Is Deflation Costly After All? The Perils of Erroneous Historical Classifications

Online Appendix

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This Online Appendix provides technical background (Section A), discussion of the data (Section B), additional results (Section C), and robustness tests (Section D).

A Technical appendix

Following Kane et al. (1999) and Black et al. (2000) I derive GMM estimators for (i) a linear model with an error-ridden binary regressor; (ii) an extension to binary covariates; (iii) using an alternative identifying assumption allowing for correlated measurement errors. In addition, I show how to estimate the bias introduced due to misclassification and discuss how to control for serial correlation in the error term.

A.1 Model and misclassification bias

Suppose that the true model reads

$$y_t = \alpha + \beta d_t + \varepsilon_t \ ,$$

where $d_t \equiv \mathbf{1}_{\{\pi_t<0\}}$ denotes a binary deflation indicator and ε_t an i.i.d. error term. Further, suppose that we have at our disposal two error-ridden indicators $x_t \equiv \mathbf{1}_{\{\tilde{\pi}_t<0\}}$, and $z_t \equiv \mathbf{1}_{\{\hat{\pi}_t<0\}}$.

Based on information of only one of the indicators, the model is not identified and we will not be able to recover the true coefficients (see Aigner 1973). For example, using only the information in x_t we can estimate three independent moments from the data, namely the expectation of y_t conditional

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on each outcome of x_t and the population probability of $x_t = 1$:

$$\begin{aligned}
E[y_t|x_t = 1] &= \alpha + \beta P[d_t = 1|x_t = 1] \\
E[y_t|x_t = 0] &= \alpha + \beta P[d_t = 1|x_t = 0] \\
P[x_t] &= P[x_t = 1|d_t = 1]P[d_t = 1] + P[x_t = 1|d_t = 0]P[d_t = 0] \\
&= (1 - P[x_t = 0|d_t = 1])P[d_t = 1] + P[x_t = 1|d_t = 0](1 - P[d_t = 1]) \, .
\end{aligned}$$

We can apply Bayes' theorem to rewrite the conditional probabilities in the conditional expectation as:

$$\begin{aligned}
P[d_t = 1|x_t = 1] &= \frac{P[x_t = 1|d_t = 1]P[d_t = 1]}{P[x_t]} \\
&= \frac{(1 - P[x_t = 0|d_t = 1])P[d_t = 1]}{(1 - P[x_t = 0|d_t = 1])P[d_t = 1] + P[x_t = 1|d_t = 0]P[d_t = 0]} \, .
\end{aligned}$$

Let us define the population probability of deflation as $p \equiv P[d_t = 1]$, as well as the probability that a deflationary and inflationary episode is misclassified as $\eta_x \equiv P[x_t = 0|d_t = 1]$ and $\nu_x \equiv P[x_t = 1|d_t = 0]$, respectively. Then, we can rewrite the moments as:

$$\begin{aligned}
E[y_t|x_t = 1] &= \alpha + \beta \frac{(1 - \eta_x)p}{(1 - \eta_x)p + \nu_x(1 - p)} \\
E[y_t|x_t = 0] &= \alpha + \beta \frac{\eta_x p}{\eta_x p + (1 - \nu_x)(1 - p)} \\
P[x_t] &= (1 - \eta_x)p + \nu_x(1 - p) \, .
\end{aligned} \tag{A.1}$$

We have to estimate five coefficients from three moments: the model is not identified.

If $\eta_x > 0$ or $\nu_x > 0$ the OLS estimate is biased. Using Eq. (A.1), we can show that the bias of the OLS estimator of α and β amount to:

$$\begin{aligned}
plim \; \hat{\alpha}_{ols} - \alpha &= E[y_t|x_t = 0] - \alpha \\
&= \beta \frac{\eta_x p}{\eta_x p + (1 - \nu_x)(1 - p)} \\
plim \; \hat{\beta}_{ols} - \beta &= E[y_t|x_t = 1] - E[y_t|x_t = 0] - \beta \\
&= -\beta \left(\frac{\eta_x p}{\eta_x p + (1 - \nu_x)(1 - p)} + \frac{\nu_x(1 - p)}{(1 - \eta_x)p + \nu_x(1 - p)} \right) \, .
\end{aligned} \tag{A.2}$$

If $\beta < 0$ the OLS estimate of α (β) is downward (upward) biased.

A.2 Consistent estimator

Exploiting the information of two binary indicators allows to consistently estimate β using GMM (see Kane et al. 1999; Black et al. 2000). We can estimate the mean of y_t conditional on four joint realizations of the two indicators. In addition, we can compute the share of deflationary periods conditional on all combinations of outcomes yielding another three independent moments. To show how many coefficients we have to estimate let us first rewrite the conditional expectation of y_t as a function of the misclassification probabilities. For $x_t = 1$ and $z_t = 1$ we obtain:

$$E[y_t|x_t = 1, z_t = 1] = \alpha + \beta P[d_t = 1|x_t = 1, z_t = 1] \, .$$

Applying Bayes' theorem we can rewrite the conditional probability as

$$P[d_t = 1|x_t = 1, z_t = 1] \;=\; \frac{P[x_t = 1, z_t = 1|d_t = 1]P[d_t = 1]}{P[x_t = 1, z_t = 1]} \;.$$

Using the assumption that x_t and z_t are independent conditional on the actual outcome of d_t we have:

$$\begin{aligned} P[d_t = 1|x_t = 1, z_t = 1] &= \frac{P[x_t = 1|d_t = 1]P[z_t = 1|d_t = 1]P[d_t = 1]}{P[x_t = 1, z_t = 1]} \\ &= \frac{(1 - P[x_t = 0|d_t = 1])(1 - P[z_t = 0|d_t = 1])P[d_t = 1]}{P[x_t = 1, z_t = 1]} \;. \end{aligned} \tag{A.3}$$

The same strategy can be applied to rewrite the denominator as:

$$\begin{aligned} P[x_t = 1, z_t = 1] &= P[x_t = 1, z_t = 1|d_t = 1]P[d_t = 1] \\ &\quad + P[x_t = 1, z_t = 1|d_t = 0]P[d_t = 0] \\ &= (1 - P[x_t = 0|d_t = 1])(1 - P[z_t = 0|d_t = 1])P[d_t = 1] \\ &\quad + P[x_t = 1|d_t = 0]P[z_t = 1|d_t = 0](1 - P[d_t = 1]) \;. \end{aligned} \tag{A.4}$$

Plugging Eq. (A.4) into Eq. (A.3), and using previously introduced notation, yields the conditional expectation in terms of seven parameters. The other six moment conditions can be derived in an analogous way:

$$\begin{aligned} E[y_t|x_t = 1, z_t = 1] &= \alpha + \beta \frac{(1-\eta_x)(1-\eta_z)p}{(1-\eta_x)(1-\eta_z)p + \nu_x\nu_z(1-p)} \\ E[y_t|x_t = 0, z_t = 1] &= \alpha + \beta \frac{\eta_x(1-\eta_z)p}{\eta_x(1-\eta_z)p + (1-\nu_x)\nu_z(1-p)} \\ E[y_t|x_t = 1, z_t = 0] &= \alpha + \beta \frac{(1-\eta_x)\eta_z p}{(1-\eta_x)\eta_z p + \nu_x(1-\nu_z)(1-p)} \\ E[y_t|x_t = 0, z_t = 0] &= \alpha + \beta \frac{\eta_x\eta_z p}{\eta_x\eta_z p + (1-\nu_x)(1-\nu_z)(1-p)} \\ P[x_t = 1, z_t = 1] &= (1-\eta_x)(1-\eta_z)p + \nu_x\nu_z(1-p) \\ P[x_t = 0, z_t = 1] &= \eta_x(1-\eta_z)p + (1-\nu_x)\nu_z(1-p) \\ P[x_t = 1, z_t = 0] &= (1-\eta_x)\eta_z p + \nu_x(1-\nu_z)(1-p) \;. \end{aligned}$$

We have to estimate four misclassification probabilities (η_x, ν_x, η_z, ν_z), two model parameters (α, β), and the probability of deflation p. Because we can estimate seven moments from the data the model is just identified. If we obtain M noisy binary indicators this yields $2 \times 2^M - 1$ moment conditions and $3 + 2 \times M$ coefficients to estimate. With more than two indicators the model is over-identified.

After estimating the parameters with GMM, we can use Eq. (A.2) to back out the size of the bias if we would only use one indicator. We can also estimate the bias in the mean during deflation periods and the bias in the mean during inflation periods. For inference, we can use the delta method.

A.3 Binary covariates

Let q_t be a well-measured binary covariate in the regression equation:

$$y_t = \alpha + \beta x_t + \delta q_t + \epsilon_t \, .$$

We can compute the conditional expectation of y_t for every combination of outcomes of the three variables. For $x_t = 1$, $z_t = 1$, and $q_t = 1$ we have:

$$\begin{aligned} E[y_t|x_t = 1, z_t = 1, q_t = 1] &= \alpha + \beta P[d_t = 1|x_t = 1, z_t = 1, q_t = 1] \\ &\quad + \delta P[q_t = 1|x_t = 1, z_t = 1, q_t = 1] \, . \end{aligned}$$

Assuming that the covariate is accurately measured implies $P[q_t = 1|x_t = 1, z_t = 1, q_t = 1] = 1$. Therefore, we can proceed applying Bayes' theorem:

$$P[d_t = 1|x_t = 1, z_t = 1, q_t = 1] = \frac{P[x_t = 1, z_t = 1, q_t = 1|d_t = 1]P[d_t = 1]}{P[x_t = 1, z_t = 1, q_t = 1]} \, .$$

If we assume that all three indicators are conditionally independent we obtain:

$$P[d_t = 1|x_t = 1, z_t = 1, q_t = 1] = \frac{(1-\eta_x)(1-\eta_z)(1-\eta_q)p}{(1-\eta_x)(1-\eta_z)(1-\eta_q)p + \nu_x\nu_z\nu_q(1-p)} \, ,$$

where $\eta_q \equiv P[q_t = 0|d_t = 1]$ and $\nu_q \equiv P[q_t = 1|d_t = 0]$.

Conditioning on all possible combinations with $M = 2$ binary indicators yields $2 \times 2^{M+1} - 1 = 15$ moments. However, we only have to estimate $3 + 1 + 2 \times (M + 1) = 10$ coefficients. The model is over-identified.

The moment conditions read:

$$\begin{aligned}
E[y_t|x_t = 1, z_t = 1, q_t = 1] &= \alpha + \beta\frac{(1-\eta_x)(1-\eta_z)(1-\eta_q)p}{(1-\eta_x)(1-\eta_z)(1-\eta_q)p + \nu_x\nu_z\nu_q(1-p)} + \delta \\
E[y_t|x_t = 0, z_t = 1, q_t = 1] &= \alpha + \beta\frac{\eta_x(1-\eta_z)(1-\eta_q)p}{\eta_x(1-\eta_z)(1-\eta_q)p + (1-\nu_x)\nu_z\nu_q(1-p)} + \delta \\
E[y_t|x_t = 1, z_t = 0, q_t = 1] &= \alpha + \beta\frac{(1-\eta_x)\eta_z(1-\eta_q)p}{(1-\eta_x)\eta_z(1-\eta_q)p + \nu_x(1-\nu_z)\nu_q(1-p)} + \delta \\
E[y_t|x_t = 0, z_t = 0, q_t = 1] &= \alpha + \beta\frac{\eta_x\eta_z(1-\eta_q)p}{\eta_x\eta_z(1-\eta_q)p + (1-\nu_x)(1-\nu_z)\nu_q(1-p)} + \delta \\
E[y_t|x_t = 1, z_t = 0, q_t = 1] &= \alpha + \beta\frac{(1-\eta_x)\eta_z(1-\eta_q)p}{(1-\eta_x)\eta_z(1-\eta_q)p + \nu_x(1-\nu_z)\nu_q(1-p)} + \delta \\
E[y_t|x_t = 0, z_t = 1, q_t = 0] &= \alpha + \beta\frac{(\eta_x(1-\eta_z)\eta_q p}{\eta_x(1-\eta_z)\eta_q p + (1-\nu_x)\nu_z(1-\nu_q)(1-p)} \\
E[y_t|x_t = 1, z_t = 1, q_t = 0] &= \alpha + \beta\frac{(1-\eta_x)(1-\eta_z)\eta_q p}{(1-\eta_x)(1-\eta_z)\eta_q p + \nu_x\nu_z(1-\nu_q)(1-p)} \\
E[y_t|x_t = 0, z_t = 0, q_t = 0] &= \alpha + \beta\frac{\eta_x\eta_z\eta_q p}{\eta_x\eta_z\eta_q p + (1-\nu_x)(1-\nu_z)(1-\nu_q)(1-p)} \\
P[x_t = 1, z_t = 1, q_t = 1] &= (1-\eta_x)(1-\eta_z)(1-\eta_q)p + \nu_x\nu_z\nu_q(1-p) \\
P[x_t = 0, z_t = 1, q_t = 1] &= \eta_x(1-\eta_z)(1-\eta_q)p + (1-\nu_x)\nu_z\nu_q(1-p) \\
P[x_t = 1, z_t = 0, q_t = 1] &= (1-\eta_x)\eta_z(1-\eta_q)p + \nu_x(1-\nu_z)\nu_q(1-p)
\end{aligned}$$

$$
\begin{aligned}
P[x_t = 0, z_t = 0, q_t = 1] &= \eta_x \eta_z (1-\eta_q) p + (1-\nu_x)(1-\nu_z)\nu_q (1-p) \\
P[x_t = 1, z_t = 0, q_t = 1] &= (1-\eta_x)\eta_z (1-\eta_q) p + \nu_x (1-\nu_z)\nu_q (1-p) \\
P[x_t = 0, z_t = 1, q_t = 0] &= \eta_x (1-\eta_z)\eta_q p + (1-\nu_x)\nu_z (1-\nu_q)(1-p) \\
P[x_t = 1, z_t = 1, q_t = 0] &= (1-\eta_x)(1-\eta_z)\eta_q p + \nu_x \nu_z (1-\nu_q)(1-p) \; .
\end{aligned}
$$

A.4 Identification under conditional dependence

Without the conditional independence assumption the model is not identified. Let us derive the conditional expectation of y_t for $x_t = 1, z_t = 1$:

$$
E[y_t | x_t = 1, z_t = 1] = \alpha + \beta \frac{\eta_{11} p}{\eta_{11} p + \nu_{11}(1-p)} \; .
$$

where $\eta_{ij} \equiv P[x_t = i, z_t = j | d_t = 1]$ and $\nu_{ij} \equiv P[x_t = i, z_t = j | d_t = 0]$ for $i \in \{0, 1\},\ j \in \{0, 1\}$. Conditioning on every combination of outcomes we can estimate seven independent moments from the data:

$$
\begin{aligned}
E[y_t | x_t = 1, z_t = 1] &= \alpha + \beta \frac{\eta_{11} p}{\eta_{11} p + \nu_{11}(1-p)} \\
E[y_t | x_t = 0, z_t = 1] &= \alpha + \beta \frac{(1 - \eta_{11} - \eta_{10} - \eta_{00}) p}{(1 - \eta_{11} - \eta_{10} - \eta_{00}) p + (1 - \nu_{00} - \nu_{10} - \nu_{11})(1-p)} \\
E[y_t | x_t = 1, z_t = 0] &= \alpha + \beta \frac{\eta_{10} p}{\eta_{10} p + \nu_{10}(1-p)} \\
E[y_t | x_t = 0, z_t = 0] &= \alpha + \beta \frac{\eta_{00} p}{\eta_{00} p + \nu_{00}(1-p)} \\
P[x_t = 1, z_t = 1] &= \eta_{11} p + \nu_{11}(1-p) \\
P[x_t = 0, z_t = 1] &= (1 - \eta_{11} - \eta_{10} - \eta_{00}) p + (1 - \nu_{00} - \nu_{10} - \nu_{11})(1-p) \\
P[x_t = 1, z_t = 0] &= \eta_{10} p + \nu_{10}(1-p) \; .
\end{aligned}
$$

However, we have to estimate nine coefficients: the two model parameters, the probability of deflation, and three joint conditional probabilities for every outcome of the true deflation indicator ($\eta_{11}, \eta_{10}, \eta_{00}, \nu_{11}, \nu_{10}, \nu_{00}$). We therefore need two additional assumptions. Two of the parameters measure the conditional probability that both indicators misclassify a deflationary period or inflationary period:

$$
\begin{aligned}
\eta_{00} &= P[x_t = 0, z_t = 0 | d_t = 1] \\
\nu_{11} &= P[x_t = 1, z_t = 1 | d_t = 0] \; .
\end{aligned}
$$

If we fix those probabilities at sensible values, we only have to estimate seven parameters and the model is identified.

The two probabilities should be relatively small. The reason is that, even if the measurement errors are correlated, the probability that they simultaneously misclassify an episode is smaller than the probability that they misclassify an episode individually. Therefore, we can fix these parameters to a reasonably small value and then examine the robustness of the results by varying those values. Fixing the probabilities at zero is equivalent to the conditional independence assumption. Fixing them at positive values allows for correlated measurement errors.

Extending this estimator to a well-measured binary covariate is straightforward if we are willing to assume that the binary covariate is conditionally independent from the deflation indicators. For example, for the case that $x_t = 1, z_t = 1$ we have:

$$E[y_t|x_t = 1, z_t = 1, q_t = 1] \;=\; \alpha + \beta \frac{\eta_{11}(1-\eta_q)p}{\eta_{11}(1-\eta_q)p + \nu_{11}\nu_q(1-p)} + \delta \; .$$

I do not list all moments for brevity. As for the case with conditional independence, the model is over-identified.

We can derive expressions for the bias of α and β, if we use only one indicator x_t:

$$\begin{aligned} plim\ \hat{\alpha} - \alpha &= \beta \frac{(\eta_{00} + \eta_{01})p}{(\eta_{00} + \eta_{01})p + (\nu_{00} + \nu_{01})(1-p)} \\ plim\ \hat{\beta} - \beta &= -\beta \left(\frac{(\nu_{10} + \nu_{11})(1-p)}{(\eta_{10} + \eta_{11})p + (\nu_{10} + \nu_{11})(1-p)} + \frac{(\eta_{00} + \eta_{01})p}{(\eta_{00} + \eta_{01})p + (\nu_{00} + \nu_{01})(1-p)} \right) \end{aligned} \tag{A.5}$$

A.5 Serial correlation in the error term

So far, we assumed that the error term is i.i.d. This assumption is violated if we fail to control for serial correlation in the dependent variable. As a consequence, our estimation approach may fail (Cosslett and Lee 1985). Black et al. (2000) suggest that their bounding approach allows to include additional controls as long as they are independent from the measurement errors in the mismeasured binary indicator. Under this assumption, we can therefore estimate β_{11} in:

$$y_t \;=\; \alpha + \beta_{11}\mathbf{1}_{\{x_t=1,z_t=1\}} + \beta_{10}\mathbf{1}_{\{x_t=1,z_t=0\}} + \beta_{01}\mathbf{1}_{\{x_t=0,z_t=1\}} + \phi y_{t-1} + \epsilon_t \;\; ,$$

which gives an upper bound for the shortfall of real activity during deflation. If $0 < \phi < 1$ we can show that the conditional mean of y_t during inflation and deflation, as well as the average shortfall during deflation, amount to

$$\begin{aligned} E[y_t|d_t = 0] &= \frac{\alpha}{1-\phi} \\ E[y_t|d_t = 1] &= \frac{\alpha+\beta}{1-\phi} \\ E[y_t|d_t = 1] - E[y_t|d_t = 0] &= \frac{\beta}{1-\phi} \end{aligned}$$

Note that the true model reads $y_t = \phi y_{t-1} + \alpha + \beta d_t + \varepsilon_t$. We can therefore express the conditional expectation in terms of its lagged value:

$$E[y_t|d_t = 0] = \phi E[y_{t-1}|d_t = 0] + \alpha \; .$$

Recursively replacing the lagged value of y_t yields:

$$E[y_t|d_t = 0] = \sum_{j=0}^{H} \alpha\phi^j + \phi^{H+1} E[\phi y_{t-H-1}|d_t = 0] \; .$$

Assuming $0 < \phi < 1$, computing the limit for $H \to \infty$, and applying standard results for geometric series yields $E[y_t|d_t = 0] = \alpha/(1-\phi)$. Analogous derivations apply for the other two conditional expectations.

B Data

This section provides an overview over the main deficiencies, an assessment of sampling error in historical price data, as well as additional information on the data sources to construct the modern replications, and the proxy.

B.1 Main deficiencies of retrospectively estimated CPIs

Table B.1 — Methodological deficiencies

Deficiency	Source	Time span	Comments
Wholesale prices	David and Solar (1977)	1774-1851	Wholesale prices approximate retail prices before 1800
Geographical coverage	David and Solar (1977)	1774-1851	Prices for Philadelphia (before 1800) and prices paid by Vermont farmers (until 1851)
Sample size	Hoover (1960)	1851-1860	Weeks Report shows many missing observations and small number of individual price quotes
	Long (1960)	1880-1890	Little information on retail prices for the entire decade after the Weeks Report ends
Reproduction cost index	Lebergott (1964)	1860-1880	Rent approximated by prices of construction materials and wages of low-skilled workers
Few services	Lebergott (1964)	1860-1880	
Linear interpolation	Long (1960)	1880-1890	Several items interpolated over the entire decade (particularly rent)

Notes: The time span represents the segment used in the composite CPI by Officer and Williamson (2016) reported in Officer (2014).

B.2 Number of observations in historical CPI data

A low number of observations is an additional deficiency in 19th century CPIs. Unfortunately, we do not observe the sampling standard error for retrospective CPI estimates. We can investigate, however, how many price quotes underlie a typical historical CPI and how much the modern sampling standard error would increase if we estimate a modern CPI based on a smaller number of individual price quotes.

Such an analysis requires an estimate of the sampling standard error of the modern CPI inflation rate; the number of observations underlying a modern CPI; an assumption on how the sampling standard error depends on the number of observations; and the number of individual price quote observations underlying retrospective estimates of historical CPI inflation.

The sampling standard error for modern CPI inflation is published by the U.S. Bureau of Labor Statistics (BLS) (see Shoemaker 2014). Because of the large number of observations, sampling error is a minor issue. Currently, the BLS collects more than 80,000 price quotes each month to calculate the CPI inflation rate. Therefore, the annual average inflation rate is based on more than 1,000,000 price quotes.[1] For a typical 12-month inflation rate in 2014, the sampling standard error amounts to 0.07%. A 95% confidence interval around a 1% 12-month inflation rate amounts to [0.9%, 1.1%].[2]

We can derive a relationship between the sampling standard error and the number of observations based on two simplifying assumptions. First, assume that the aggregate CPI inflation rate is the unweighed average of the individual price changes, and second, that those individual price changes are i.i.d. with finite variance s^2. Then, the estimated CPI inflation rate approximately follows a normal distribution, with mean equal to the true inflation rate and sampling variance equal to $\sigma^2 = s^2/N$. Because the BLS publishes an estimate of the sampling standard error (σ) as well as the number of observations (N), this formula allows us to back out the standard deviation of individual price changes (s). Then we can gauge the sampling standard error when reducing the number of individual price quotes.[3]

From the careful description by Hoover (1960) we can gauge the number of price quotes from the Weeks (1886) Report. The retrospective survey asked respondents to provide an average price for each year from 1851 to 1880. Hoover (1960) notes on p. 146 that "*This is by far the most extensive compilation of retail prices available for the nineteenth century.*". Nevertheless, the number of observations was substantially lower than for a modern CPI.[4]

Data for the Weeks Report was collected from one or two respondents in more than 40 cities. They were asked to retrospectively provide average annual prices for the years 1851-1880. If no average price could be provided, they could instead report the price of 1 June. The Weeks Report covered 60 items and Hoover (1960) added 14 additional items from other data sources.[5] If we assume that the annual average price reported is equivalent to 12 monthly observations and all respondents reported all items, the number of price quotes in a typical year exceeds 75,000 (see Table B.2).

Under a more realistic but still optimistic scenario, the number of monthly observations amounts to just over 8,000 each year. This scenario mimicks the situation towards the end of the Weeks Report

[1] I assume that if the BLS records a price quote it also observes the price change. Therefore, the actual number of observations used in calculating the inflation rate is lower if new products are introduced. This is a conservative assumption because missing data affects data collection for the 19th century more strongly than todays professionally organized survey schemes.

[2] I use the sampling standard error of the 12-month inflation rate to approximate the sampling standard error of the annual average inflation rate. Simulations indicate that this is a reasonable and inconsequential approximation.

[3] We have to examine, however, whether the simplifying assumptions reasonably approximate the more complicated methodology that is used to construct the CPI. Note that Shoemaker (2014) indicates that the primary reason for the higher standard error of the regional rather than at the U.S. inflation rate is that the sample size is smaller. A typical 1-month inflation rate in 2014 is based on a sample of approximately 87,000 price quotes, and the sampling standard error amounts to 0.04%. The formula would predict that the standard error for the Northeast region, which is based on a sample of approximately 18,400 price quotes, should amount to 0.09%. This is smaller than but close to the value reported by the BLS (0.10%). Repeating the exercise for several years since 2008 yields similar results and suggests that the simple formula yields sensible predictions.

[4] There is evidence that the information from Hoover (1960) gives us an upper bound to the number of observations used to compute retrospective CPIs for other segments. Other researchers have discarded almost half of the price quotes from the Weeks Report because prices for June 1 are not representative of the entire annual average and because price series were not continuously reported (see Officer 2014). Moreover, Long (1960) notes that the retail price data for the period 1880-1890, when the Weeks Report ended and before the BLS started to collect monthly data on food items, is even thinner.

[5] She added price data from less-representative sources for fruit, shoe repairs and physician fees. Fruit prices were estimated from wholesale prices for Philadelphia, whereas shoe repairs and physician fees stem from Adams (1939) prices paid by Vermont farmers. Moreover, she collected prices for newspapers from the library of congress.

Table B.2 — Price quotes underlying 19th century CPI inflation

	No missing	Optimistic	Pessimistic
Cities	43	43	43
Respondents	2	1.5	1.5
Items reported	74	74/3	74/4
Implied monthly observations.	12	12/2+1/2	12/4+3/4
Fraction of years	1	0.8	0.5
Number of price quotes	76,368	8,273	2,237

Note: Estimated number of price quotes in a typical year underlying the CPI constructed by Hoover (1960) if there are no missing observations, for an optimistic scenario on the number of missing observations, and for a pessimistic scenario. The total number of observations amounts to the product of the individual elements.

and is based on the discussion on p. 146 in Hoover (1960) of the completeness of the survey. I assume that, on average, 1.5 surveyed individuals responded, and they reported prices for 1/3 of the items. Half of the respondents are assumed to accurately report the average price over the year, equivalent to 12 monthly observations. The other half reported only one monthly observation. In this scenario, the vast majority of years are reported, which is in line with the fact that the survey was mostly complete for the last 5 to 10 years.

The number of observations tumbles to just over 2,000 each year under a pessimistic scenario, which reflects the situation for the early period of the Weeks Report. I set the share of items reported to 1/4. Moreover, I take into account that many of the annual averages reported were informed guesses based on partially available information rather than averages of accurate monthly information. I thus reduce the share of implicit monthly observations to 1/4 and assume that the remaining 3/4 correspond to one monthly price quote representative of the entire year. Finally, I assume that only half of all annual observations were reported, which is line with the fact that the data are particularly scanty at the beginning of the sample period.

In a typical price index for the 19th century the number of price quotes therefore ranges from about 2,000 to just over 8,000. If we assume that the number of observations lies in the middle of an optimistic and pessimistic scenario, namely just over 5,000 observations, the sampling standard error for the 12-month inflation rate increases to 0.99%. A 95% confidence interval for the measured CPI inflation rate of 1% amounts to [-0.9%, 2.9%], which is considerably wider than in 2014.

At least since 1921, the number of individual price quotes was sizeable. A BLS bulletin from 1923 allows us to gauge a bound for the number of monthly price quotes collected in a year (see BLS 1923). By 1921, food prices were collected in 51 cities and for 28 items. The number of price quotes varied with the size of the cities from 10-15 (smaller cities) to 20-30 (larger cities). Assuming that, on average, 20 price quotes were collected each month for each item, the sample size amounted to 342,000 price quote observations each year. For most other items, there is no reliable information on how many price quotes were collected. We know, however, that by 1919 the number of items was substantial (see BLS 1941). The BLS collected prices for the following commodity groups (number of items in parentheses): clothing (65), fuel and lighting (6), rent (1), house furnishings (24), and miscellaneous goods and services (39). Officer (2014) reports that all items other than food were collected in 32 urban areas. Few prices, however, were collected on a monthly basis, because the CPI was published only for selected months of the year. If we conservatively assume that only one price quote was collected each quarter for each item in each of the 32 urban areas, this yields another 17,000 price quotes each year. Thus, by 1921, we obtain a conservative estimate of the number of price quotes underlying the annual CPI inflation rate of 360,000 observations. For the early 20th century, sampling error arising from a small number of price quotes is therefore a minor issue. If we gauge the sampling error using the formula derived before, a confidence interval around a 1% inflation rate amounts to $[0.8\%, 1.2\%]$, which is not much wider than in 2014.

B.3 Data sources for replication and proxy

Table B.3 shows the CPI basket by Hoover (1960), the modern series used to replicate the 19th century CPI, and the data series used to construct the proxy (see also Figure B.1). The identifiers are abbreviations of those used by the BLS, the Federal Reserve Bank of St. Louis (fred.stlouisfed.org), and Historical Statistics of the United States (hsus.cambridge.org).[6]

Table B.3 — Basket and data sources

Hoover (1960)		Replication				Proxy		
Description	Weight	Weight 2017	Description	Identifier	Period	Description	Identifier	Period
Food	**57.4**	**13.384**				Processed foods and feeds	WPU02	1947-2017
						Foods	Cc115	1798-1941
Flour	9.7	0.04	Flour and prepared flour mixes	CSEFA01	1980-2017			
			Flour and prepared flour mixes	MSE0101	1977-1998			
			Cereals and pastry products	MSA1111	1935-1998			
Corn meal	1.1	0.119	Rice, pasta and cornmeal	CSEFA03	1977-2017			
Rice	0.2		Rice, pasta and cornmeal	MSE0103	1977-1997			
			Cereals and pastry products	MSA1111	1935-1998			
Beef, fresh	5.8	0.436	Beef and veal	CSEFC	1935-2017			
Beef, corned	0.6							
Veal	1.5							
Pork, fresh	1.3	0.30	Pork	CSEFD	1935-2017			
Pork, other	1.5							
Mutton	1.5	0.227	Other meats	CSEFE	1953-2017			
			Meats	SA111211	1947-1997			
Fish	1.4	0.25	Fish and seafood	CSEFG	1935-2017			

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[6] For the modern CPI they can be accessed under download.bls.gov/pub/time.series/cu/cu.series, for the discontinued series under download.bls.gov/pub/time.series/mu/mu.item, for the PPI data under fred.stlouisfed.org/categories/33583, and for the historical wholesale price data under hsus.cambridge.org/HSUSWeb/toc/showTable.do?id=Cc66-204. The modern CPI weights for 2017 stem from www.bls.gov/cpi/tables/relative-importance/2017.txt. All links accessed on 21 October 2019.

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Hoover (1960)			Replication			Proxy		
Description	Weight	Weight 2017	Description	Identifier	Period	Description	Identifier	Period
Milk	3.0	0.207	Fresh whole milk	CSS09011	1939-2017			
Butter	5.9	0.0305	Butter	CSS10011	1939-2017			
Cheese	0.5	0.242	Cheese and related products	CSEFJ02	1977-2017			
			Dairy products	MSA1113	1935-1990			
Potatoes	4.6	0.478	Fresh vegetables	CSEFL	1950-2017			
Beans	2.4							
Fruit	2.5	0.555	Fresh fruits and melons (wholesale)	WPU0111	1950-2017			
			Sweet potatoes (wholesale)	WPU011303	1950-2017			
			Potatoes (retail)	CSEFL01	1950-2017			
Eggs	2.3	0.097	Eggs	CSEFH	1935-2017			
Tea	1.3	0.093	Other beverage materials including tea	CSEFP02	1997-2017			
			Nonalcoholic beverages	MSE17	1947-1998			
Coffee	4.0	0.168	Coffee	CSEFP01	1980-2017			
			Roasted coffee	MSS17031	1939-1997			
Lard	1.5	0.0305	Margarine	CSS16011	1939-2017			
Sugar	4.5	0.282	Sugar and sweets	CSEFR	1935-2017			
Molasses	0.2							
Sirup	0.1							
Clothing	11.0	3.018				Apparel	WPU0381	1947-2017
						Textile products	Cc117	1798-1941
Mousselines de laine	2.2	1.044	Women's apparel	CSEAC	1990-2018			
Satinets	1.2		Women's and girls' apparel	MSA3112	1947-1998			
Overalls	0.9	0.581	Men's apparel	CSEAA	1990-2017			
Shirtings	0.4		Men's apparel	MSA3111	1947-1998			
Cotton flannel	2.3							
Boots	4.0	0.671	Footwear	CSEAE	1990-2017			
			Footwear	MSE40	1947-1998			
Household commodities	4.0	1.778				Textile house furnishings	WPU0382	1947-2017
						House furnishing goods	Cc122	1840-1941
Soap, common	0.3	0.0.704	Personal care products	CSEGB	1980-2017			

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Hoover (1960)			Replication			Proxy		
Description	Weight	Weight 2017	Description	Identifier	Period	Description	Identifier	Period
			Toilet goods and personal care appliances	CSE64	1947-1998			
Starch	0.2	0.835	Housekeeping supplies	CSEHN	1980-2017			
			Housekeeping supplies	MSE33	1967-1998			
			Soaps and detergents	MSS33011	1950-1997			
Sheetings	1.2	0.261	Window and floor coverings and other linens	CSEHH	1997-2017			
Prints	2.1		Textile housefurnishings	MSE28	1950-1997			
Tickings	0.2							
Fuel and light	7.0	4.679	Energy commodities	CSACE	1980-2017	Fuels and related products and power	PPIENG	1926-2017
			Energy commodities	MSACE	1957-1998	Fuel and lighting	Cc118	1798-1941
			Fuel oil and other household fuel commodities	M102SE25	1940-1997			
Services less rent	2.9							
Newspapers	1.1	0.067	Newspapers and magazines	CSERG01	1997-2017			
			Newspapers	MSE5901	1947-1997			
Shoe repairs	0.7	0.104	Repair of household items	CSEHP04	1997-2017			
			Household maintenance and repair services	MSE23	1963-1998			
			Services	MSAS	1956-1997			
Medical care	1.1	6.924	Medical care services	CSAM2	1980-2017			
			Medical care services	MSA512	1950-1998			
Rent	17.7	30.099	Rent of primary residence	CSEHA	1930-2017			

Figure B.1 — Price series for replication

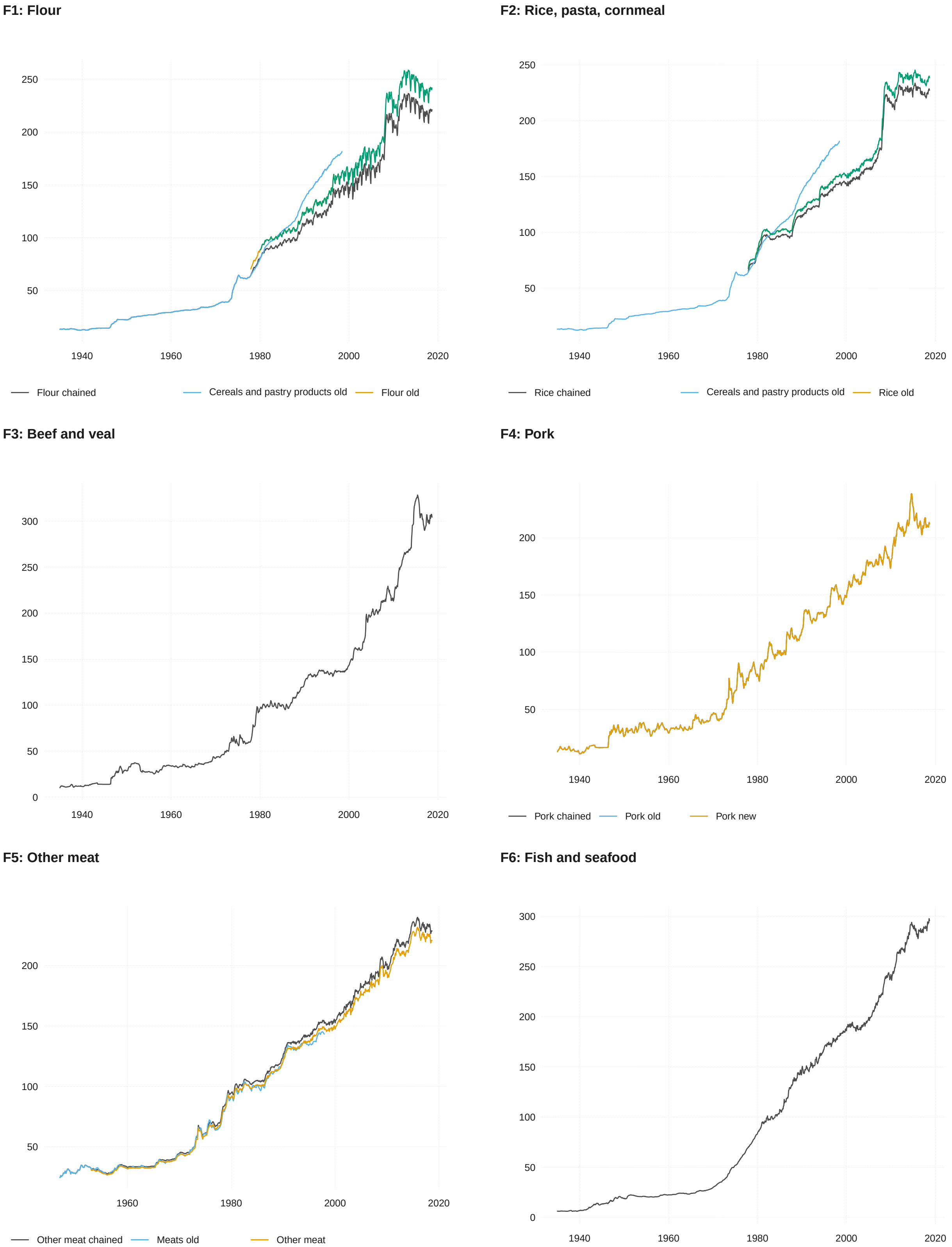
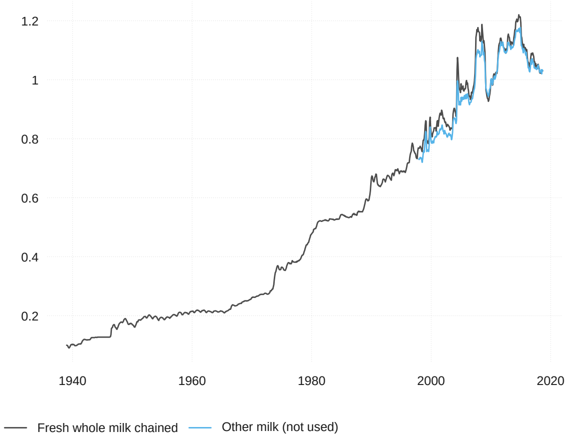


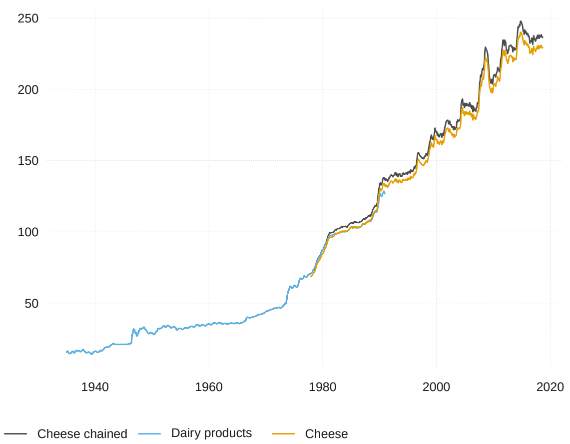
Figure B.1 — Price series for replication (continued)

F7: Fresh whole milk

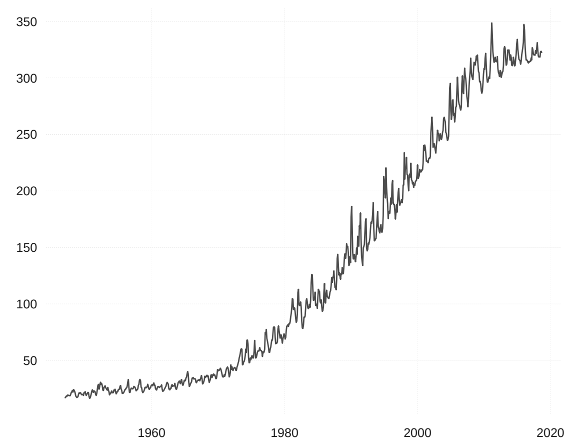


F8: Butter

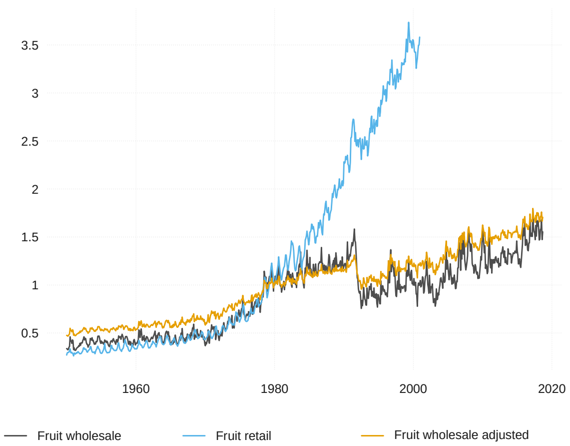
F9: Cheese



F10: Fresh vegetables



F10: Fruit wholesale and retail prices



F12: Eggs

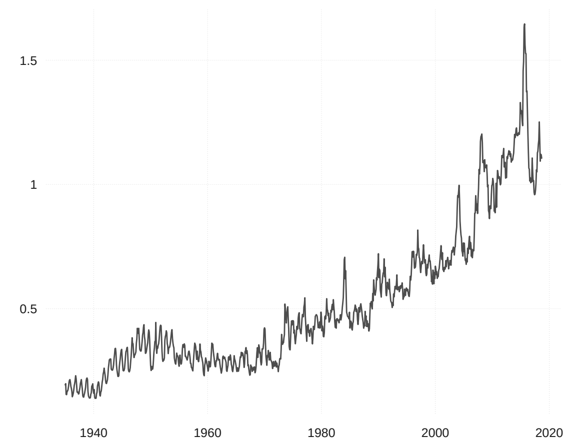


Figure B.1 — Price series for replication (continued)

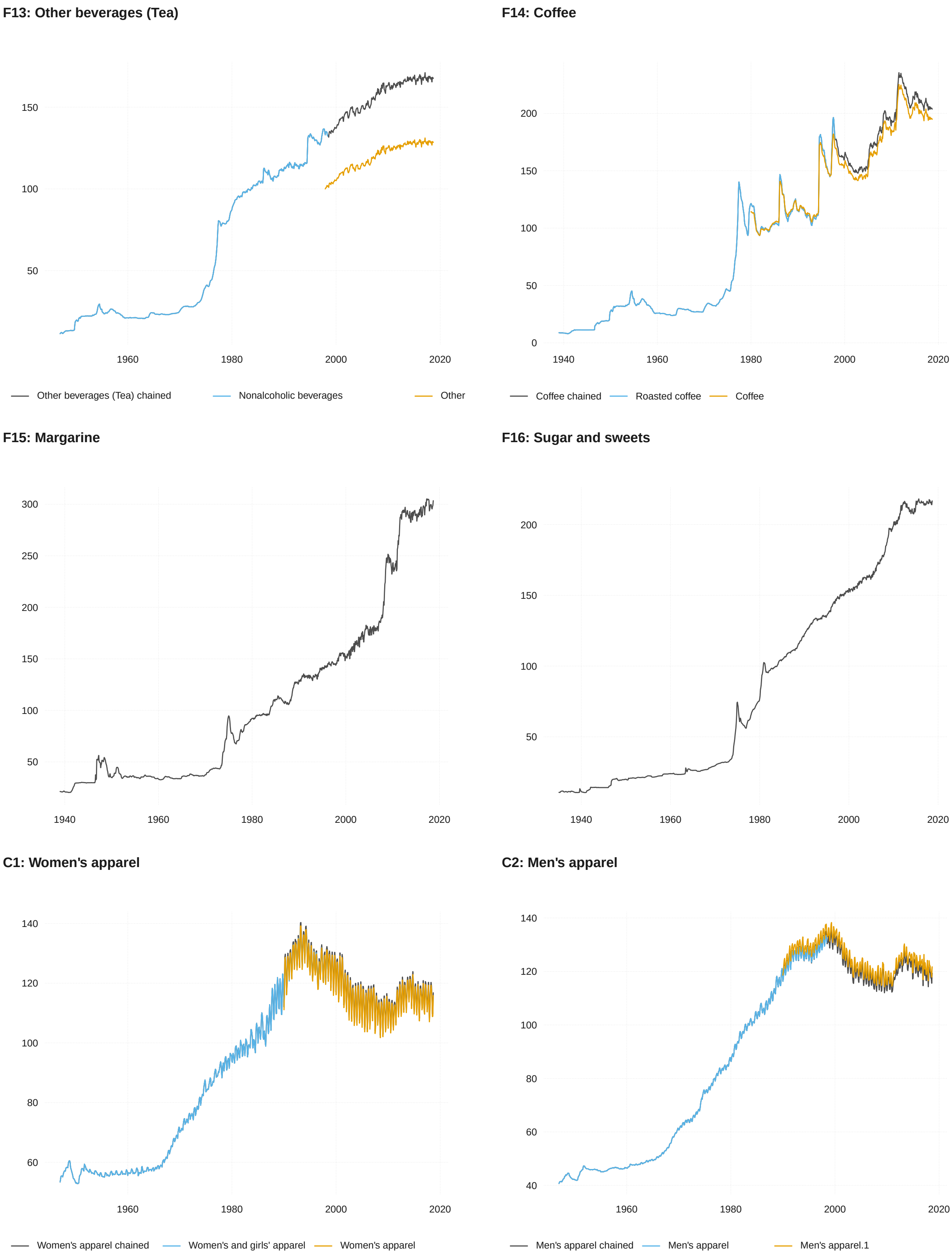
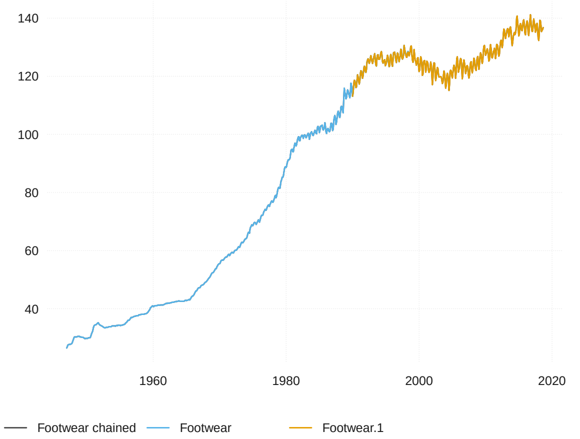
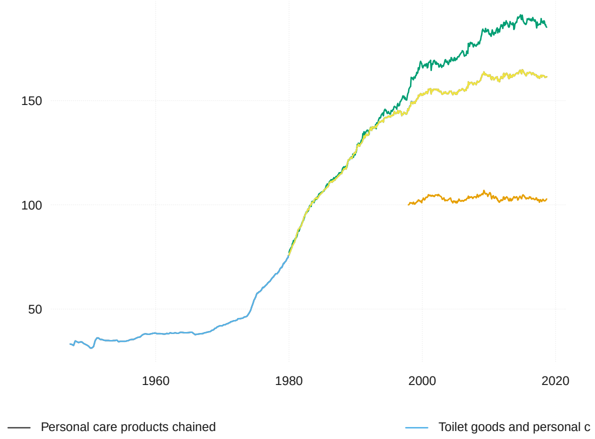


Figure B.1 — Price series for replication (continued)

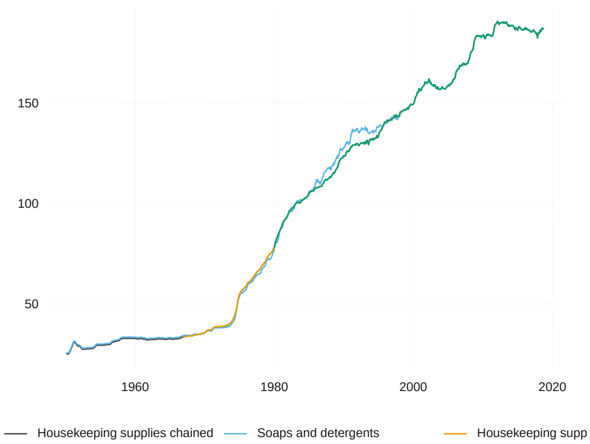
C3: Footwear



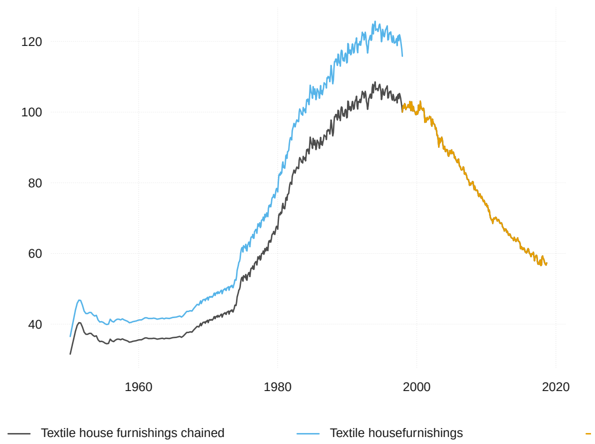
H1: Personal care products (Soap)



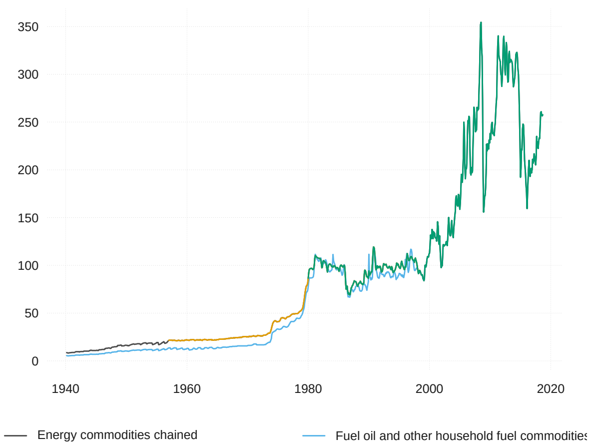
H2: Housekeeping supplies (Starch)



H3: Textile house furnishings



E1: Energy commodities (fuel and light)



O1: Newspapers

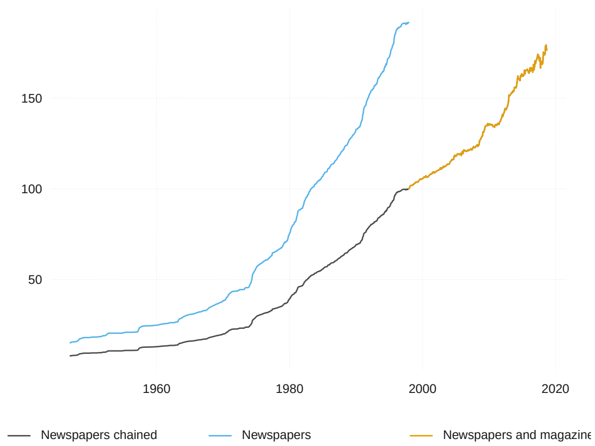
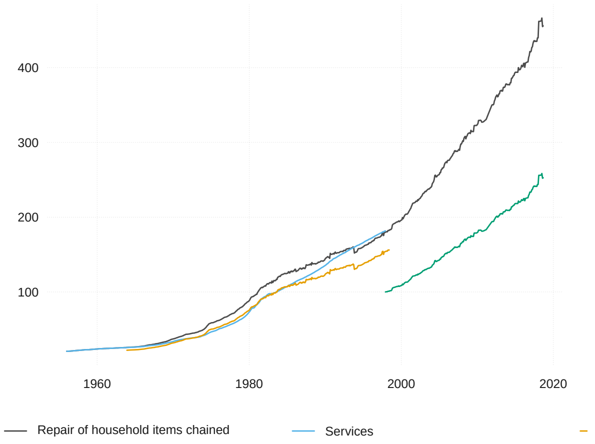
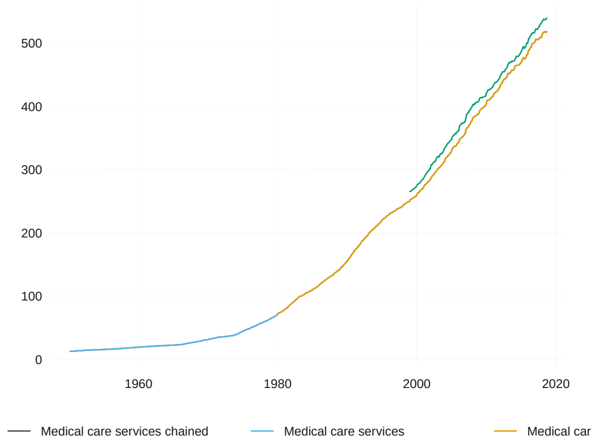


Figure B.1 — Price series for replication (continued)

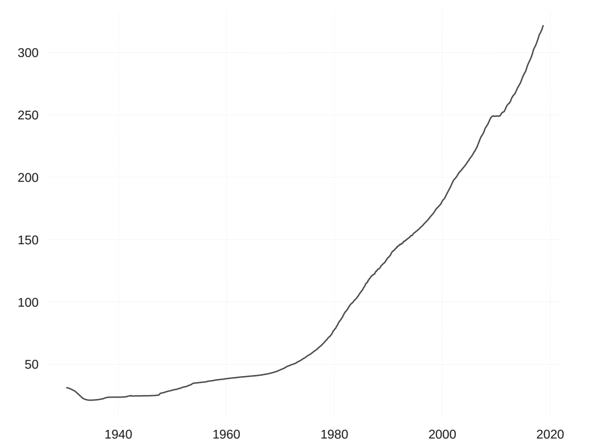
O2: Repair of household items (Shoe repair)



O3: Medical care services



R1: Rent



B.4 Construction of the proxy

For the proxy, I use the Warren and Pearson (1933) data from Hanes (2006). I link these series with the consistent replication by Hanes (1998) based on BLS data in 1890. Each wholesale price index is then attributed to an aggregate item in the CPI basket: food, clothing, household items, fuel and light (see Table B.3). The proxy is then calculated as a fixed weight price index with weights by Hoover (1960), for the 19th century, and with CPI weights in December 2017, for the post-WWII period. The index is a rough approximation to a CPI because I do not adjust the volatility of wholesale prices. In addition, the price series for house furnishings starts only in 1840. I therefore splice this series with the series for textile products.

Ideally, the proxy and the composite CPI by Officer and Williamson (2016) use distinct data sources. If this is the case, their measurement errors are to some extent independent. Indeed, this is the case for most of the 19th century. The Warren and Pearson (1933) data up to 1890 stem from New York newspapers supplemented by prices published in the Finance Report (1863) (see also Hanes 2006). After 1890, Hanes (1998) provides prices series consistent with Warren and Pearson (1933) based on BLS data.

By contrast, from 1800 to 1851, the composite CPI uses retail prices for some benchmark years and prices paid by Vermont farmers to interpolate in between (see David and Solar 1977). From 1851 to 1860, Hoover (1960) mainly uses retail prices. Although she also uses some wholesale prices, the sources are distinct: Hoover (1960) uses prices for five fruit items from Philadelphia and from the Aldrich Report (1893). The Lebergott (1964) segment from 1860-1880 mainly uses prices from the Weeks Report. Only for building materials, which are not included in the proxy, Lebergott (1964) uses some wholesale prices. Although retail prices are scarce from 1880 to 1890, Long (1960) does not use wholesale prices. From 1890–1914, the underlying data sources of the composite CPI and the proxy show some overlap. The CPI segment by Rees (1961) from 1890-1914 uses wholesale prices for eleven items from the BLS (1923). For most of the 19th century, however, the data sources underlying the CPI and the proxy are therefore distinct.

B.5 Construction of the CPI replication

To replicate the CPI by Hoover (1960) I match 19th century expenditure items with modern counterparts (see Table B.3).[7] When the modern price series start after 1960 I link them with closely corresponding discontinued BLS price series (either a close match or the next higher aggregate). The overlapping series usually closely match each other (see Figure B.1). If there is no overlap, I linearly interpolate the series in between making sure that the indices have the same base year.

Some remarks are in order. For milk, the modern weight covers all types of milk, not only whole milk. For butter and margarine no separate modern CPI weights are available. I therefore equally split this weight. For rent, the modern weight includes rent for the primary residence and owner-occupied rent-equivalent.

I follow Hoover (1960) and use the wholesale price for fruit with a rough adjustment accounting for the higher volatility of prices at the wholesale stage. I use the relative inflation volatility for potatoes at the wholesale and retail stage to adjust the volatility of fruit wholesale price inflation, keeping the

[7] I use a slightly higher level of aggregation than Hoover (1960).

average inflation rate unchanged.

For clothing, I attributed the raw materials presumably used for producing men's (shirtings, cotton flannel) and women's clothing (mousselines de laine, satinets) to the corresponding finished goods CPI category. I attributed the raw materials likely used for production of other textiles than clothing (sheetings, prints, tickings) to textile house furnishings. For shoe repairs, I linked repair of household items with household maintenance and repair services. Because the latter starts only in 1963, I additionally link this series with service prices.

All series start at least in 1960. The replication is then calculated as a fixed weight price index with the corresponding CPI weights in December 2017. This takes into account that information on expenditure weights were usually available only for single years during the 19th century.

B.6 Properties of the modern replications

Table B.4 shows descriptive statistics of the various inflation measures, as well as for the corresponding measurement errors. The errors are on average positive, suggesting that the proxy and replication on average underestimate inflation. The standard deviation of CPI inflation amounts to 2.8. Meanwhile, the standard deviation of the errors ranges from 1.1–3.1. Therefore, the signal-to-noise ratio ranges from 0.8–6.1.

Table B.4 — Descriptive statistics (1960-2017)

Statistic	Mean	St. Dev.	Min	Max
CPI	3.82	2.82	−0.36	13.55
Proxy	3.07	4.50	−6.97	17.75
Replication	3.47	2.20	−1.84	10.62
CPI−replication	0.35	1.14	−1.63	3.37
CPI−proxy	0.75	3.11	−9.35	6.61

Notes: Inflation rates measured in percent, differences measured in percentage points. All statistics calculated using annual data from 1960-2017.

Table B.5 provides regression results to show how the proxy and replication relate to actual inflation and whether the measurement errors are i.i.d. The first two columns regress the two error-ridden measures of inflation on actual CPI inflation, in line with the functional form assumed in the simulation exercise (see Eq. 5 in the main text). For the replication, the slope is smaller than one and the constant positive. Therefore, the measurement errors are not of the classical type, which would require that the slope is one and the constant is zero. Meanwhile, the R^2 is relatively high (0.86) suggesting that the variance of the remaining measurement errors is low. For the proxy, the opposite pattern emerges. The slope is larger than one—although the difference is not statistically significant—, the constant significantly negative, and the R^2 is lower than for the replication. The third and fourth columns show that the measurement errors are not i.i.d. The error of the replication is significantly related to its own lag and also related to past errors of the proxy.

Table B.5 — Properties of measurement errors (1960-2017)

	Replication	Proxy	CPI-replication	CPI-proxy
CPI	0.72^{***}	1.17^{***}		
	(0.04)	(0.20)		
CPI-proxy			0.05	
			(0.06)	
CPI-replication				0.76
				(0.98)
CPI-proxy (t-1)			-0.15^{***}	0.34^{***}
			(0.03)	(0.09)
CPI-replication (t-1)			0.73^{***}	-0.03
			(0.11)	(0.91)
Constant	0.71^{***}	-1.38^{**}	0.15	0.24
	(0.23)	(0.63)	(0.13)	(0.41)
N	57	57	56	56
R^2	0.86	0.53	0.60	0.14

Notes: Linear regressions of the replication and the proxy on the CPI (first and second columns). Regressions of the measurement errors of the replication and the proxy on current and lagged values of the measurement errors (third and fourth columns). The measurement errors are calculated as the difference between actual inflation and the error-ridden inflation rates. HAC-robust standard errors are given in parentheses. Coefficients with superscripts ***,**,* are statistically significant at the 1%, 5%, 10% level.

B.7 Other data sources

Table B.6 — Other data sources

Name	Time	Identifier	Comments
United States			
CPI	1774-2018		Officer and Williamson (2016)
Real GDP	1790-2018		Johnston and Williamson (2016); per capita series available
Industrial production	1790-1915		Davis (2004)
	1899-1937	Dd495	Atack and Bateman (2006) linked with Fabricant (1940)
	1919-2015	INDPRO	`fred.stlouisfed.org`
Banking crises	1825-1929		Jalil (2015); 1833-1834, 1837-1839, 1857, 1873, 1893, 1907
Stock prices	1802-1870	Cj797	Rousseau (2006); index of common stocks
	1870-2015		Jordà et al. (2016) and Knoll et al. (2017)
Money supply	1867-1947		M2 by Friedman and Schwartz (1963) as reported by Anderson (2003)
United Kingdom			Hills et al. (2019)
Real GDP per capita	1800-2016	A1-B	Longer series available
Total production and construction	1800-1913	A14-Q	Longer series available
Private consumption	1830-2016	A1-P	
Investment	1830-2016	A1-Q	
Unemployment rate	1759-2016	A1-AB	
CPI	1800-2016	A1-AO	Used for baseline
WPI	1830-2016	A47-J	Used as proxy
Norway			
Real GDP per capita	1830-2017		Grytten (2004b)
CPI	1800-2017		Grytten (2004a). Used for baseline
Proxy	1777-1920		Klovland (2013). Own calculations. Wholesale prices weighed by CPI weights
Finland			Hjerppe (1989)
Real GDP per capita	1860-1949		
CPI	1860-1949		Cost-of-living index. Used for baseline
WPI	1860-1949		Used as proxy
Switzerland			
Real GDP per capita	1851-1913	Q.1a-G	HSSO (2012a). Two measures available. I use the one deflated by GDP deflator. See Stohr (2004) and Ritzmann and David (2012) for the underlying data
CPI	1800-2017		Studer and Schuppli (2008) and Kaufmann (2019) based on various data sources

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TABLE B.6 – *continued from previous page*

Name	Time	Identifier	Comments
Proxy	1813-1928		HSSO (2012b). Own calculations. Wholesale prices weighed by CPI weights similar to Norway
Spain			Carreras and Tafunell (2005)
Real GDP per capita	1850-1899	T17.8	
CPI	1800-1936	T16.9	Various CPI series. First series used as baseline, second as proxy
Austria			Bank of Greece, Bulgarian National Bank, National Bank of Romania, Oesterreichische Nationalbank (2014)
Real GDP per capita	1870-1913	AH 6.1_A	Large jump in 1900
CPI	1850-1936	AH 5_A	
Chile			Braun et al. (2000)
Real GDP per capita	1810-1995	T1.1	
CPI	1810-1995	T4.1	Interpolation until 1830. Therefore, start in 1830
Hungary			Bank of Greece, Bulgarian National Bank, National Bank of Romania, Oesterreichische Nationalbank (2014)
Real GDP per capita	1870-1913	AH 6.1_A	Large jump in 1900
GDP deflator	1870-1936	AH 6.1_A	Implicit GDP deflator calculated from real and nominal GDP
Portugal			Mata and Valério (2011)
Real GDP per capita	1865-1931		
GDP deflator	1865-1931		CPI series almost identical to GDP deflator
Sweden			
Real GDP per capita	1800-2014	II.A4.1-K	Edvinsson (2014). Longer series available
CPI	1800-2014	I.A8.1-C	Edvinsson and Söderberg (2010). Longer series available

Figure B.2 — International data

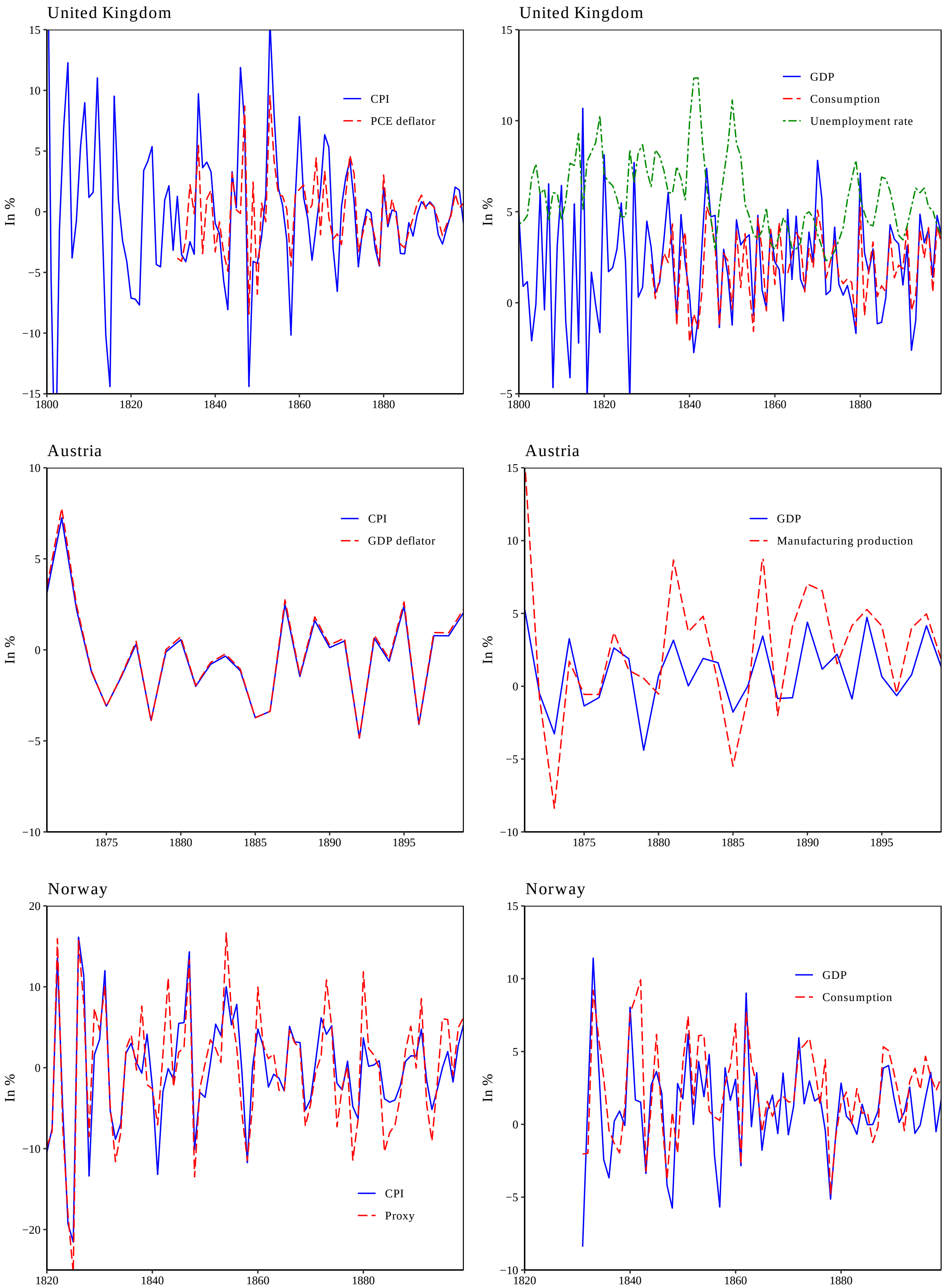


Figure B.2 — International data (continued)

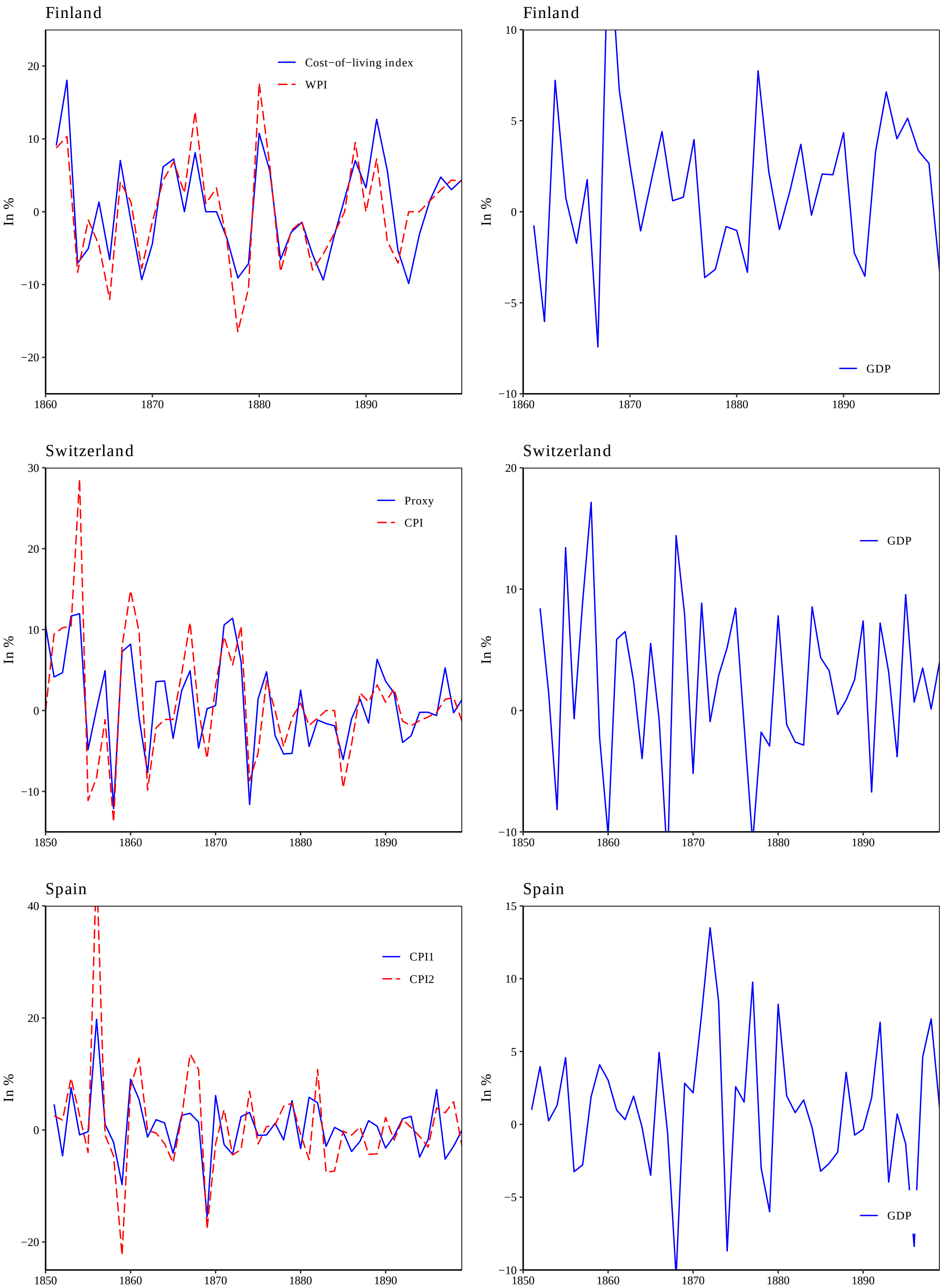


Figure B.2 — International data (continued)

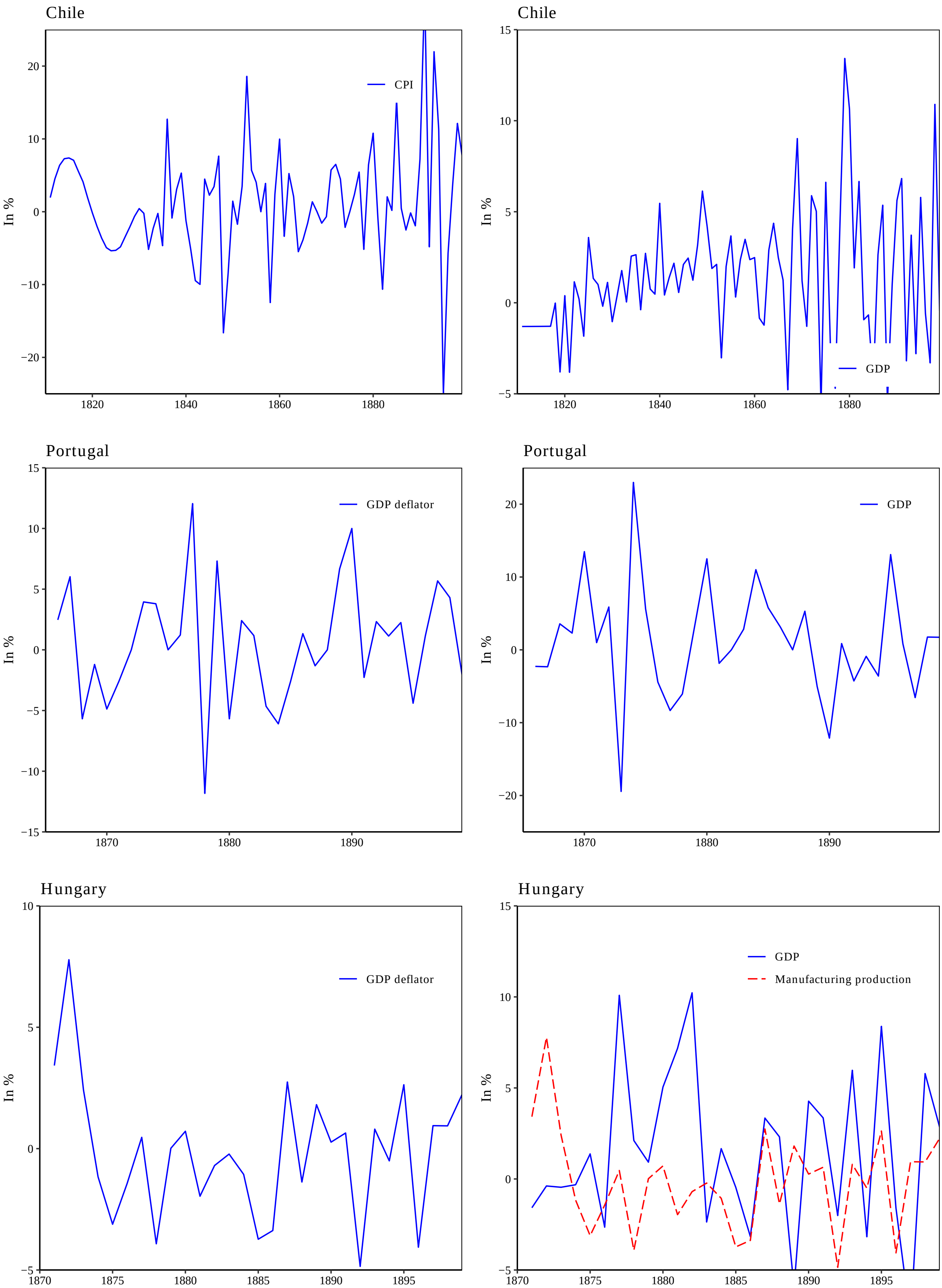
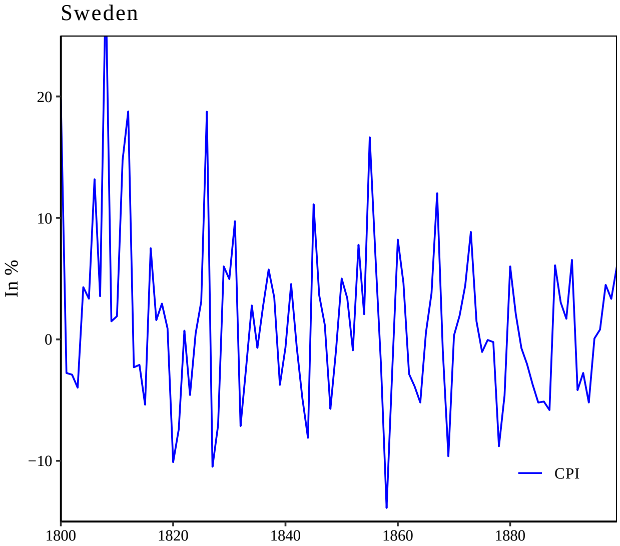


Figure B.2 — International data (continued)



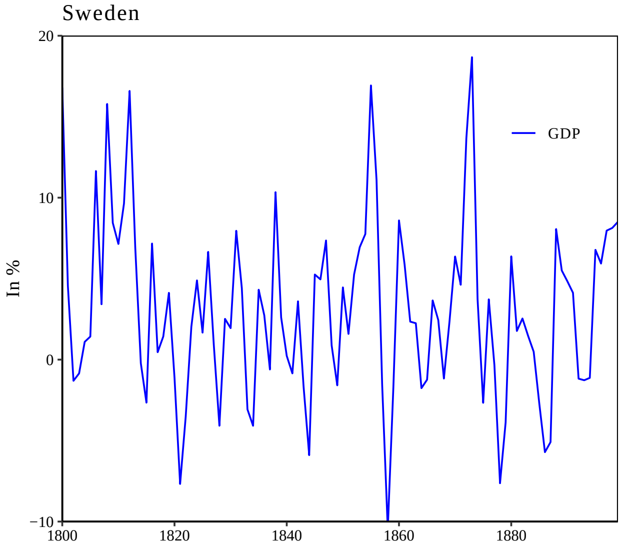
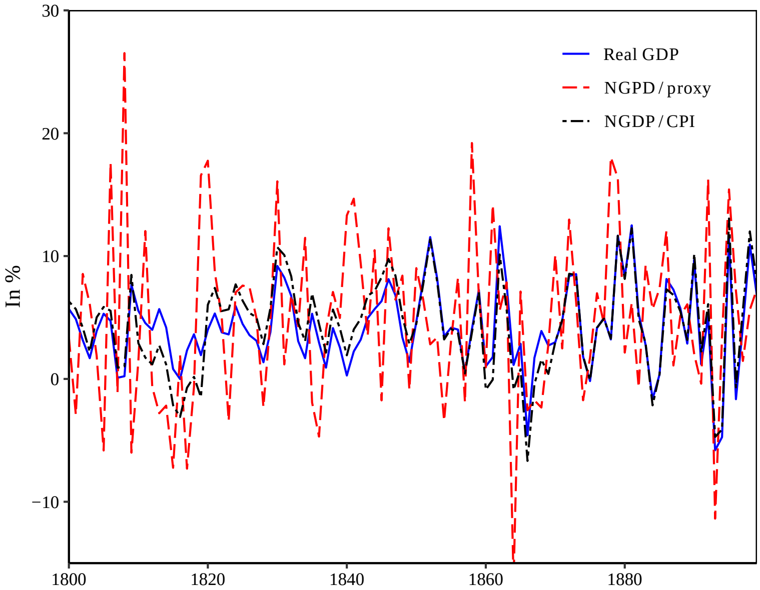


Figure B.3 — U.S. real GDP growth with various deflators



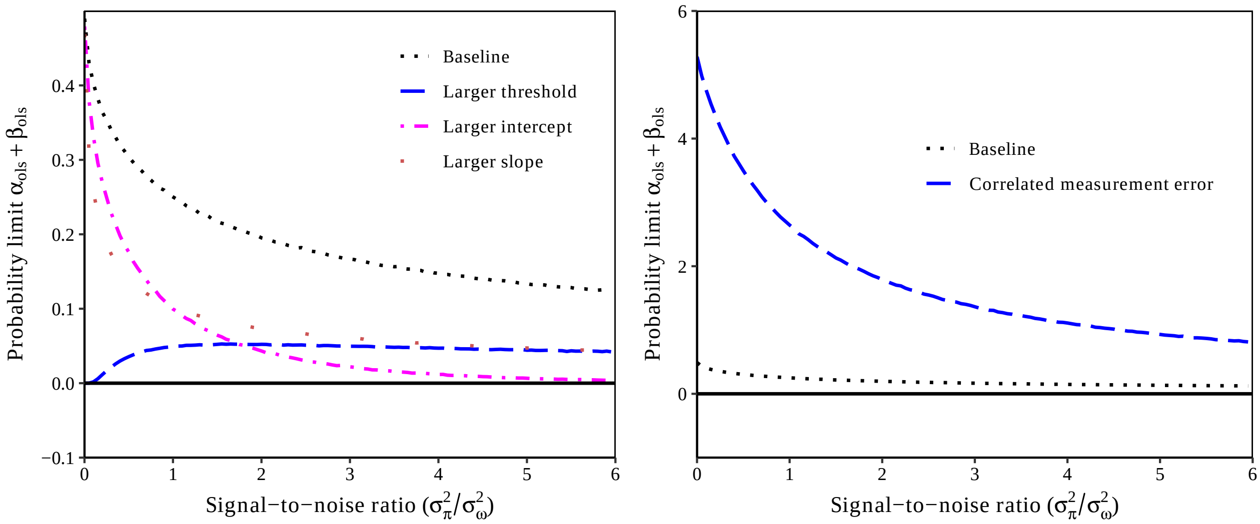
Notes: U.S. real GDP (Johnston and Williamson 2019); nominal GDP (Johnston and Williamson 2019) deflated by error-ridden proxy; nominal GDP deflated by the CPI (Officer and Williamson 2016).

C Additional results

Figure C.1 — Simulated probability limit

(a) $\alpha + \beta$ with mismeasured inflation

(b) $\alpha + \beta$ with mismeasured economic activity and inflation



Notes: The figure shows the probability limit of the OLS estimator of $y_t = \alpha + \beta x_t + \epsilon_t$, where $x_t \equiv \mathbf{1}_{\{\tilde{\pi}_t < c\}}$, as a function of the signal-to-noise ratio ($\sigma^2_\pi/\sigma^2_\omega$). The dotted horizontal lines give the true value of $\alpha + \beta = 0$. The error-ridden inflation rate depends linearly on the well-measured inflation rate ($\tilde{\pi}_t = \rho_0 + \rho_1 \pi_t + \omega_t$). The well-measured inflation rate (π_t) and the measurement errors (ω_t) are assumed to be identically and independently normally distributed with zero mean. The baseline simulation assumes $c = 0, \rho_0 = 0, \rho_1 = 1$, and $\sigma = \sqrt{\sigma^2_\pi + \sigma^2_\omega} = 6$ (dashed line). The other simulations assume a larger threshold ($c = 5$), a larger intercept ($\rho_0 = 5$), and a larger slope ($\rho_1 = 3$). Panels (a) and (b) show the probability limit with uncorrelated measurement errors in the dependent variable. Panels (c) and (d) show the probability limit with negatively correlated measurement errors in the dependent variable.

Table C.1 — U.S. industrial production growth during inflation and deflation (1800–1869)

	Baseline	Cond. independence			Cond. dependence
Model parameters:					
$\alpha = E[y \mid \pi > 0]$	5.65***	6.87***	6.33***	9.19***	7.95***
	(1.15)	(1.17)	(1.82)	(2.18)	(1.82)
$\beta = E[y \mid \pi < 0] - E[y \mid \pi > 0]$	-0.52	-2.85*	-2.09	-8.37*	-4.96*
	(1.71)	(1.54)	(3.29)	(4.36)	(2.54)
$\alpha + \beta = E[y \mid \pi < 0]$	5.13***	4.02***	4.24**	0.82	2.98**
	(1.18)	(1.04)	(1.94)	(2.55)	(1.51)
$P[\pi < 0]$			0.44		0.51**
			(0.40)		(0.22)
Bias estimates:					
$plim\ \hat{\alpha} - \alpha$			-0.68		-2.03
			(1.14)		(1.58)
$plim\ \hat{\beta} - \beta$			1.21		2.46*
			(0.83)		(1.45)
$plim\ \hat{\alpha} + \hat{\beta} - \alpha - \beta$			0.89		2.15*
			(1.17)		(1.10)
N	71	71	71	71	71
Bound	Upper	Upper	Point	Lower	Point
Method	OLS	OLS	GMM	IV	GMM
Indicator	CPI	CPI, proxy	CPI, proxy	CPI, proxy	CPI, proxy

Notes: Model: $y_t = \alpha + \beta x_t + \epsilon_t$ where $x_t \equiv \mathbf{1}_{\{\tilde{\pi}_t < 0\}}$. Model parameters: Mean growth rate during inflation α; shortfall during deflation β; mean growth rate during deflation $\alpha + \beta$; probability of deflation ($P[\pi < 0]$). Bias estimates: Difference between the probability limit if we would only use the CPI; calculated based on the underlying GMM estimates, with standard errors computed using the delta method. Baseline: OLS estimates using the CPI. Conditional independence: Bounds and a point estimates using the CPI and proxy, assuming conditional independence. Conditional dependence: Point estimates using the CPI and proxy, assuming that the joint misclassification probabilities equal 0.15. HAC-robust standard errors are given in parentheses. ***,**,* denotes significance at the 1%, 5%, 10% level.

Table C.2 — U.S. industrial production growth during inflation and deflation (1870–1899)

	Baseline	Cond. independence			Cond. dependence
Model parameters:					
$\alpha = E[y\|\pi > 0]$	9.31***	10.72***	11.13***	13.42***	17.32***
	(1.79)	(1.80)	(2.05)	(3.48)	(6.66)
$\beta = E[y\|\pi < 0] - E[y\|\pi > 0]$	-6.19**	-8.36***	-9.03***	-13.04**	-16.48**
	(2.67)	(2.33)	(2.30)	(5.13)	(6.65)
$\alpha + \beta = E[y\|\pi < 0]$	3.12*	2.36	2.10	0.38	0.83
	(1.65)	(1.44)	(1.50)	(2.08)	(1.73)
$P[\pi < 0]$			0.61***		0.71***
			(0.16)		(0.21)
Bias estimates:					
$plim\ \hat{\alpha} - \alpha$			-1.82		-7.23
			(1.86)		(6.74)
$plim\ \hat{\beta} - \beta$			1.22		7.37
			(1.43)		(6.63)
$plim\ \hat{\alpha} + \hat{\beta} - \alpha - \beta$			1.02		2.29
			(1.48)		(1.76)
N	30	30	30	30	30
Bound	Upper	Upper	Point	Lower	Point
Method	OLS	OLS	GMM	IV	GMM
Indicator	CPI	CPI, proxy	CPI, proxy	CPI, proxy	CPI, proxy

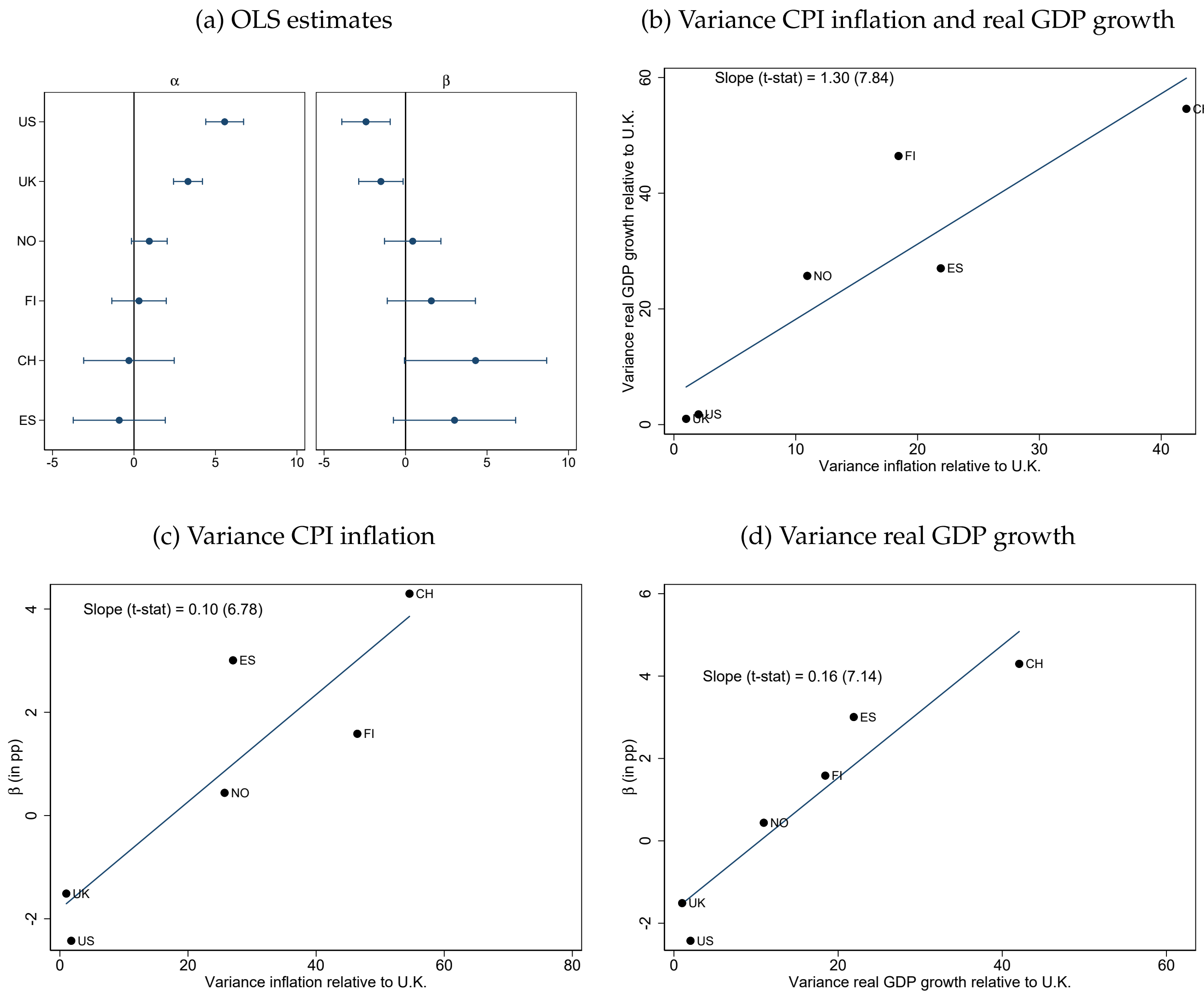
Notes: Model: $y_t = \alpha + \beta x_t + \epsilon_t$ where $x_t \equiv \mathbf{1}_{\{\tilde{\pi}_t < 0\}}$. Model parameters: Mean growth rate during inflation α; shortfall during deflation β; mean growth rate during deflation $\alpha + \beta$; probability of deflation ($P[\pi < 0]$). Bias estimates: Difference between the probability limit if we would only use the CPI; calculated based on the underlying GMM estimates, with standard errors computed using the delta method. Baseline: OLS estimates using the CPI. Conditional independence: Bounds and a point estimates using the CPI and proxy, assuming conditional independence. Conditional dependence: Point estimates using the CPI and proxy, assuming that the joint misclassification probabilities equal 0.15. HAC-robust standard errors are given in parentheses. ***,**,* denotes significance at the 1%, 5%, 10% level.

Table C.3 — U.S. GDP growth during inflation and deflation (1800–1899)

	Baseline	Cond. independence			Cond. dependence
Model parameters:					
$\alpha = E[y\|\pi > 0]$	4.95***	5.57***	5.70***	6.44***	6.69***
	(0.49)	(0.59)	(0.71)	(0.96)	(1.01)
$\beta = E[y\|\pi < 0] - E[y\|\pi > 0]$	-1.50**	-2.43***	-2.63***	-4.53***	-4.25***
	(0.72)	(0.76)	(0.81)	(1.70)	(1.30)
$\alpha + \beta = E[y\|\pi < 0]$	3.45***	3.14***	3.07***	1.91**	2.44***
	(0.53)	(0.49)	(0.52)	(0.90)	(0.67)
$P[\pi < 0]$			0.56***		0.58***
			(0.15)		(0.13)
Bias estimates:					
$plim\ \hat{\alpha} - \alpha$			-0.75		-1.54*
			(0.51)		(0.80)
$plim\ \hat{\beta} - \beta$			0.67*		1.78**
			(0.36)		(0.73)
$plim\ \hat{\alpha} + \hat{\beta} - \alpha - \beta$			0.38		1.01*
			(0.41)		(0.56)
N	100	100	100	100	100
Bound	Upper	Upper	Point	Lower	Point
Method	OLS	OLS	GMM	IV	GMM
Indicator	CPI	CPI, proxy	CPI, proxy	CPI, proxy	CPI, proxy

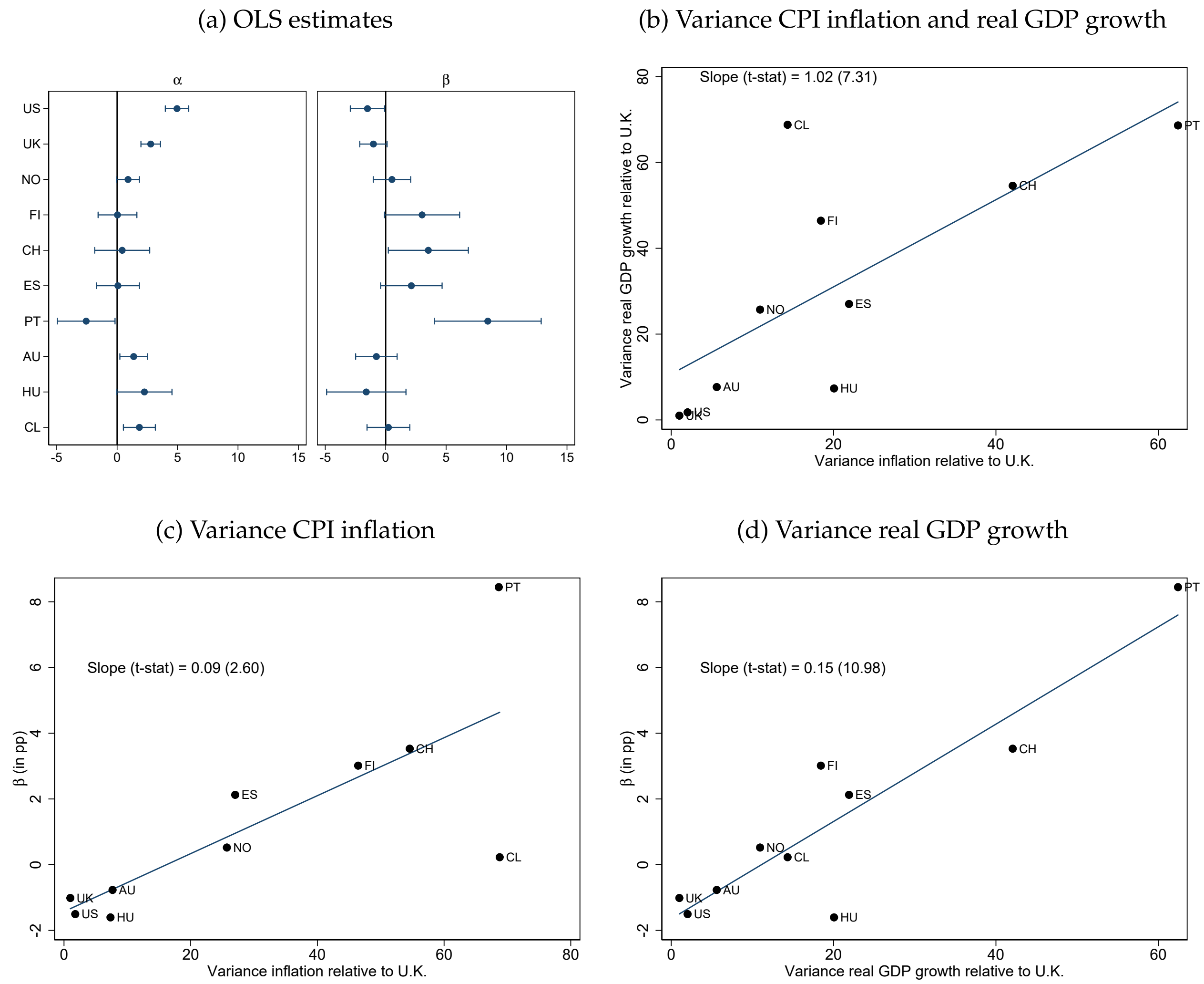
Notes: Model: $y_t = \alpha + \beta x_t + \epsilon_t$ where $x_t \equiv \mathbf{1}_{\{\tilde{\pi}_t < 0\}}$. Model parameters: Mean growth rate during inflation α; shortfall during deflation β; mean growth rate during deflation $\alpha + \beta$; probability of deflation ($P[\pi < 0]$). Bias estimates: Difference between the probability limit if we would only use the CPI; calculated based on the underlying GMM estimates, with standard errors computed using the delta method. Baseline: OLS estimates using the CPI. Conditional independence: Bounds and a point estimates using the CPI and proxy, assuming conditional independence. Conditional dependence: Point estimates using the CPI and proxy, assuming that the joint misclassification probabilities equal 0.15. HAC-robust standard errors are given in parentheses. ***,**,* denotes significance at the 1%, 5%, 10% level.

Figure C.2 — Volatility of historical data and OLS estimates based on two indicators



Notes: Model: $y_t = \alpha + \beta x_t + \epsilon_t$ where $x_t \equiv \mathbf{1}_{\{\tilde{\pi}_t<0\}}$. Panel (a): real GDP growth during inflation (α) and shortfall during deflation (β), with HAC-robust 95% confidence intervals. Panel (b): Relationship between the volatility of inflation and GDP growth. Panels (c)-(d): Relationship between the volatility of macroeconomic data and the OLS estimate of the shortfall in GDP growth during deflation (β). All volatilities normalized by the volatility of U.K. data.

Figure C.3 — Volatility of historical data and OLS estimates excluding Sweden



Notes: Model: $y_t = \alpha + \beta x_t + \epsilon_t$ where $x_t \equiv \mathbf{1}_{\{\tilde{\pi}_t<0\}}$. Panel (a): real GDP growth during inflation (α) and shortfall during deflation (β), with HAC-robust 95% confidence intervals. Panel (b): Relationship between the volatility of inflation and GDP growth. Panels (c)-(d): Relationship between the volatility of macroeconomic data and the OLS estimate of the shortfall in GDP growth during deflation (β). All volatilities normalized by the volatility of U.K. data.

D Robustness tests

Table D.1 — U.S. industrial production growth with controls (1800-1899)

	Cond. independence				Cond. dependence	
Growth inflation	13.41***	10.32***	9.78***	10.19***	12.98***	13.01***
	(1.50)	(1.35)	(1.28)	(1.52)	(2.48)	(2.58)
Shortfall deflation	-6.06***	-5.31***	-6.49***	-5.51***	-11.27***	-9.16***
	(2.23)	(1.51)	(1.66)	(1.37)	(2.82)	(2.40)
Stock price decline	-4.40**	-3.15**		-4.35***		-5.63**
	(2.15)	(1.54)		(1.36)		(2.36)
Banking crisis	-4.29	-2.60	3.21		5.48**	
	(8.26)	(3.00)	(2.20)		(2.51)	
M2 slowdown	-4.13*					
	(2.32)					
N	32	75	75	97	75	97
Bound	Upper	Upper	Point	Point	Point	Point
Method	OLS	OLS	GMM	GMM	GMM	GMM
Indicator	CPI, proxy	CPI, proxy	CPI, proxy	CPI, proxy	CPI, proxy	CPI, proxy
p-value J-test			0.494	0.777	0.674	0.785
p-value equal shortfall			0.00	0.50	0.00	0.15

Notes: Model: $y_t = \alpha + \beta x_t + \delta q_t + \epsilon_t$ where $x_t \equiv \mathbf{1}_{\{\tilde{\pi}_t<0\}}$. Model parameters: Mean growth rate during inflation α; shortfall during deflation β; Shortfall during banking crisis, stock price decline, or money growth smaller than its unconditional mean δ. Conditional independence: Bounds and a point estimates using the CPI and proxy, assuming conditional independence. Conditional dependence: Point estimates using the CPI and proxy, assuming that the joint misclassification probabilities equal 0.15. HAC-robust standard errors are given in parentheses. The lower panel reports the p-value for the J-statistic according to Hansen (1982) and the p-value for the null hypothesis that $\beta = \delta$. ***,**,* denotes significance at the 1%, 5%, 10% level.

Table D.2 — U.S. industrial production growth during deflation with three indicators (1841–1891)

	Baseline			Cond. independence			
Growth inflation	7.97***	9.10***	7.06***	9.00***	7.81***	8.24***	8.24***
	(1.37)	(1.28)	(1.30)	(1.48)	(1.51)	(1.53)	(1.53)
Shortfall deflation	-3.99**	-5.07***	-2.14	-5.79***	-4.01**	-4.99***	-5.53***
	(1.68)	(1.56)	(1.70)	(1.74)	(1.92)	(1.81)	(1.85)
N	51	51	51	51	51	51	51
Indicator	CPI	Proxy	Falkner	CPI, Proxy	CPI, Falkner	Proxy, Falkner	All
Bound	Upper	Upper	Upper	Upper	Upper	Upper	Upper

Notes: Model: $y_t = \alpha + \beta x_t + \epsilon_t$ where $x_t \equiv \mathbf{1}_{\{\tilde{\pi}_t < 0\}}$. Model parameters: Mean growth rate during inflation α; shortfall during deflation β. Baseline: OLS estimates using one indicator. Conditional independence: Bounds using multiplice indicators, assuming conditional independence. HAC-robust standard errors are given in parentheses. ***,**,* denotes significance at the 1%, 5%, 10% level.

Table D.3 — U.S. industrial production growth alternative classifications (1800-1899)

	Baseline	Cond. independence			Cond. dependence
(A) Severe deflation (π <-3%)					
$\alpha = E[y\|\pi > 0]$	6.36***	7.29***	7.62***	8.03***	8.03***
	(0.83)	(0.95)	(1.18)	(1.24)	(1.18)
$\beta = E[y\|\pi < 0] - E[y\|\pi > 0]$	-3.08**	-4.38***	-4.77***	-9.03**	-11.61*
	(1.30)	(1.48)	(1.54)	(3.82)	(6.57)
N	100	100	100	100	100
Bound	Upper	Upper	Point	Lower	Point
Method	OLS	OLS	GMM	IV	GMM
Indicator	CPI	CPI, proxy	CPI, proxy	CPI, proxy	CPI, proxy
(B) Persistent deflation of at least two years					
$\alpha = E[y\|\pi > 0]$	6.12***	7.66***	7.88***	9.36***	8.82***
	(0.88)	(1.00)	(1.37)	(1.66)	(1.49)
$\beta = E[y\|\pi < 0] - E[y\|\pi > 0]$	-1.58	-3.89***	-3.94**	-9.89***	-6.90***
	(1.48)	(1.46)	(1.90)	(3.76)	(2.58)
N	100	100	100	100	100
Bound	Upper	Upper	Point	Lower	Point
Method	OLS	OLS	GMM	IV	GMM
Indicator	CPI	CPI, proxy	CPI, proxy	CPI, proxy	CPI, proxy

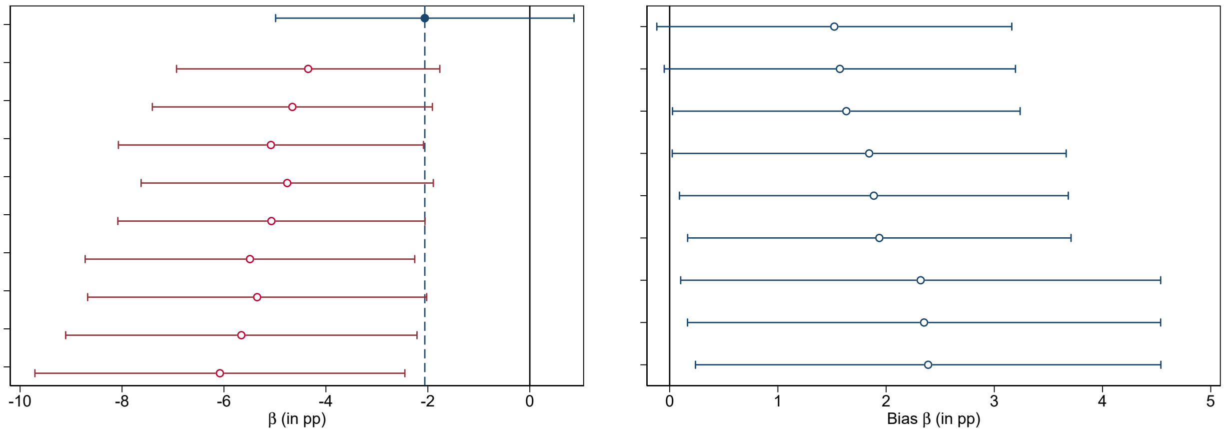
Notes: Model: $y_t = \alpha + \beta x_t + \epsilon_t$ where $x_t \equiv \mathbf{1}_{\{\tilde{\pi}_t < 0\}}$. Model parameters: Mean growth rate during inflation α; shortfall during deflation β; Baseline: OLS estimates using the CPI. Conditional independence: Bounds and a point estimates using the CPI and proxy, assuming conditional independence. Conditional dependence: Point estimates using the CPI and proxy, assuming that the joint misclassification probabilities equal 0.15. HAC-robust standard errors are given in parentheses. ***,**,* denotes significance at the 1%, 5%, 10% level.

Table D.4 — Controlling for serial correlation in U.S. industrial production growth (1800-1899)

	Baseline	Conditional independence	
Shortfall deflation	-2.00	-4.35***	-9.21***
	(1.54)	(1.39)	(2.87)
Lagged dep. variable	0.02	0.00	-0.20
	(0.12)	(0.10)	(0.18)
N	100	100	100
Bound	Upper	Upper	Lower
Method	OLS	OLS	IV
Indicator	CPI	CPI, proxy	CPI, proxy

Notes: Model: $y_t = \alpha + \beta x_t + \phi y_{t-1} + \epsilon_t$ where $x_t \equiv \mathbf{1}_{\{\tilde{\pi}_t < 0\}}$. Model parameters: Shortfall during deflation $\beta/(1-\phi)$; persistence of dependent variable ϕ. Baseline: OLS estimates using the CPI. Conditional independence: Bounds and a point estimates using the CPI and proxy, assuming conditional independence. HAC-robust standard errors are given in parentheses. ***,**,* denotes significance at the 1%, 5%, 10% level.

Figure D.1 — Varying misclassification assumptions



Notes: Model: $y_t = \alpha + \beta x_t + \epsilon_t$ where $x_t \equiv \mathbf{1}_{\{\tilde{\pi}_t < 0\}}$. Model parameters: Shortfall during deflation β; bias estimate based on the underlying GMM coefficients (see Eq. A.5). The estimates are based on all possible combinations of joint misclassification rates between 0 and 15% in steps of 5%. The dashed vertical line marks the OLS estimate using only the CPI. Point estimates are displayed as circles, 95% confidence intervals based on HAC-robust standard errors, computed with the delta method, are given as horizontal lines.

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