List of Tables and Figures of "Estimation and Inference for Spatial Models with Heterogeneous Coefficients: An Application to U.S. House Prices"

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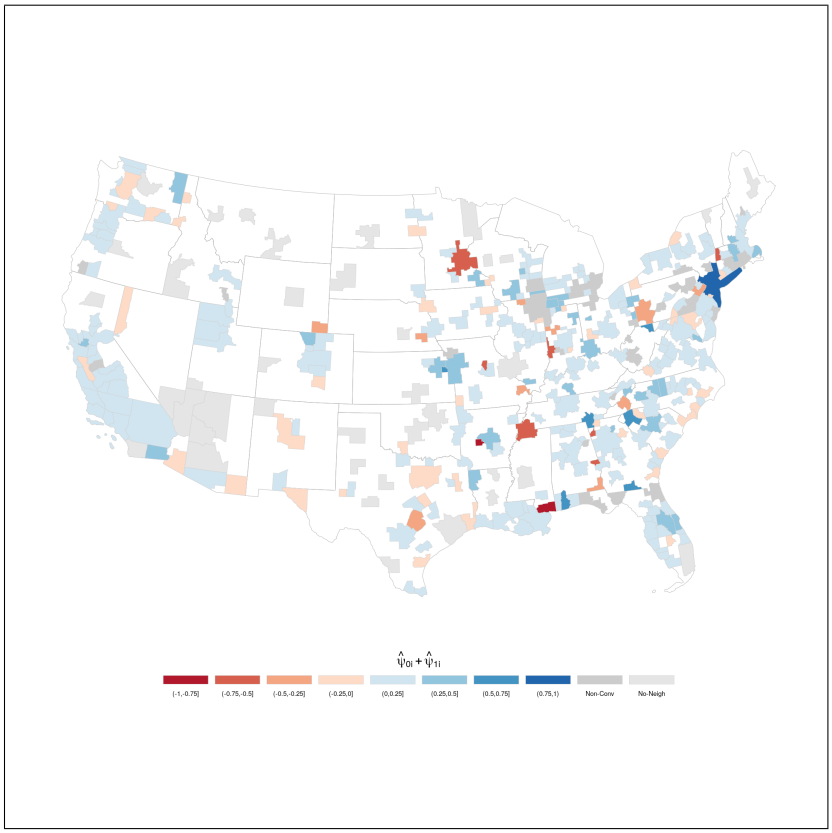
Figure 1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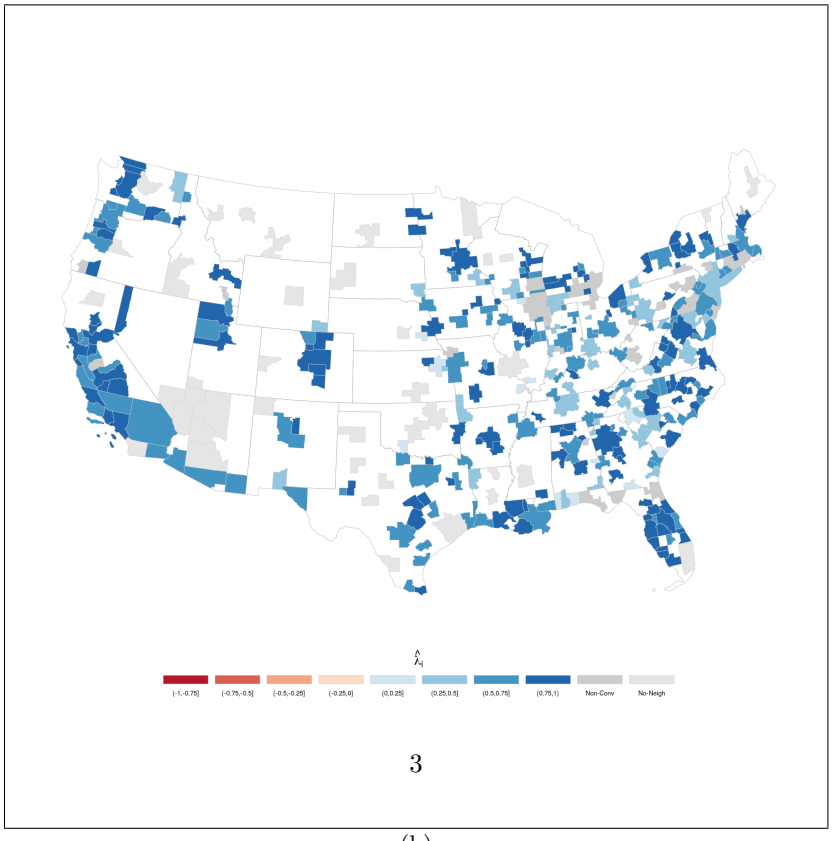
(-0.25, 0]
(0, 0.25]
(0.25, 0.5]
(0.5, 0.75]
(0.75, 1]
Non-Conv
No-Neigh

(a)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(-0.5, -0.25]
(-0.25, 0]
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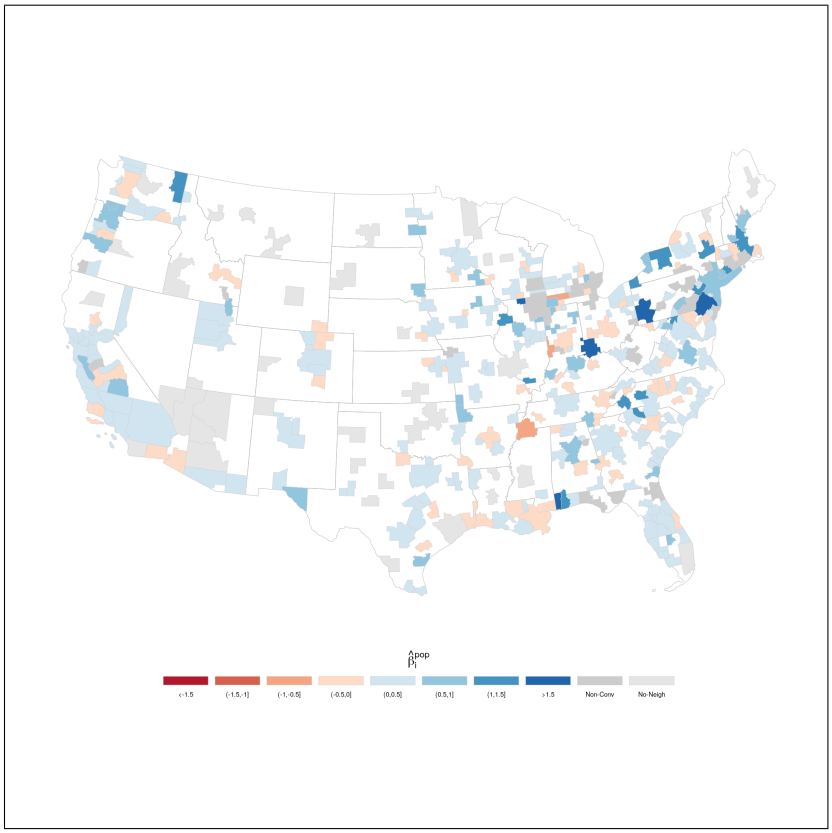
(0.25, 0.5]
(0.5, 0.75]
(0.75, 1]
Non-Conv
No-Neigh

(b)

Figure 2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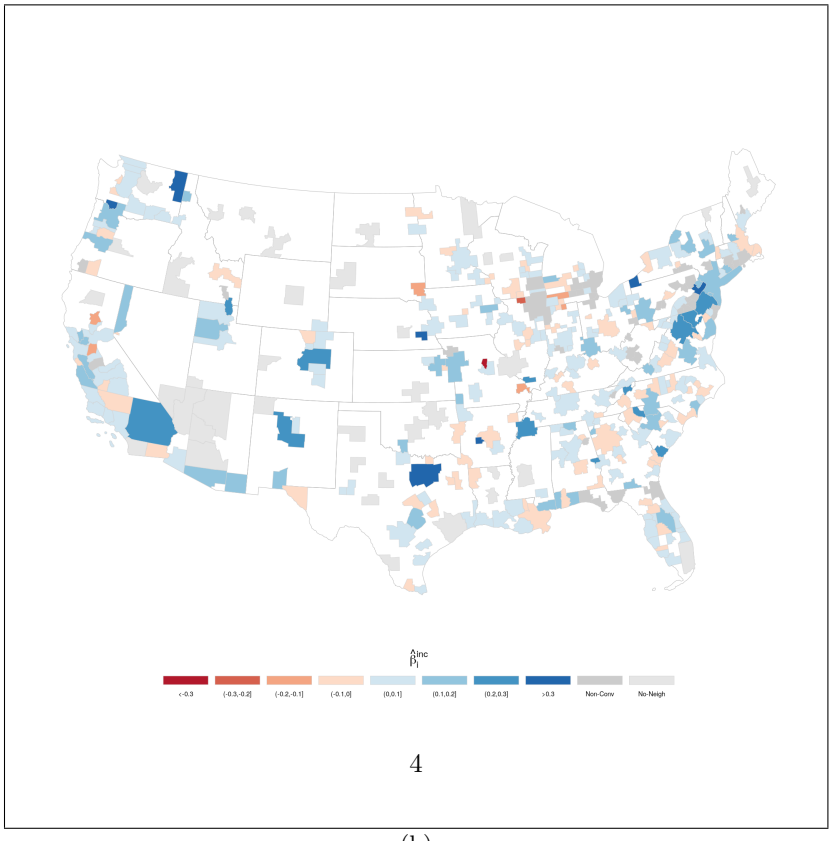
(0,0.5]
(0.5,1]
(1,1.5]
>1.5
Non-Conv
No-Neigh

(a)


$\hat{\beta}_i^{inc}$
<-0.3
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(-0.2,-0.1]
(-0.1,0]
(0,0.1]
(0.1,0.2]
(0.2,0.3]
>0.3
Non-Conv
No-Neigh

(b)

Table 1

T	25		50		100		200	
Parameter	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ψ_{i0}								
$\psi_{1,0} = 0.1261$	-0.0056	0.1891	0.0005	0.1230	-0.0023	0.0851	0.0010	0.0592
$\psi_{2,0} = 0.3883$	-0.0051	0.2495	-0.0058	0.1687	-0.0006	0.1148	-0.0003	0.0803
$\psi_{3,0} = 0.4375$	-0.0115	0.2436	-0.0022	0.1499	0.0034	0.1041	-0.0003	0.0743
$\psi_{4,0} = 0.5059$	0.0050	0.1769	-0.0040	0.1221	-0.0028	0.0820	-0.0010	0.0571
$\psi_{5,0} = 0.7246$	-0.0109	0.2089	-0.0031	0.1502	-0.0009	0.1071	0.0006	0.0721
β_{i0}								
$\beta_{1,0} = 0.9649$	0.0125	0.2236	0.0069	0.1472	0.0024	0.1008	-0.0020	0.0717
$\beta_{2,0} = 0.9572$	0.0100	0.2674	0.0068	0.1833	-0.0022	0.1272	-0.0025	0.0892
$\beta_{3,0} = 0.2785$	0.0078	0.2908	-0.0012	0.1806	-0.0026	0.1252	0.0022	0.0907
$\beta_{4,0} = 0.9134$	-0.0020	0.2195	0.0072	0.1461	0.0012	0.1000	0.0000	0.0684
$\beta_{5,0} = 0.8147$	0.0104	0.2842	0.0108	0.1950	0.0081	0.1341	0.0003	0.0911

T	25	50	100	200	25	50	100	200
Parameter	size				power			
ψ_{i0}								
$\psi_{1,0} = 0.1261$	0.1040	0.0675	0.0535	0.0515	0.3410	0.4665	0.7060	0.9065
$\psi_{2,0} = 0.3883$	0.0950	0.0690	0.0560	0.0580	0.2515	0.3525	0.4900	0.7315
$\psi_{3,0} = 0.4375$	0.0935	0.0620	0.0560	0.0510	0.2245	0.3355	0.5115	0.7975
$\psi_{4,0} = 0.5059$	0.0835	0.0740	0.0660	0.0485	0.3430	0.5025	0.7355	0.9345
$\psi_{5,0} = 0.7246$	0.0660	0.0670	0.0645	0.0530	0.2450	0.3610	0.5410	0.7975
β_{i0}								
$\beta_{1,0} = 0.9649$	0.0900	0.0645	0.0525	0.0530	0.2845	0.3825	0.5360	0.8075
$\beta_{2,0} = 0.9572$	0.0930	0.0725	0.0625	0.0570	0.2165	0.2885	0.4380	0.6535
$\beta_{3,0} = 0.2785$	0.0960	0.0710	0.0515	0.0585	0.2585	0.3000	0.4565	0.6375
$\beta_{4,0} = 0.9134$	0.0865	0.0630	0.0565	0.0485	0.3055	0.3845	0.5715	0.8245
$\beta_{5,0} = 0.8147$	0.0890	0.0705	0.0555	0.0510	0.2005	0.2570	0.3700	0.6080

Table 2

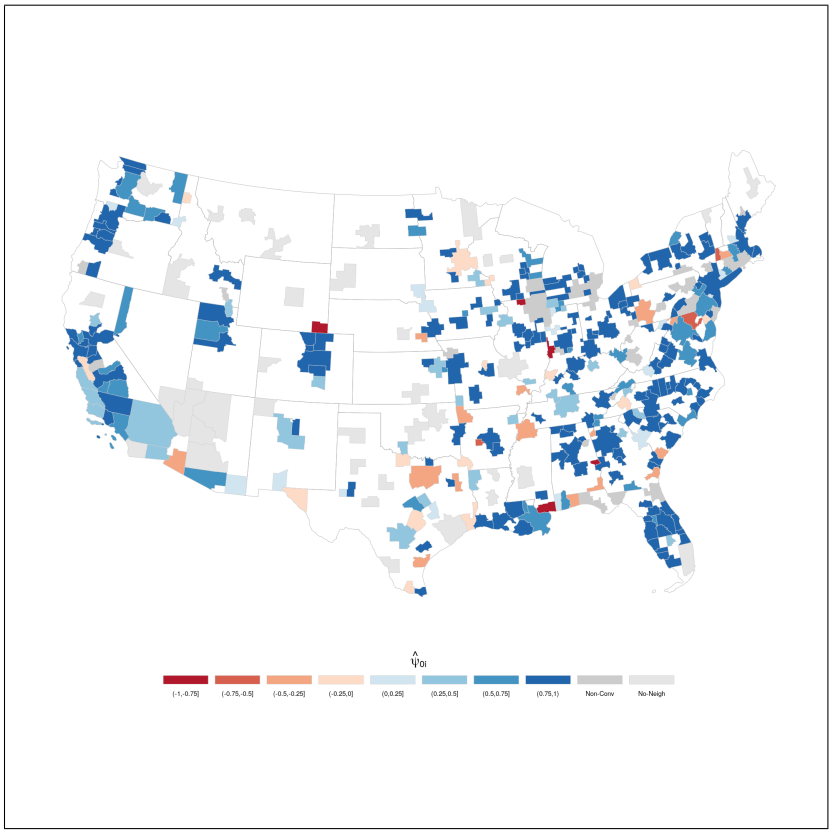
T	25		50		100		200	
Parameter	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ψ_{i0}								
$\psi_{1,0} = 0.0244$	-0.0005	0.3152	-0.0049	0.2138	0.0021	0.1415	-0.0001	0.1010
$\psi_{2,0} = 0.0255$	-0.0330	0.5189	0.0034	0.3674	-0.0137	0.2641	-0.0033	0.1794
$\psi_{3,0} = 0.0397$	0.0129	0.3509	-0.0017	0.2448	-0.0014	0.1681	0.0013	0.1183
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\psi_{51,0} = 0.3927$	-0.0027	0.2912	0.0038	0.2056	0.0009	0.1395	0.0005	0.0960
$\psi_{52,0} = 0.3987$	0.0001	0.1994	-0.0031	0.1381	0.0029	0.0921	0.0005	0.0638
$\psi_{53,0} = 0.4004$	-0.0112	0.3063	0.0075	0.2049	0.0033	0.1392	-0.0015	0.0991
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\psi_{98,0} = 0.7695$	-0.0031	0.1621	0.0018	0.1149	0.0055	0.0824	-0.0003	0.0586
$\psi_{99,0} = 0.7705$	-0.0530	0.2903	-0.0126	0.1895	0.0002	0.1401	0.0003	0.1041
$\psi_{100,0} = 0.7904$	-0.0125	0.1716	-0.0094	0.1275	0.0011	0.0897	0.0008	0.0613
β_{i0}								
$\beta_{1,0} = 0.1978$	0.0089	0.2782	0.0017	0.1771	0.0007	0.1192	-0.0073	0.0824
$\beta_{2,0} = 0.7060$	0.0252	0.3699	0.0016	0.2359	-0.0005	0.1608	0.0049	0.1144
$\beta_{3,0} = 0.4173$	0.0107	0.2541	0.0034	0.1733	0.0000	0.1157	0.0028	0.0821
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\beta_{51,0} = 0.9448$	0.0060	0.1924	-0.0024	0.1294	0.0018	0.0896	0.0009	0.0634
$\beta_{52,0} = 0.1190$	0.0046	0.1824	0.0026	0.1259	-0.0005	0.0853	0.0021	0.0578
$\beta_{53,0} = 0.7127$	0.0026	0.2630	-0.0050	0.1654	0.0038	0.1201	0.0012	0.0831
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\beta_{98,0} = 0.1067$	0.0041	0.1688	-0.0031	0.1115	0.0010	0.0762	-0.0002	0.0550
$\beta_{99,0} = 0.4588$	0.0207	0.2643	0.0039	0.1788	0.0033	0.1232	0.0027	0.0888
$\beta_{100,0} = 0.3674$	0.0056	0.1691	0.0032	0.1179	0.0009	0.0830	0.0004	0.0560

T	25	50	100	200	25	50	100	200
Parameter	size				power			
ψ_{i0}								
$\psi_{1,0} = 0.0244$	0.0890	0.0810	0.0520	0.0590	0.1820	0.2200	0.3290	0.5480
$\psi_{2,0} = 0.0255$	0.0705	0.0595	0.0555	0.0490	0.0945	0.0895	0.1495	0.2140
$\psi_{3,0} = 0.0397$	0.0905	0.0745	0.0585	0.0575	0.1555	0.1895	0.2805	0.4450
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\psi_{51,0} = 0.3927$	0.0950	0.0645	0.0590	0.0535	0.1785	0.2625	0.3590	0.5810
$\psi_{52,0} = 0.3987$	0.0850	0.0620	0.0625	0.0505	0.3050	0.4390	0.6285	0.8660
$\psi_{53,0} = 0.4004$	0.0885	0.0785	0.0570	0.0585	0.1995	0.2490	0.3745	0.5800
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\psi_{98,0} = 0.7695$	0.0635	0.0630	0.0660	0.0610	0.3340	0.4755	0.6935	0.9145
$\psi_{99,0} = 0.7705$	0.0300	0.0285	0.0370	0.0495	0.1455	0.2045	0.3095	0.5205
$\psi_{100,0} = 0.7904$	0.0545	0.0570	0.0625	0.0505	0.3120	0.4665	0.6455	0.8845
β_{i0}								
$\beta_{1,0} = 0.1978$	0.1160	0.0700	0.0610	0.0505	0.2405	0.3040	0.4380	0.7115
$\beta_{2,0} = 0.7060$	0.1025	0.0580	0.0505	0.0545	0.1725	0.2095	0.2710	0.4510
$\beta_{3,0} = 0.4173$	0.0950	0.0780	0.0550	0.0595	0.2450	0.3160	0.4655	0.7080
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\beta_{51,0} = 0.9448$	0.0910	0.0685	0.0590	0.0570	0.3260	0.4665	0.6500	0.8880
$\beta_{52,0} = 0.1190$	0.0970	0.0800	0.0505	0.0440	0.3500	0.4840	0.7030	0.9150
$\beta_{53,0} = 0.7127$	0.1075	0.0660	0.0665	0.0515	0.2420	0.3150	0.4410	0.6810
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\beta_{98,0} = 0.1067$	0.0960	0.0660	0.0530	0.0545	0.3950	0.5500	0.7605	0.9475
$\beta_{99,0} = 0.4588$	0.0725	0.0615	0.0545	0.0595	0.2015	0.2775	0.4225	0.6415
$\beta_{100,0} = 0.3674$	0.0935	0.0660	0.0695	0.0540	0.3605	0.5025	0.7255	0.9370

Table 3

r	Name	N_r	$\hat{\psi}_{MG,r}$	$\hat{\psi}_{MG0,r}$	$\hat{\psi}_{MG1,r}$	$\hat{\lambda}_{MG,r}$	$\hat{\beta}^{pop}_{MG,r}$	$\hat{\beta}^{inc}_{MG,r}$
1 & 2	New England & Mideast	35	0.067	0.499‡	-0.432‡	0.645‡	0.629‡	0.085‡
			(0.044)	(0.087)	(0.069)	(0.026)	(0.172)	(0.020)
3	Great Lakes	48	0.115‡	0.714‡	-0.599‡	0.629‡	0.224‡	0.025†
			(0.030)	(0.064)	(0.047)	(0.026)	(0.068)	(0.012)
4	Plains	26	0.040	0.525‡	-0.484‡	0.599‡	0.252‡	0.039
			(0.058)	(0.083)	(0.060)	(0.046)	(0.073)	(0.030)
5	Southeast	106	0.105‡	0.669‡	-0.564‡	0.655‡	0.194‡	0.038‡
			(0.024)	(0.044)	(0.032)	(0.019)	(0.034)	(0.008)
6 & 7	Southwest & Rocky Mountain	40	0.017	0.325‡	-0.308‡	0.713‡	0.162‡	0.072‡
			(0.021)	(0.078)	(0.064)	(0.020)	(0.038)	(0.016)
8	Far West	41	0.126‡	0.711‡	-0.585‡	0.759‡	0.183‡	0.067‡
			(0.017)	(0.040)	(0.035)	(0.012)	(0.047)	(0.018)
	US	296	0.088‡	0.603‡	-0.515‡	0.667‡	0.250‡	0.050‡
			(0.013)	(0.027)	(0.020)	(0.010)	(0.029)	(0.006)

Figure F1


$\hat{\psi}_{0i}$
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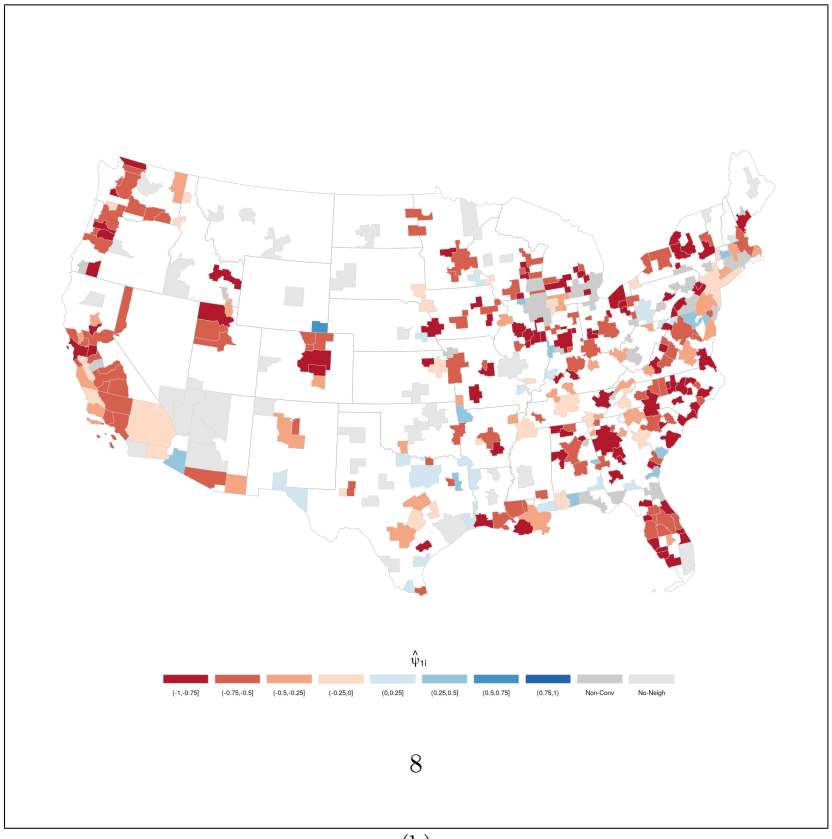
(-0.75, -0.5]
(-0.5, -0.25]
(-0.25, 0]
(0, 0.25]
(0.25, 0.5]
(0.5, 0.75]
(0.75, 1]
Non-Conv
No-Neigh

(a)


$\hat{\psi}_{1i}$
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(-0.75, -0.5]
(-0.5, -0.25]
(-0.25, 0]
(0, 0.25]
(0.25, 0.5]
(0.5, 0.75]
(0.75, 1]
Non-Conv
No-Neigh

(b)

Figure F2

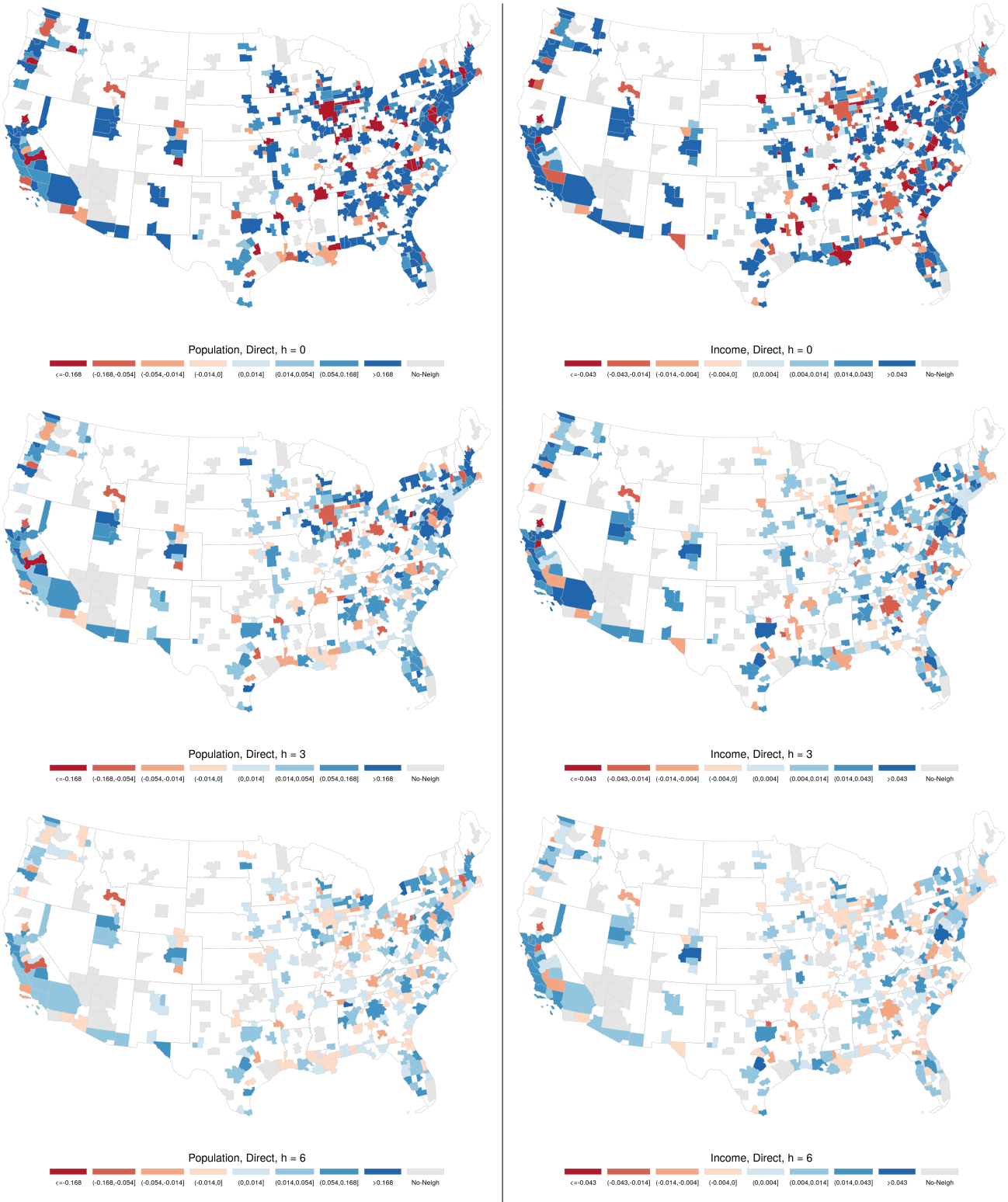
Population, Direct, h = 0
<=-0.168
(-0.168,-0.054]
(-0.054,-0.014]
(-0.014,0]
(0,0.014]
(0.014,0.054]
(0.054,0.168]
>0.168
No-Neigh
Income, Direct, h = 0
<=-0.043
(-0.043,-0.014]
(-0.014,-0.004]
(-0.004,0]
(0,0.004]
(0.004,0.014]
(0.014,0.043]
>0.043
No-Neigh
Population, Direct, h = 3
Income, Direct, h = 3
Population, Direct, h = 6
Income, Direct, h = 6

Figure F3

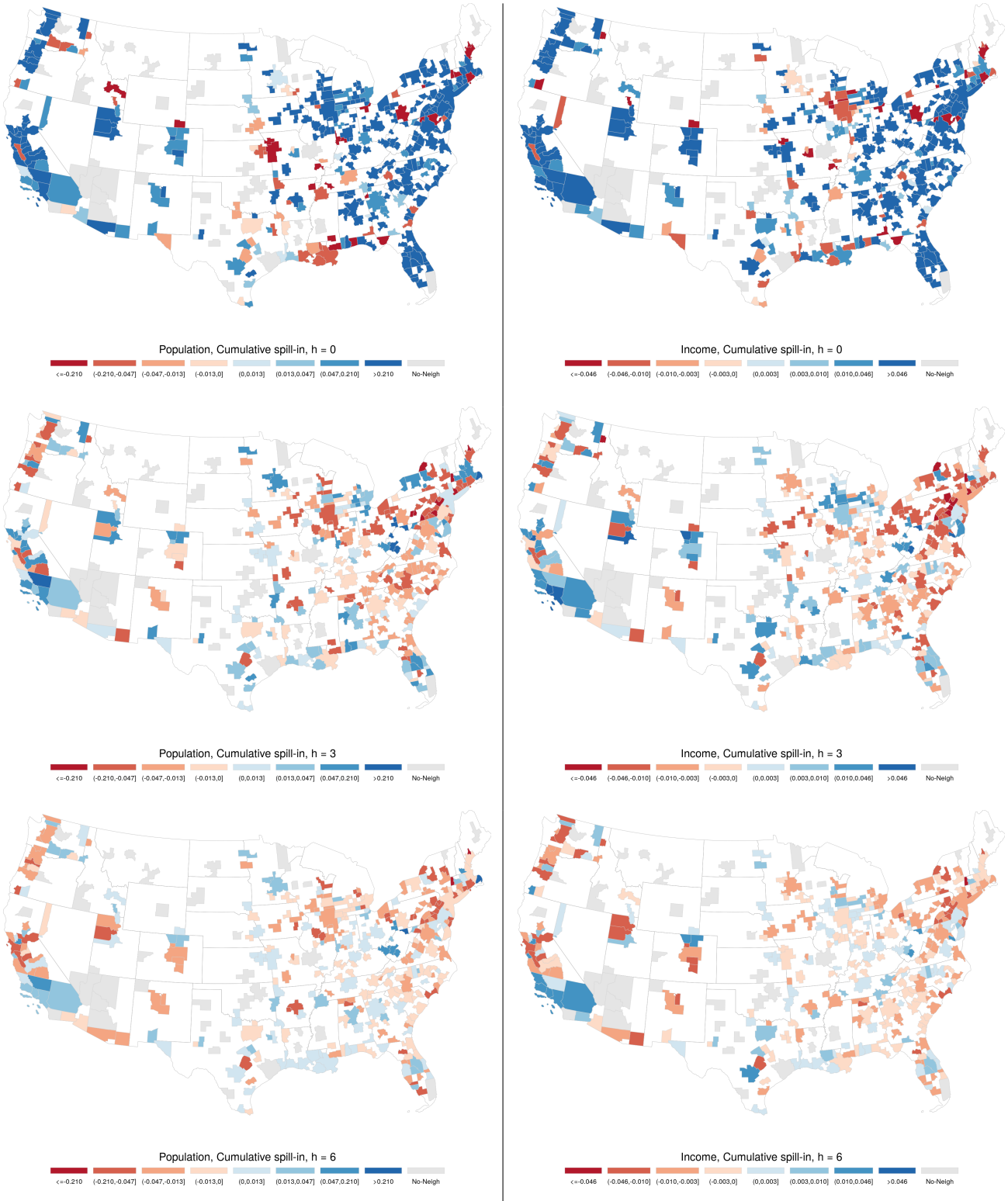
Population, Cumulative spill-in, h = 0
<=-0.210
(-0.210,-0.047]
(-0.047,-0.013]
(-0.013,0]
(0,0.013]
(0.013,0.047]
(0.047,0.210]
>0.210
No-Neigh
Income, Cumulative spill-in, h = 0
<=-0.046
(-0.046,-0.010]
(-0.010,-0.003]
(-0.003,0]
(0,0.003]
(0.003,0.010]
(0.010,0.046]
>0.046
No-Neigh
Population, Cumulative spill-in, h = 3
<=-0.210
(-0.210,-0.047]
(-0.047,-0.013]
(-0.013,0]
(0,0.013]
(0.013,0.047]
(0.047,0.210]
>0.210
No-Neigh
Income, Cumulative spill-in, h = 3
<=-0.046
(-0.046,-0.010]
(-0.010,-0.003]
(-0.003,0]
(0,0.003]
(0.003,0.010]
(0.010,0.046]
>0.046
No-Neigh
Population, Cumulative spill-in, h = 6
<=-0.210
(-0.210,-0.047]
(-0.047,-0.013]
(-0.013,0]
(0,0.013]
(0.013,0.047]
(0.047,0.210]
>0.210
No-Neigh
Income, Cumulative spill-in, h = 6
<=-0.046
(-0.046,-0.010]
(-0.010,-0.003]
(-0.003,0]
(0,0.003]
(0.003,0.010]
(0.010,0.046]
>0.046
No-Neigh

Figure F4

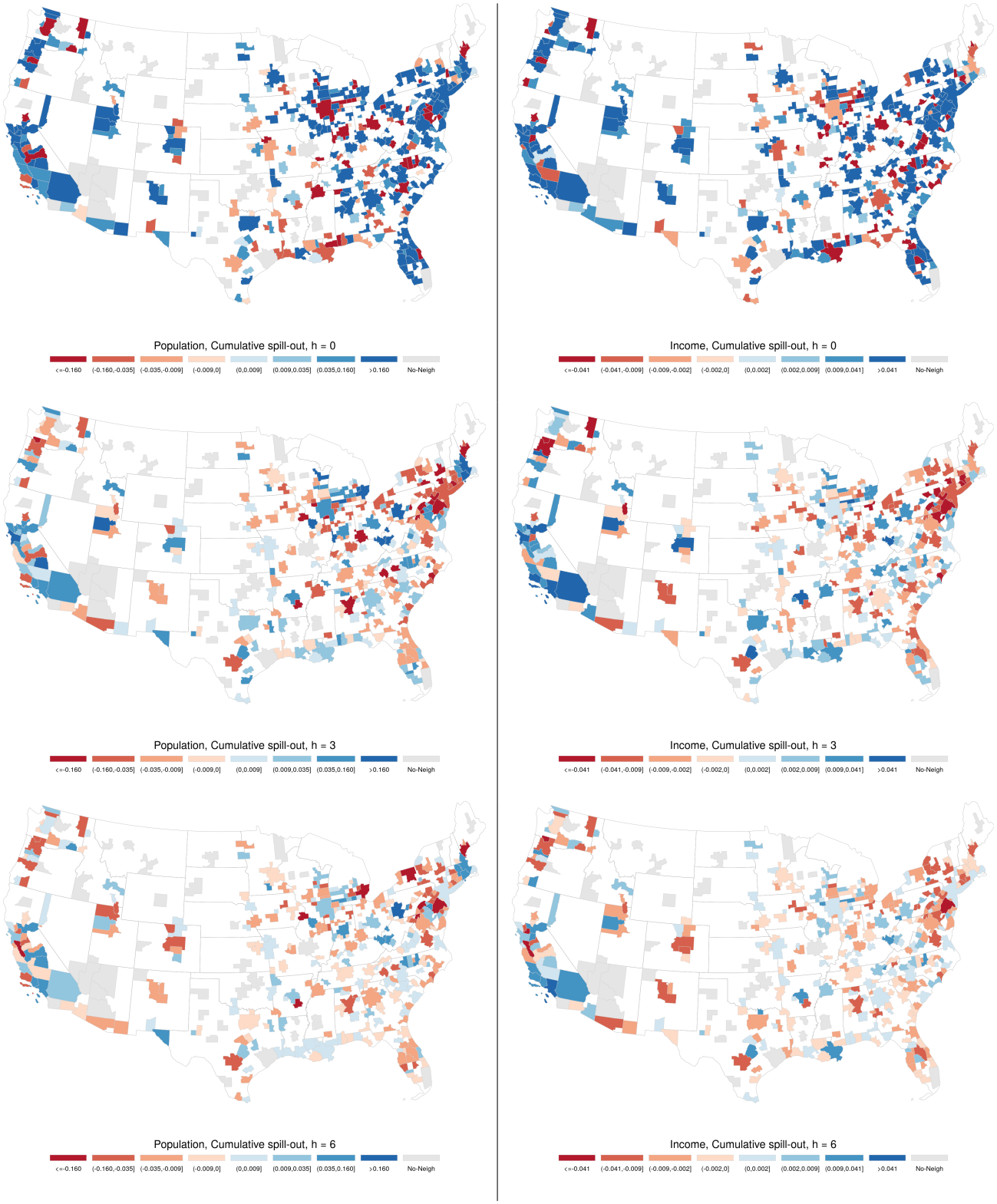
Population, Cumulative spill-out, h = 0
<=-0.160
(-0.160,-0.035]
(-0.035,-0.009]
(-0.009,0]
(0,0.009]
(0.009,0.035]
(0.035,0.160]
>0.160
No-Neigh
Income, Cumulative spill-out, h = 0
<=-0.041
(-0.041,-0.009]
(-0.009,-0.002]
(-0.002,0]
(0,0.002]
(0.002,0.009]
(0.009,0.041]
>0.041
No-Neigh
Population, Cumulative spill-out, h = 3
<=-0.160
(-0.160,-0.035]
(-0.035,-0.009]
(-0.009,0]
(0,0.009]
(0.009,0.035]
(0.035,0.160]
>0.160
No-Neigh
Income, Cumulative spill-out, h = 3
<=-0.041
(-0.041,-0.009]
(-0.009,-0.002]
(-0.002,0]
(0,0.002]
(0.002,0.009]
(0.009,0.041]
>0.041
No-Neigh
Population, Cumulative spill-out, h = 6
<=-0.160
(-0.160,-0.035]
(-0.035,-0.009]
(-0.009,0]
(0,0.009]
(0.009,0.035]
(0.035,0.160]
>0.160
No-Neigh
Income, Cumulative spill-out, h = 6
<=-0.041
(-0.041,-0.009]
(-0.009,-0.002]
(-0.002,0]
(0,0.002]
(0.002,0.009]
(0.009,0.041]
>0.041
No-Neigh

Table F1

r	Name	N_r	Population	Area (sqr miles)	Density (per sqr mile)
1 & 2	New England & Mideast	54	55275714	83111	665.1
3	Great Lakes	57	34050860	77400	439.9
4	Plains	27	9320364	64266	145.0
5	Southeast	116	42770914	215211	198.7
6 & 7	Southwest & Rocky Mountain	41	18339459	145292	126.2
8	Far West	43	35590563	145666	244.3

Table F2

		Within-region effects					
		Direct (in levels)			Indirect (as % of Direct)		
r	Name	0	3	6	0	3	6
		Population					
1 & 2	New England & Mideast	0.558	0.112	0.035	2.171	-1.423	-2.332
3	Great Lakes	0.232	0.042	0.017	3.665	-0.216	-1.129
4	Plains	0.267	0.034	0.012	2.574	-1.710	-2.167
5	Southeast	0.224	0.041	0.016	1.512	-0.037	-0.317
6 & 7	Southwest & Rocky Mountain	0.176	0.044	0.015	1.765	0.627	-0.666
8	Far West	0.223	0.061	0.023	4.699	0.984	-1.238
		Income					
1 & 2	New England & Mideast	0.096	0.021	0.007	2.386	-1.666	-2.213
3	Great Lakes	0.035	0.007	0.003	3.283	-0.022	-0.809
4	Plains	0.046	0.007	0.003	4.444	-1.705	-2.587
5	Southeast	0.045	0.008	0.003	1.638	-0.056	-0.329
6 & 7	Southwest & Rocky Mountain	0.082	0.020	0.008	1.834	0.512	-0.752
8	Far West	0.083	0.022	0.008	4.121	1.267	-0.084

Table F3

r	Name	N_r	$\hat{\psi}_{MG,r}$	$\hat{\psi}_{MG0,r}$	$\hat{\psi}_{MG1,r}$	$\hat{\lambda}_{MG,r}$	$\hat{\beta}^{pop}_{MG,r}$	$\hat{\beta}^{inc}_{MG,r}$
1 & 2	New England & Mideast	36	0.080*	0.537‡	-0.456‡	0.646‡	0.628‡	0.082‡
			(0.042)	(0.084)	(0.068)	(0.027)	(0.166)	(0.019)
3	Great Lakes	48	0.151‡	0.759‡	-0.609‡	0.635‡	0.198‡	0.029‡
			(0.027)	(0.050)	(0.040)	(0.024)	(0.060)	(0.010)
4	Plains	27	0.009	0.472‡	-0.463‡	0.608‡	0.217‡	0.043
			(0.059)	(0.097)	(0.063)	(0.041)	(0.084)	(0.028)
5	Southeast	106	0.125‡	0.742‡	-0.617‡	0.674‡	0.170‡	0.032‡
			(0.020)	(0.033)	(0.026)	(0.017)	(0.031)	(0.007)
6 & 7	Southwest & Rocky Mountain	39	0.009	0.313‡	-0.304‡	0.718‡	0.158‡	0.074‡
			(0.020)	(0.079)	(0.066)	(0.020)	(0.038)	(0.015)
8	Far West	42	0.104‡	0.682‡	-0.578‡	0.759‡	0.177‡	0.066‡
			(0.025)	(0.048)	(0.033)	(0.012)	(0.047)	(0.019)
	US	298	0.095‡	0.631‡	-0.536‡	0.676‡	0.234‡	0.049‡
			(0.012)	(0.025)	(0.019)	(0.010)	(0.028)	(0.006)

Table G1. Results in this table are generated with the same scripts as in Table 1 except for σ_{i0}^2 which here needs to be set to `v_sgmsq=(v_chisq_2df./8)+0.25` in `mc_fixed_coefficients.m`.

T	25		50		100		200	
Parameter	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ψ_{i0}								
β_{i0}								

T	25	50	100	200	25	50	100	200
Parameter	size				power			
ψ_{i0}								
β_{i0}								

Table G2. Results in this table are generated with the same scripts as in Table 2 except for σ_{i0}^2 which here needs to be set to `v_sgmsq=(v_chisq_2df./8)+0.25` in `mc_fixed_coefficients.m`.

T	25		50		100		200	
Parameter	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ψ_{i0}								
β_{i0}								

T	25	50	100	200	25	50	100	200
Parameter	size				power			
ψ_{i0}								
β_{i0}								

Figure G1

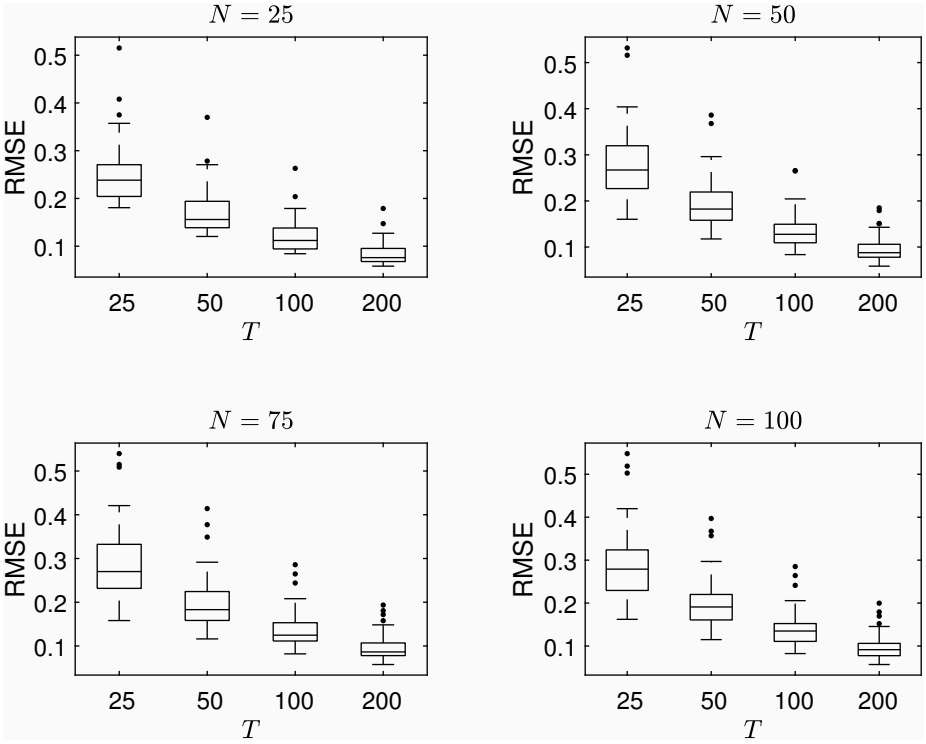

Figure G2

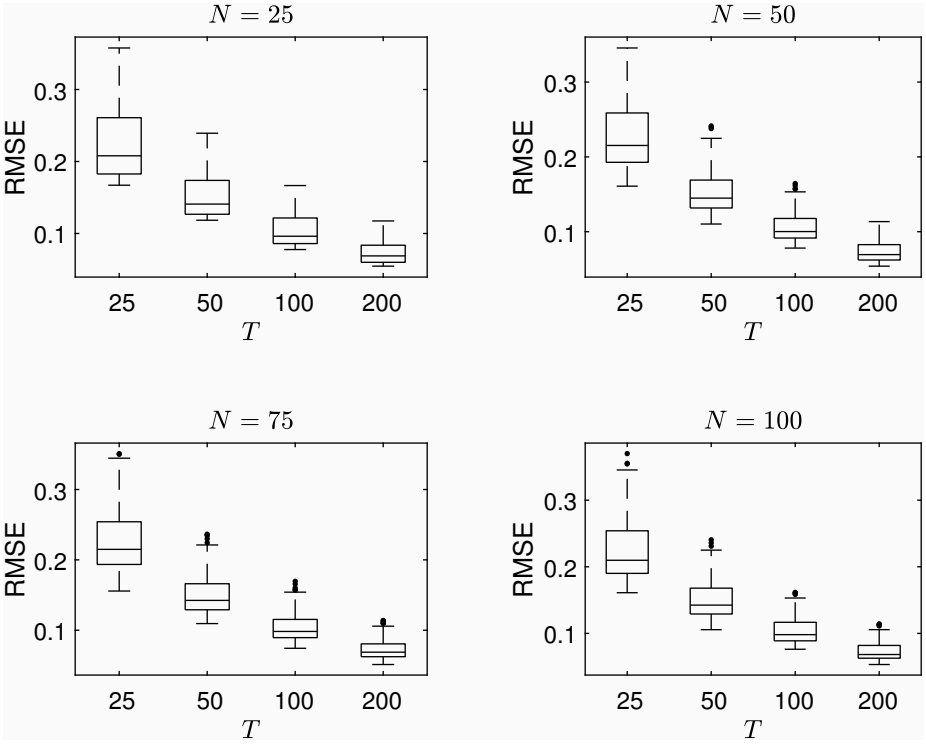

Figure G3

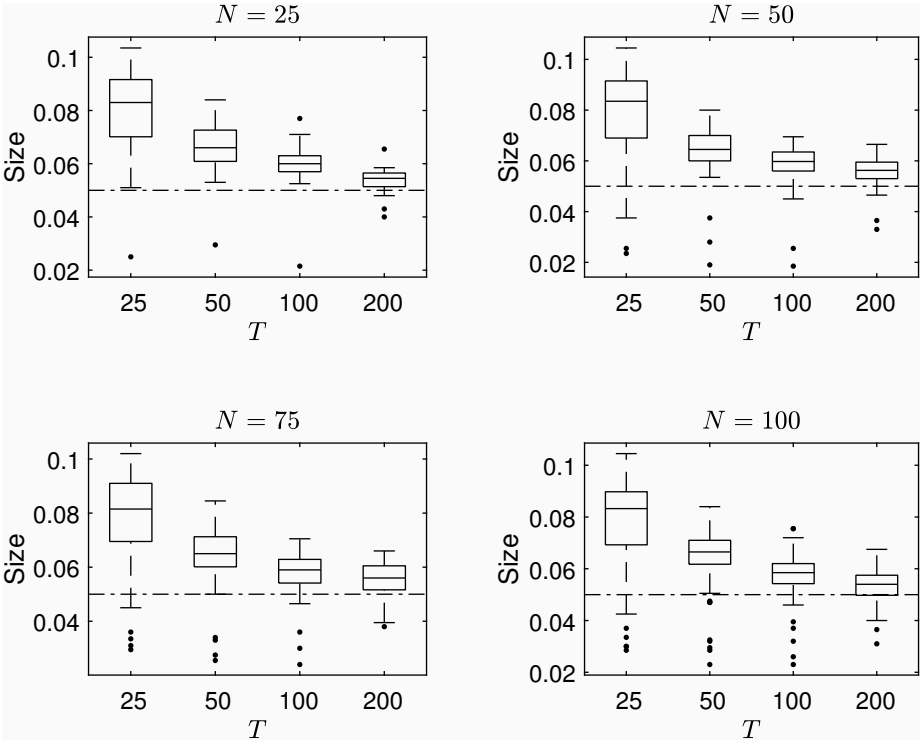

Figure G4

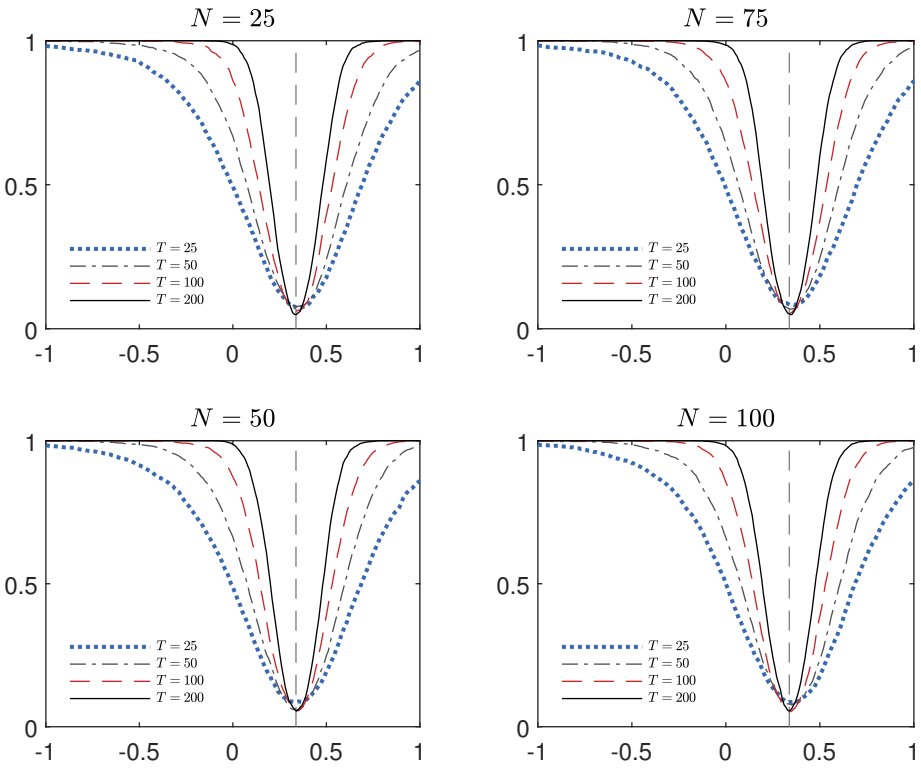

Figure G5

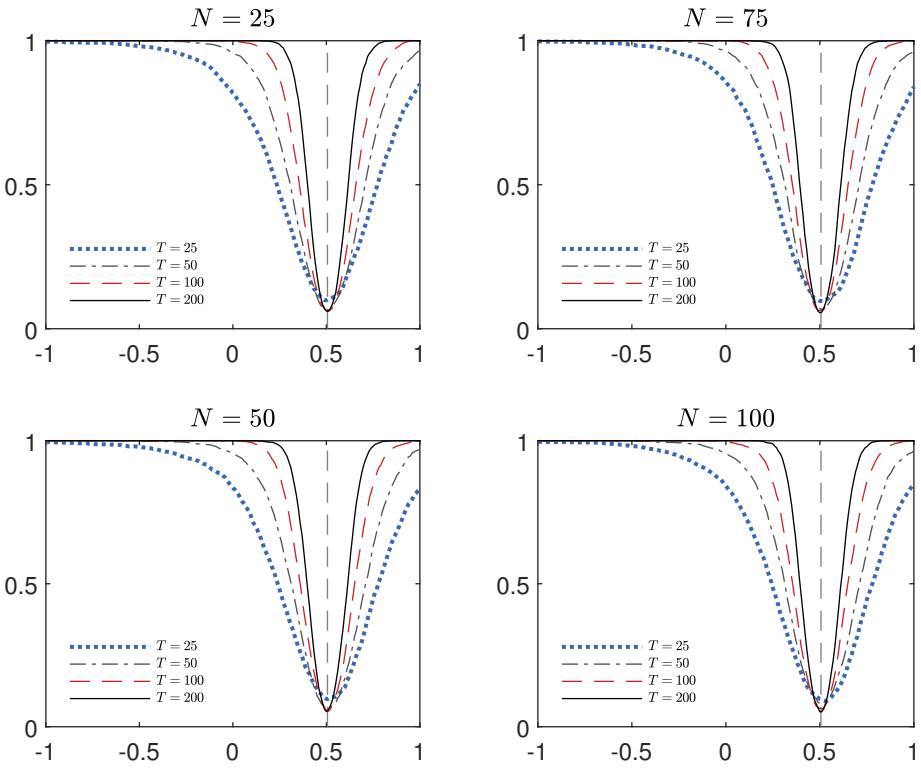

Figure G6

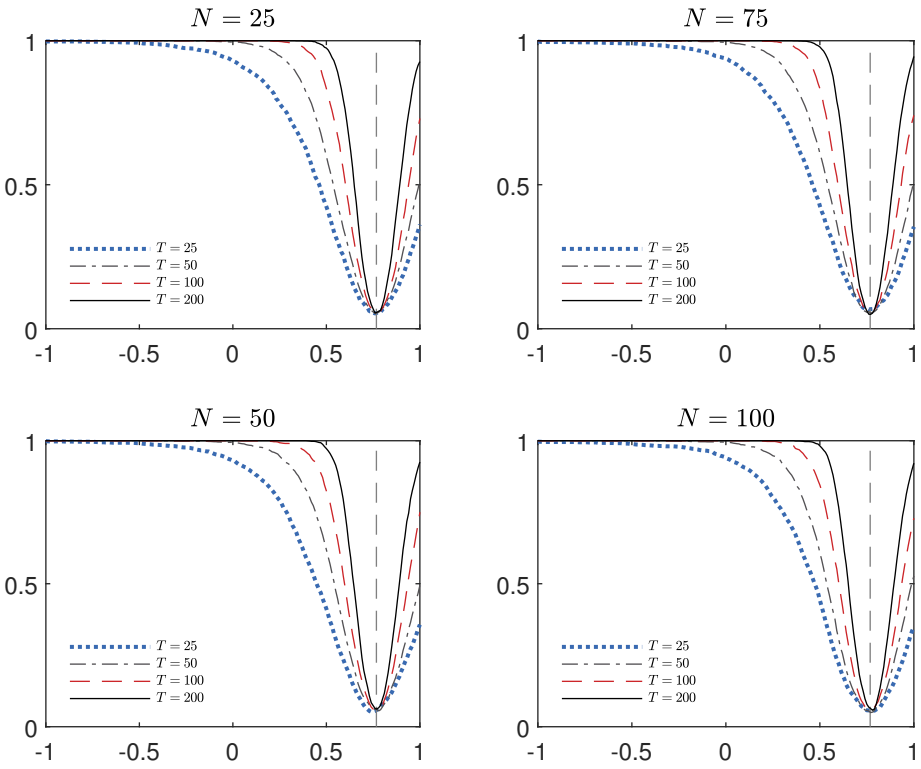

Figure G7

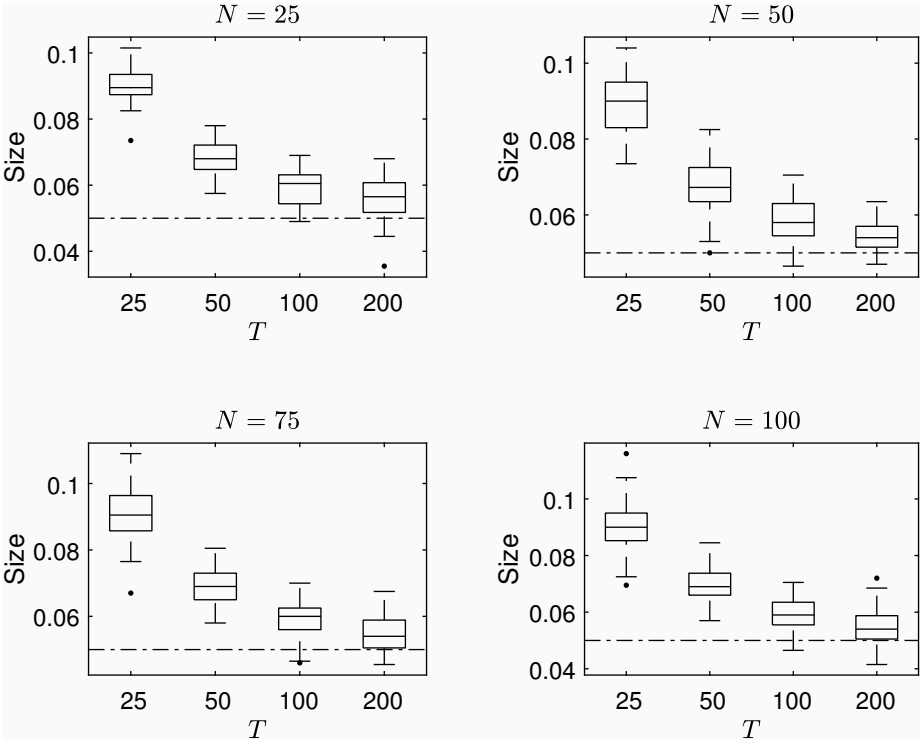

Figure G8

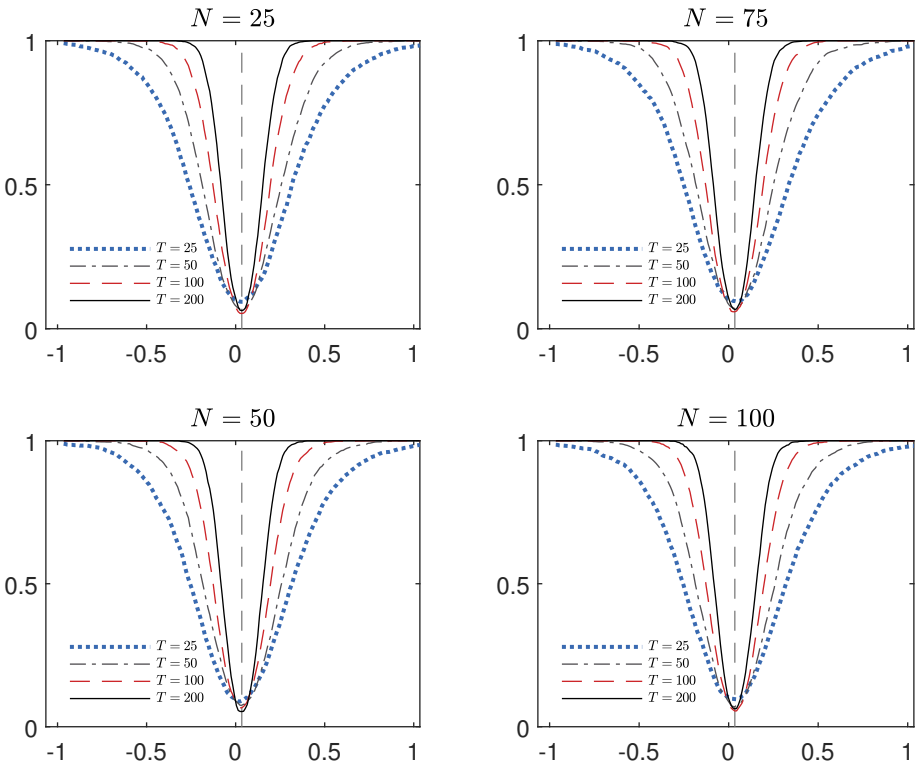

Figure G9

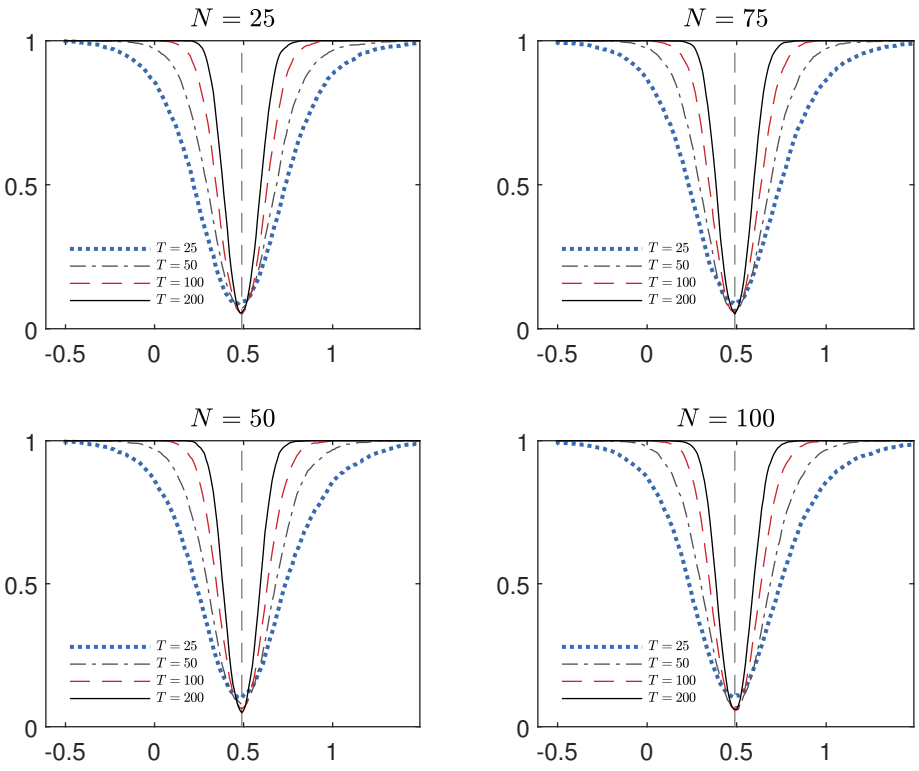

Figure G10

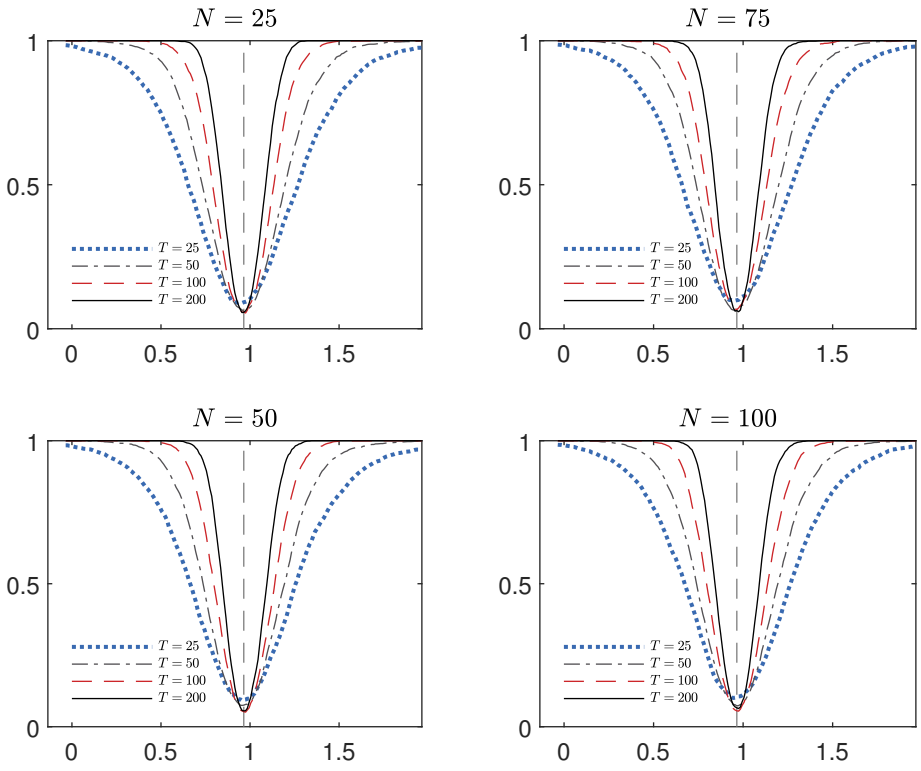

Table G3. Results in the upper panel (Gaussian errors) are generated with the same scripts as in the lower panel except for the variable `gaussian` in `mc_random_coefficients.m` which needs to be set to 1 instead of 0.

N	25				50				75				100			
T	25	50	100	200	25	50	100	200	25	50	100	200	25	50	100	200
Gaussian errors																
non-Gaussian errors																
Bias																
$\hat{\bar{\psi}}_{MG}$	-0.0102	-0.0018	-0.0010	-0.0003	-0.0088	-0.0018	-0.0007	0.0008	-0.0097	-0.0017	0.0001	-0.0007	-0.0079	-0.0010	-0.0010	-0.0005
$\hat{\bar{\beta}}_{MG}$	0.0087	0.0016	0.0025	-0.0011	0.0048	0.0013	0.0026	0.0010	0.0067	0.0023	0.0016	0.0011	0.0047	0.0028	0.0014	0.0001
RMSE																
$\hat{\bar{\psi}}_{MG}$	0.0638	0.0558	0.0508	0.0480	0.0451	0.0389	0.0361	0.0336	0.0369	0.0313	0.0285	0.0271	0.0319	0.0275	0.0250	0.0237
$\hat{\bar{\beta}}_{MG}$	0.0755	0.0648	0.0610	0.0593	0.0528	0.0461	0.0440	0.0420	0.0436	0.0372	0.0362	0.0338	0.0380	0.0325	0.0307	0.0300
Size																
$\hat{\bar{\psi}}_{MG}$	0.0330	0.0410	0.0520	0.0535	0.0280	0.0330	0.0425	0.0480	0.0260	0.0350	0.0360	0.0390	0.0225	0.0320	0.0350	0.0375
$\hat{\bar{\beta}}_{MG}$	0.0625	0.0620	0.0600	0.0600	0.0540	0.0505	0.0625	0.0560	0.0555	0.0515	0.0580	0.0510	0.0515	0.0495	0.0525	0.0540