$. Reports the posterior mean, median, 90% credible interval, and the p-value for Geweke's Separated Partial Means test. Final acceptance rate: 15%. 1,000,000 draws were made, with the first 500,000 used as a burn-in period and every 50th thinned.

Parameters	**Posterior**				
	mean	median	5 %	95%	Geweke Chi-Sq p
h	0.14	0.13	0.04	0.28	0.06
$\tilde{s}$	1.56	1.56	1.53	1.59	0.04
$\gamma^{\tau,L}$	0.23	0.22	0.08	0.45	0.81
$\gamma^{g,L}$	1.20	1.18	0.77	1.71	0.01
ρ^a	0.97	0.97	0.96	0.98	0.01
ρ^g	0.94	0.94	0.90	0.97	0.11
ρ^z	0.57	0.58	0.29	0.82	0.43
ρ^τ	0.94	0.94	0.92	0.96	0.97
$\sigma_{a,p}$	0.022	0.022	0.017	0.028	0.12
$\sigma_{g,p}$	0.041	0.041	0.034	0.049	0.14
$\sigma_{z,p}$	0.49	0.49	0.35	0.63	0.0
$\sigma_{\tau,p}$	0.026	0.026	0.022	0.032	0.97

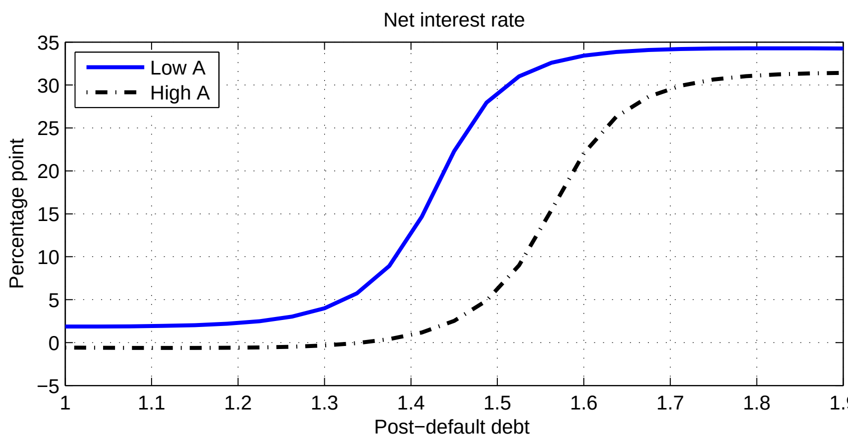

Figure 1: The pricing rule for the net interest rate at different levels of productivity.

Table 2: Posterior Estimates for $\delta^A = 0.2$. Reports the posterior mean, median, 90% credible interval, and the p-value for Geweke's Separated Partial Means test. Final acceptance rate: 15%. 1,000,000 draws were made, with the first 500,000 used as a burn-in period and every 50th thinned.

Parameters	**Posterior**				
	mean	median	5 %	95%	Geweke Chi-Sq p
h	0.11	0.11	0.04	0.21	0.51
$\tilde{s}$	1.53	1.53	1.50	1.56	0.06
$\gamma^{\tau,L}$	0.22	0.20	0.08	0.42	0.33
$\gamma^{g,L}$	1.18	1.20	0.73	1.62	0.13
ρ^a	0.97	0.97	0.96	0.98	0.11
ρ^g	0.94	0.94	0.90	0.98	0.0
ρ^z	0.45	0.44	0.23	0.75	0.03
ρ^τ	0.94	0.94	0.91	0.96	0.49
$\sigma_{a,p}$	0.022	0.022	0.017	0.027	0.61
$\sigma_{g,p}$	0.039	0.038	0.032	0.047	0.0
$\sigma_{z,p}$	0.55	0.54	0.46	0.64	0.93
$\sigma_{\tau,p}$	0.026	0.025	0.021	0.032	0.50

Table 3: Posterior Estimates for $\delta^A = 0.45$. Reports the posterior mean, median, 90% credible interval, and the p-value for Geweke's Separated Partial Means test. Final acceptance rate: 31%. 500,000 draws were made, with the first 250,000 used as a burn-in period and every 25th thinned.

Parameters	**Posterior**				
	mean	median	5 %	95%	Geweke Chi-Sq p
h	0.12	0.11	0.04	0.25	0.26
$\tilde{s}$	1.61	1.61	1.58	1.65	0.12
$\gamma^{\tau,L}$	0.26	0.23	0.08	0.55	0.07
$\gamma^{g,L}$	1.10	1.07	0.67	1.66	0.06
ρ^a	0.97 0.97	0.96	0.98	0.08	
ρ^g	0.92	0.96	0.92	0.87	0.15
ρ^z	0.54	0.55	0.29	0.79	0.60
ρ^τ	0.94	0.94	0.91	0.97	0.01
$\sigma_{a,p}$	0.016	0.016	0.012	0.21	0.12
$\sigma_{g,p}$	0.036	0.036	0.028	0.046	0.77
$\sigma_{z,p}$	0.50	0.49	0.37	0.66	0.69
$\sigma_{\tau,p}$	0.019	0.019	0.014	0.024	0.14

Table 4: Posterior Estimates for log-linearized model without default. Reports the posterior mean, median, 90% credible interval, and the p-value for Geweke's Separated Partial Means test. Final acceptance rate: 39%. 1,000,000 draws were made, with the first 500,000 used as a burn-in period and every 50th thinned.

Parameters	**Posterior**				
	mean	median	5 %	95%	Geweke Chi-Sq p
h	0.72	0.73	0.59	0.83	0.62
$\gamma^{\tau,L}$	0.64	0.62	0.30	1.05	0.07
$\gamma^{g,L}$	1.10	1.08	0.66	1.62	0.30
ρ^a	0.95 0.95	0.94	0.96	0.39	
ρ^g	0.95	0.96	0.91	0.98	0.79
ρ^z	0.62	0.63	0.38	0.82	0.93
ρ^τ	0.93	0.93	0.90	0.95	0.92
$\sigma_{a,p}$	0.019	0.019	0.015	0.024	0.66
$\sigma_{g,p}$	0.042	0.041	0.033	0.051	0.08
$\sigma_{z,p}$	0.51	0.50	0.38	0.66	0.78
$\sigma_{\tau,p}$	0.022	0.022	0.018	0.027	0.10

Table 5: **Sensitivity analysis.** The table reports posterior medians and 90% credible intervals (in brackets) for various models.

Parameters	**Models**					
	$\delta^A = 0.3$	$\delta^A = 0.2$	$\delta^A = 0.45$	Meas. Err. 30%	Meas. Err. 25%	Alt. $\gamma^{\tau,L}$, $\gamma^{g,L}$ priors
h	0.13	0.11	0.12	0.13	0.11	0.16
	[0.04, 0.28]	[0.04, 0.21]	[0.04, 0.27]	[0.04, 32]	[0.04, 0.25]	[0.05, 0.39]
$\tilde{s}$	1.56	1.53	1.61	1.58	1.58	1.58
	[1.53, 1.59]	[1.50, 1.56]	[1.58, 1.66]	[1.54, 1.62]	[1.55, 1.61]	[1.53, 1.63]
$\gamma^{\tau,L}$	0.22	0.20	0.21	0.24	0.24	0.33
	[0.08, 0.45]	[0.08, 0.42]	[0.08, 0.46]	[0.09, 0.49]	[0.09, 0.54]	[0.02, 0.72]
$\gamma^{g,L}$	1.18	1.20	1.13	1.10	1.10	0.53
	[0.77, 1.71]	[0.73, 1.62]	[0.72, 1.75]	[0.72, 1.67]	[0.71, 1.63]	[0.25, 0.81]
ρ^a	0.97	0.97	0.97	0.97	0.97	0.96
	[0.96, 0.98]	[0.96, 0.98]	[0.96, 0.98]	[0.95, 0.98]	[0.96, 0.98]	[0.95, 0.98]
ρ^g	0.94	0.94	0.92	0.92	0.93	0.89
	[0.90, 0.97]	[0.90, 0.98]	[0.87,0.96]	[0.87, 0.96]	[0.88, 0.97]	[0.82, 0.95]
ρ^z	0.58	0.44	0.57	0.61	0.58	0.57
	[0.29, 0.82]	[0.23, 0.75]	[0.29, 0.81]	[0.34, 0.82]	[0.28, 0.84]	[0.30, 0.80]
ρ^τ	0.94	0.94	0.94	0.94	0.94	0.94
	[0.92, 0.96]	[0.91, 0.96]	[0.91, 0.96]	[0.90, 0.96]	[0.91, 0.97]	[0.89, 0.97]
$\sigma_{a,p}$	0.022	0.022	0.016	0.016	0.018	0.014
	[0.017, 0.028]	[0.017, 0.027]	[0.012, 0.21]	[0.012, 0.021]	[0.013, 0.024]	[0.011, 0.018]
$\sigma_{g,p}$	0.041	0.038	0.036	0.036	0.038	0.034
	[0.034, 0.049]	[0.032, 0.047]	[0.029, 0.046]	[0.029, 0.045]	[0.030, 0.048]	[0.027, 0.044]
$\sigma_{z,p}$	0.49	0.54	0.48	0.48	0.47	0.47
	[0.35, 0.63]	[0.46, 0.64]	[0.37, 0.63]	[0.35, 0.64]	[0.35, 0.67]	[0.35, 0.65]
$\sigma_{\tau,p}$	0.026	0.025	0.019	0.019	0.023	0.014
	[0.022, 0.032]	[0.021, 0.032]	[0.014, 0.025]	[0.014, 0.025]	[0.018, 0.030]	[0.010, 0.019]

Table 6: Smoothed estimates of measurement error.

		$\frac{b_t}{y_t}$	$\frac{g_t}{y_t}$	$\frac{T_t}{y_t}$	y_t	R_t
Nonlinear $\delta^A = 0.3$	mean absolute value	0.01	0.004	0.002	0.005	0.001
	relative standard deviation	0.12	0.09	0.11	0.14	0.24
Linear	mean absolute value	0.01	0.006	0.002	0.01	0.0003
	relative standard deviation	0.09	0.13	0.08	0.29	0.10
Meas. Err. 25%	mean absolute value	0.01	0.005	0.002	0.005	0.001
	relative standard deviation	0.12	0.12	0.13	0.16	0.27
Meas. Err. 30%	mean absolute value	0.01	0.01	0.004	0.006	0.001
	relative standard deviation	0.13	0.16	0.22	0.18	0.26

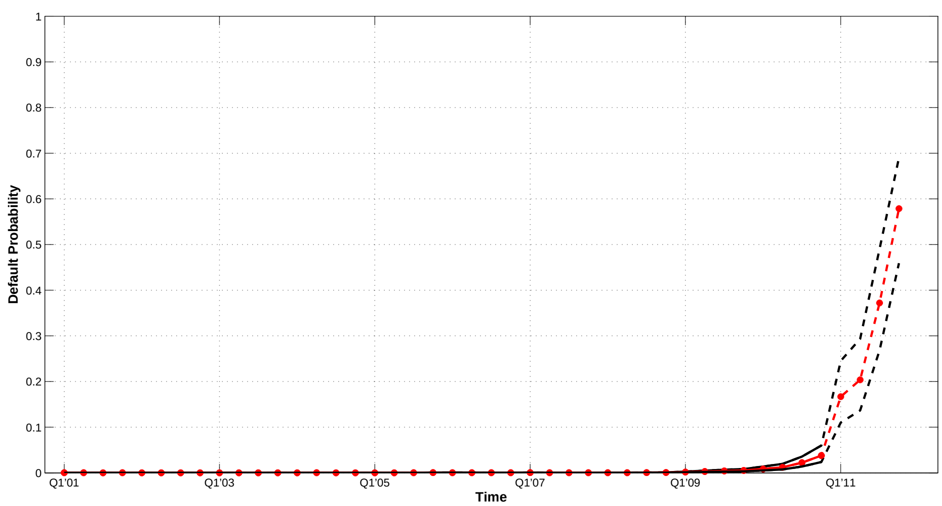

Figure 2: Model-implied sovereign default probabilities for Greece for calibration with $\delta^A = 0.45$. Solid lines denote the median and 90% confidence interval probabilities for in-sample debt-to-GDP ratios. Dashed lines denote the median and 90% confidence interval probabilities for out-of-sample debt-to-GDP ratios.

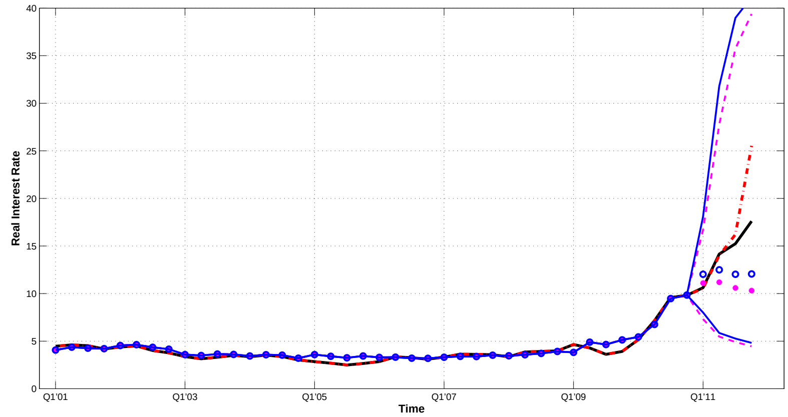

Figure 3: Data versus fitted and forecast values for the Greek interest rate. The median (blue, circled line) and 90% interval (blue, solid lines) of the interest rate forecasts for 2011 are calculated based on the posterior median parameter estimates from the calibration with $\delta^A = 0.3$. The median (magenta, starred line) and 90% interval (magenta, dashed lines) of the interest rate forecasts for 2011 are calculated based on the posterior median parameter estimates from the calibration with $\delta^A = 0.45$. The black solid line shows the BIS data, and red dotted-dashed line shows the Bloomberg data.

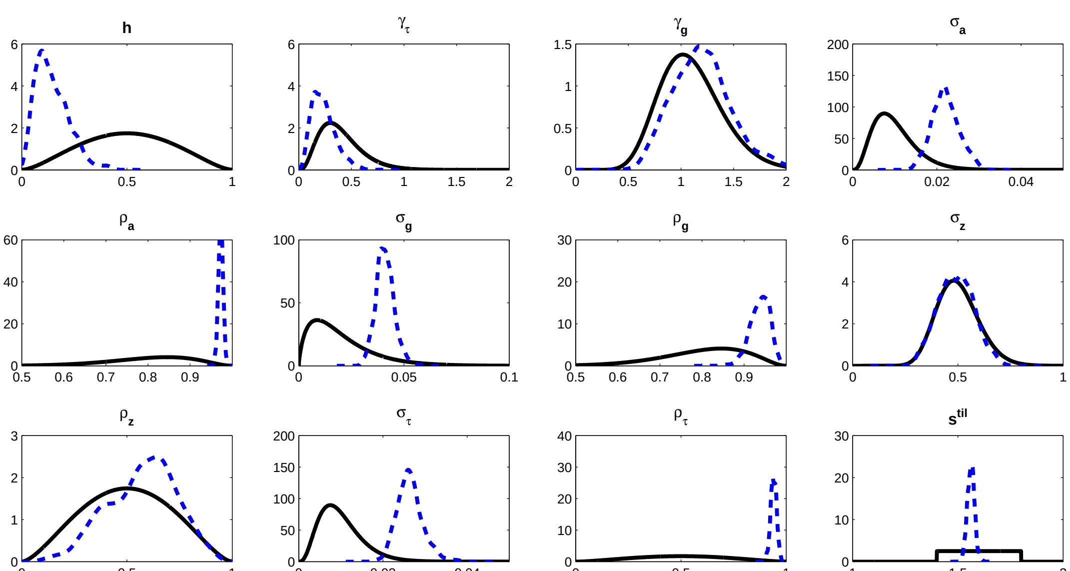

Figure 4: Prior distributions (black solid lines) versus posterior distributions (blue dashed lines) for the nonlinear model with annualized default rate of 30%, $\delta^A = 0.3$.

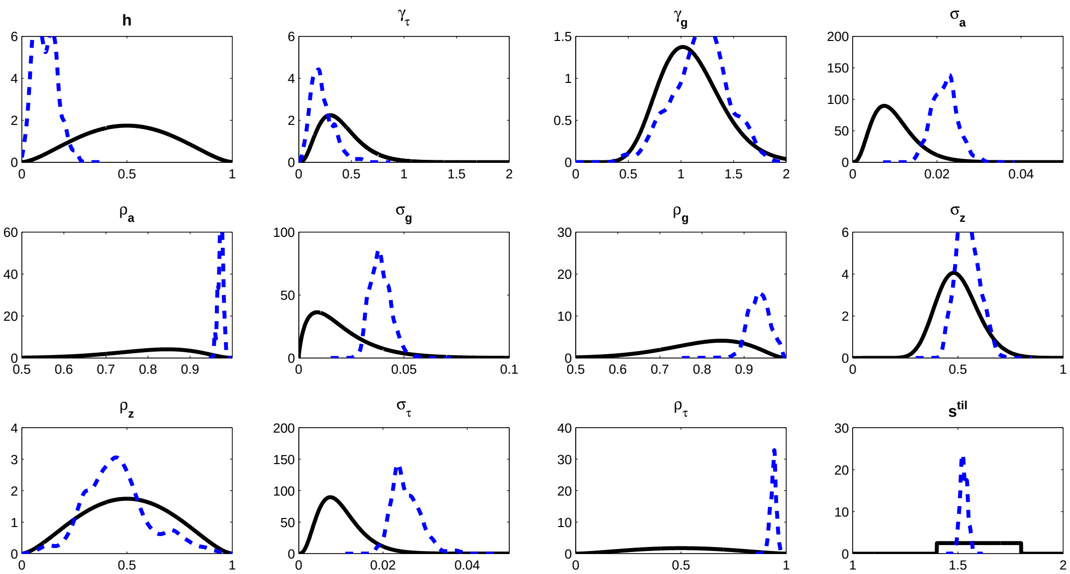

Figure 5: Prior distributions (black solid lines) versus posterior distributions (blue dashed lines) for the nonlinear model with annualized default rate of 20%, $\delta^A = 0.2$.

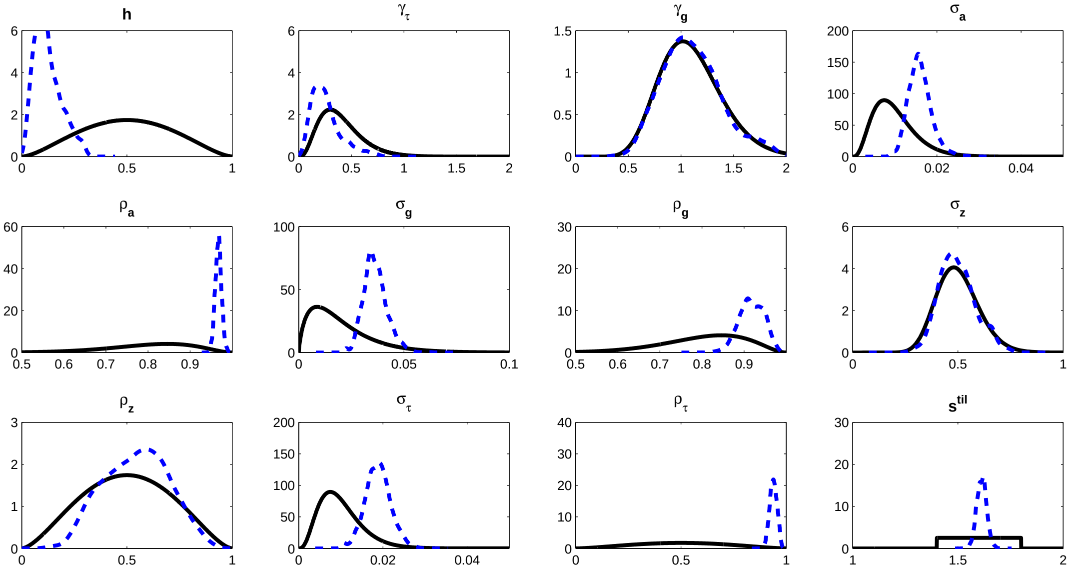

Figure 6: Prior distributions (black solid lines) versus posterior distributions (blue dashed lines) for the nonlinear model with annualized default rate of 45%, $\delta^A = 0.45$.

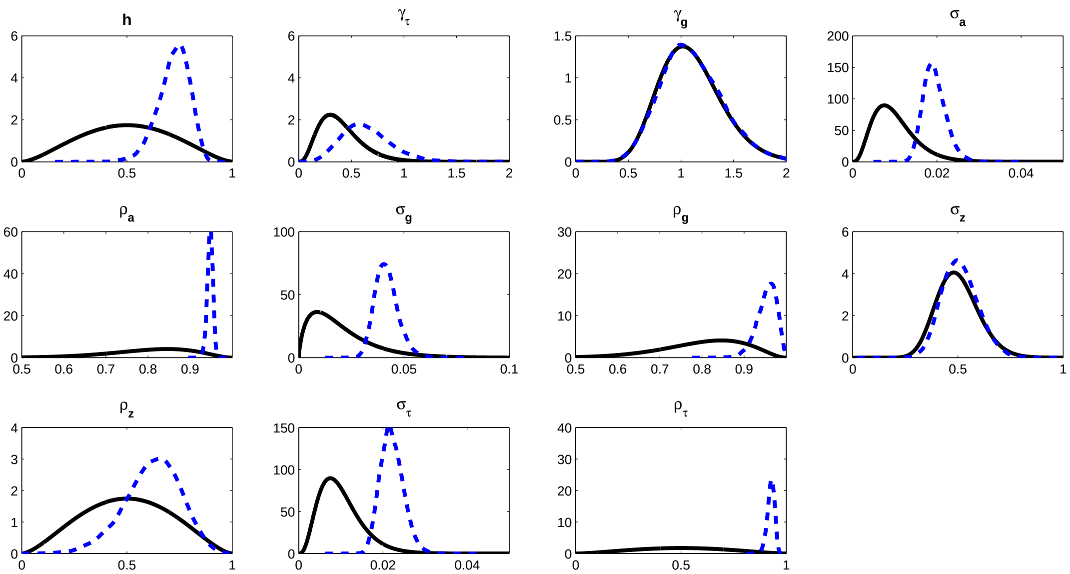

Figure 7: Prior distributions (black solid lines) versus posterior distributions (blue dashed lines) for the log-linearized model without default.

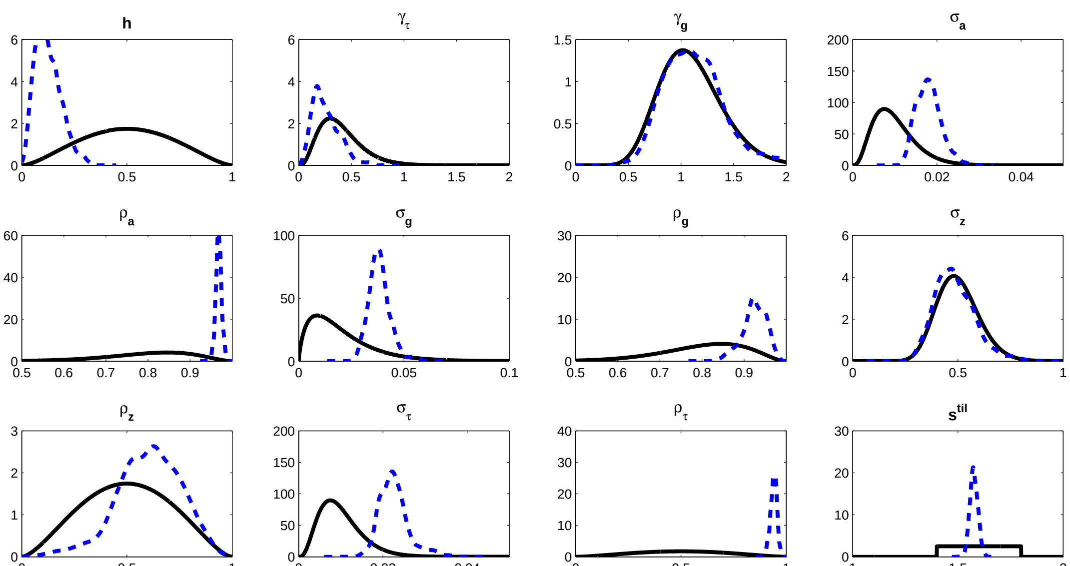

Figure 8: Prior distributions (black solid lines) versus posterior distributions (blue dashed lines) for the nonlinear model with standard deviations of measurement error set at 25% of the standard deviations of observables.

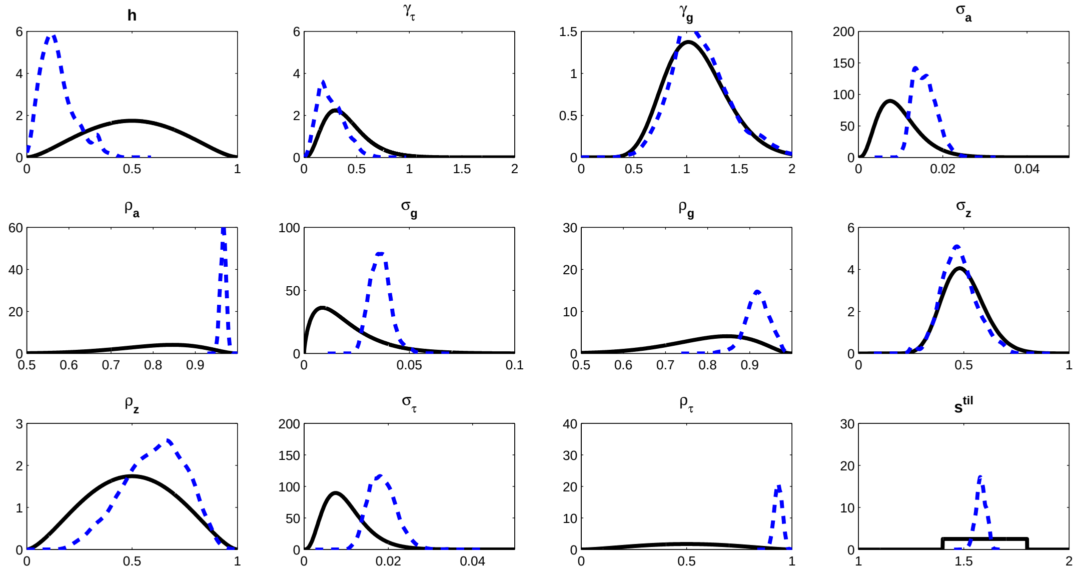

Figure 9: Prior distributions (black solid lines) versus posterior distributions (blue dashed lines) for the nonlinear model with standard deviations of measurement error set at 30% of the standard deviations of observables.

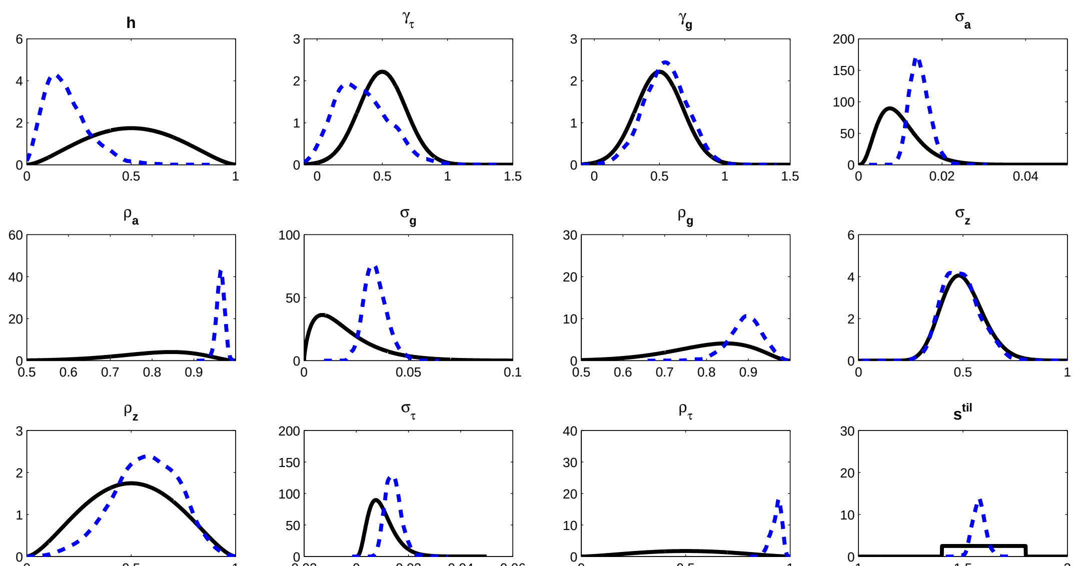

Figure 10: Prior distributions (black solid lines) versus posterior distributions (blue dashed lines) for the nonlinear model where $\gamma^{\tau,L}$ and $\gamma^{g,L}$ are distributed N(0.5, 0.18).

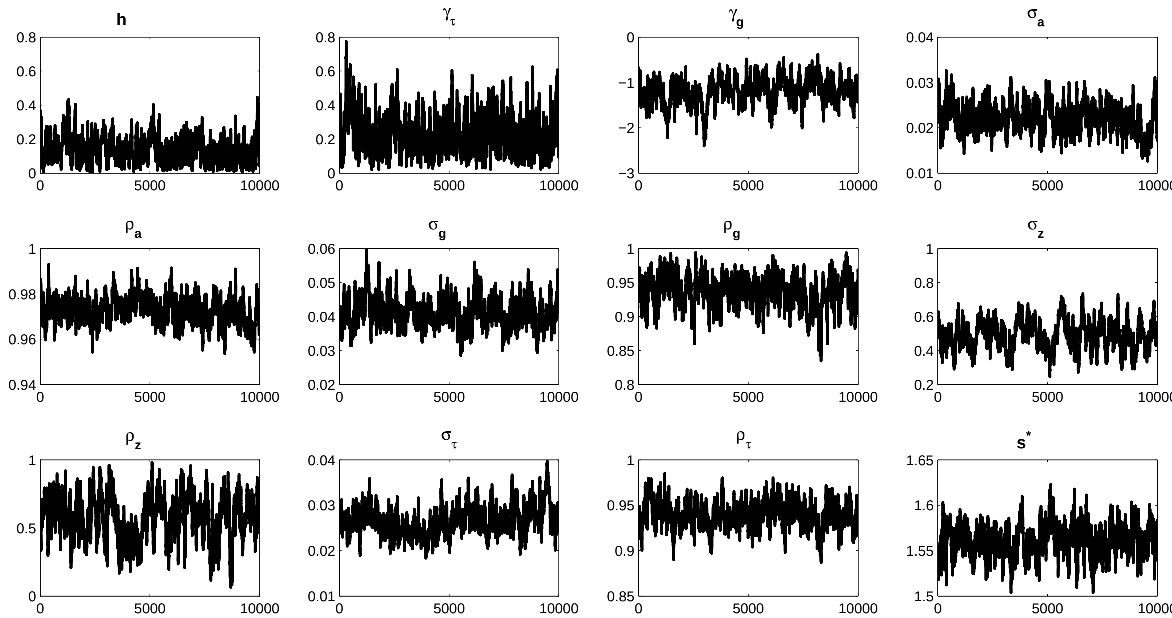

Figure 11: Trace plots for model specification with $\delta^A = 0.3$.

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Den Haan, W. J. (2010): "Comparison of Solutions to the Incomplete Markets Model with Aggregate Uncertainty," *Journal of Economic Dynamics and Control*, 34(1), 4–27.

Geweke, J. (2005): *Contemporary Bayesian Econometrics and Statistics*. John Wiley and Sons, Inc., Hoboken, NJ.

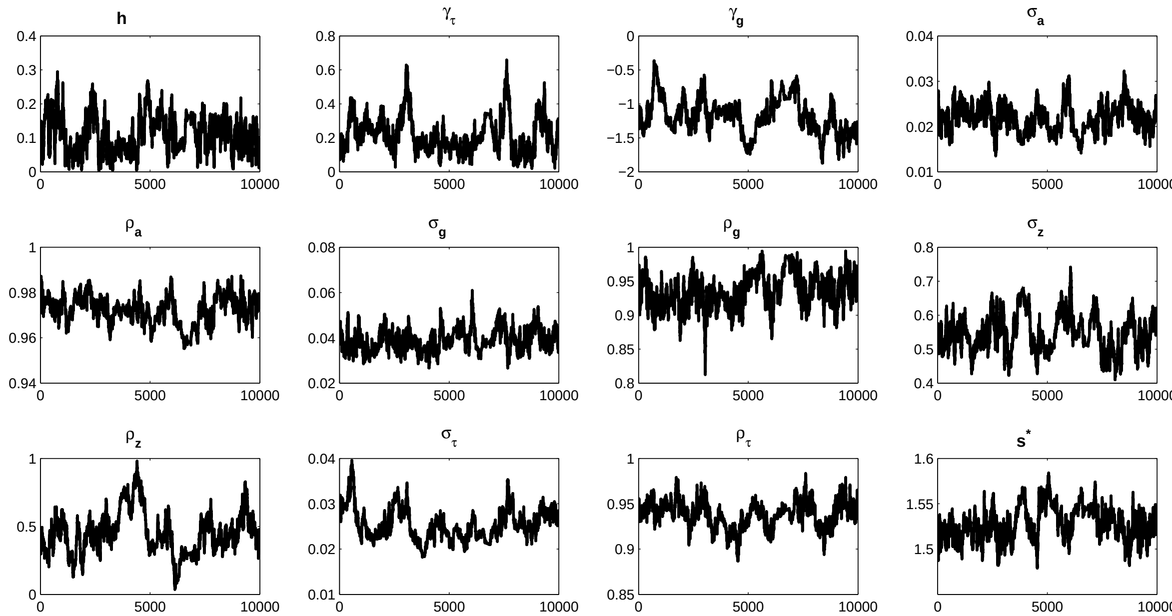

Figure 12: Trace plots for model specification with $\delta^A = 0.2$.

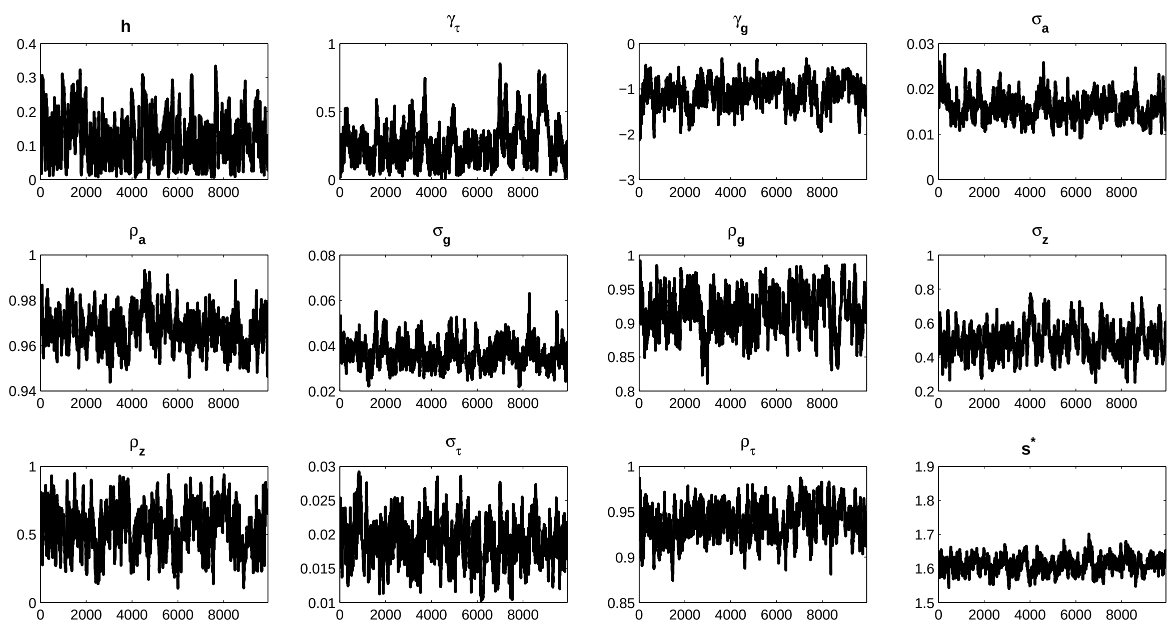

Figure 13: Trace plots for model specification with $\delta^A = 0.45$.

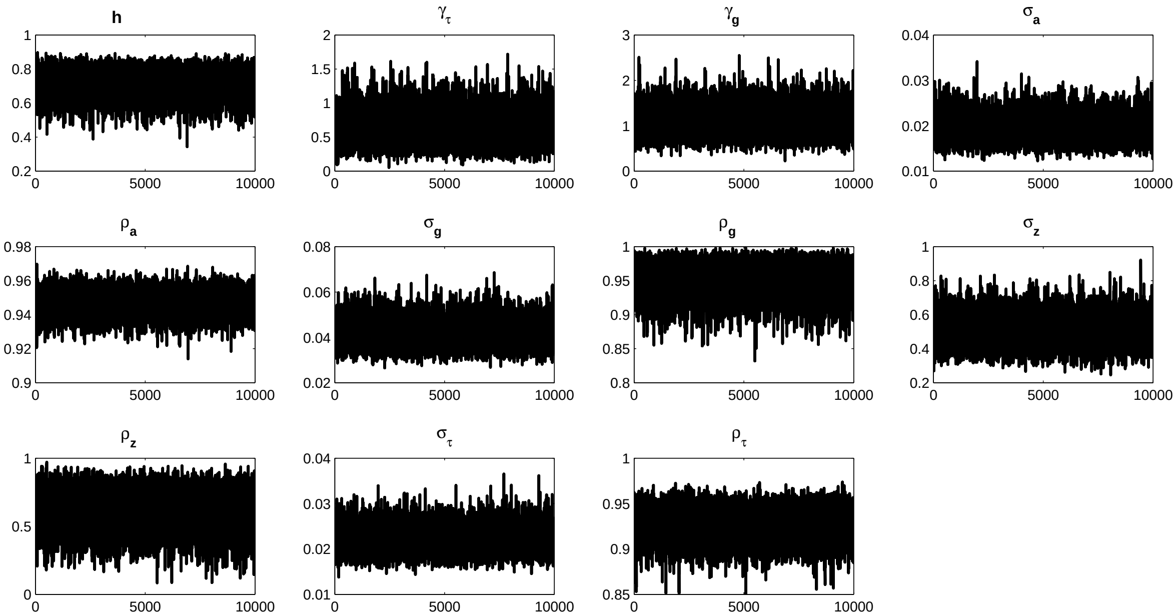

Figure 14: Trace plots for log-linearized model without default.

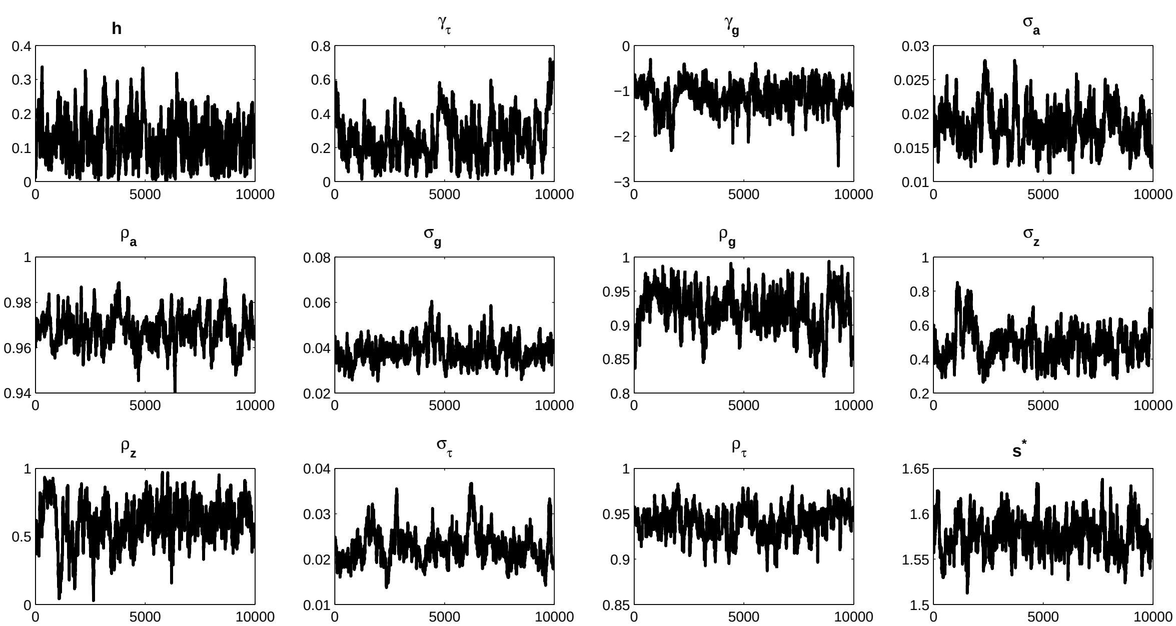

Figure 15: Trace plots for model specification with 25% relative standard deviation measurement error.

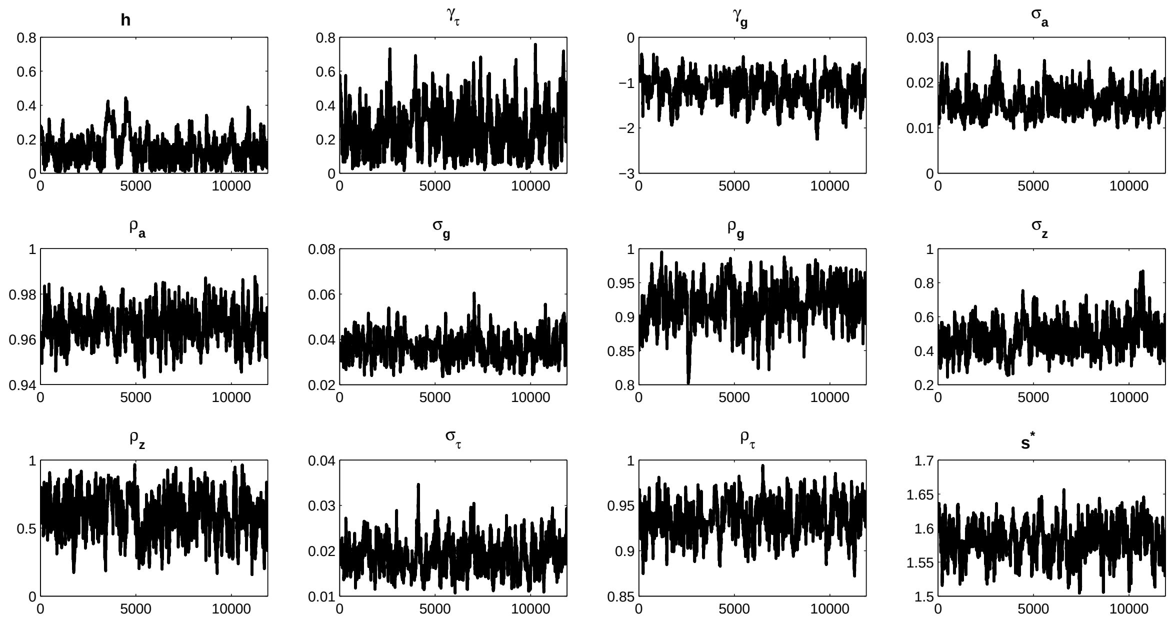

Figure 16: Trace plots for model specification with 30% relative standard deviation measurement error.

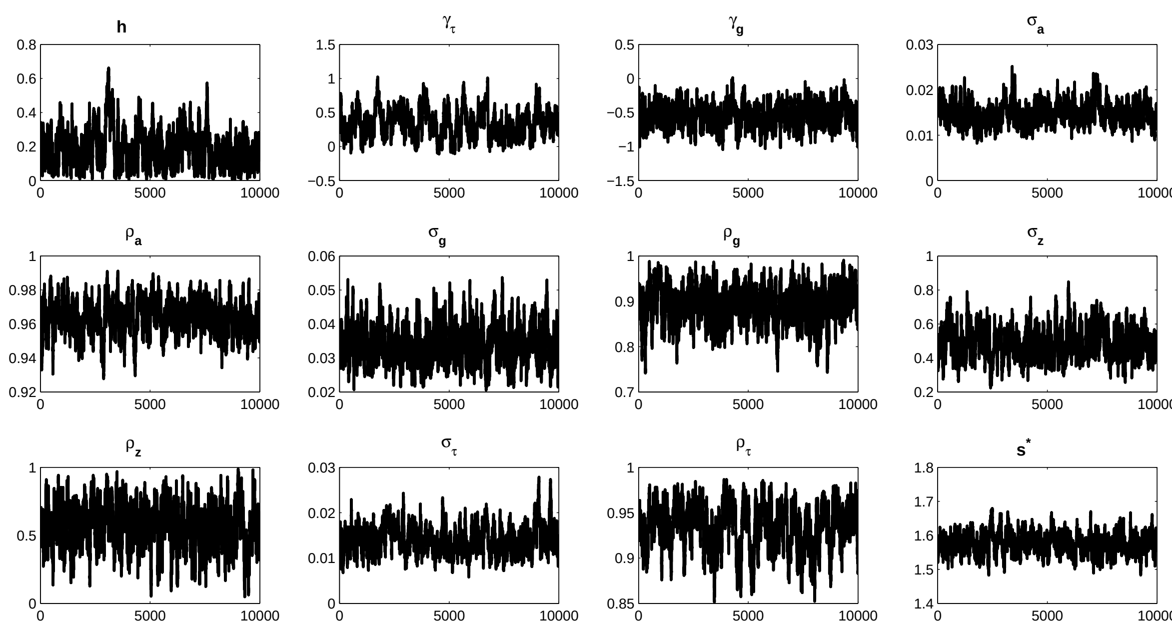

Figure 17: Trace plots for model specification where $\gamma^{\tau,L}$ and $\gamma^{g,L}$ are distributed N(0.5, 0.18).