Online Appendix for "Regional Output Growth in the United Kingdom: More Timely and Higher Frequency Estimates From 1970" by Gary Koop, Stuart McIntyre, James Mitchell and Aubrey Poon

In this set of three appendices, we describe the data (A. Data Appendix), provide full details of our econometric methods (B. Technical Appendix) and present some supplementary empirical results (C. Empirical Appendix).

A Data Appendix

This appendix summaries the data sources and construction of the estimation databases used in this paper. It describes the process of arriving at an annual dataset for nominal and real GVA for the 12 NUTS 1 regions (these are defined by the Classification of Territorial Units for Statistics) of the UK (excluding the UK Continental Shelf) from 1966 to 2017 that is as consistent as possible, given changes to accounting standards, over the time period. Our regional nominal GVA data are measured at factor cost prior to 1996 and at basic prices from 1997. Our real GVA data utilize the ONS's balanced GVA data, GBA(B), for the period 1998-2017[23]; and in the earlier period we deflate our regional nominal GVA data by the UK wide deflator. We also extend our database to incorporate a number of additional indicators into our model. These include the US dollar to British pound exchange rate, the oil price, the Bank Rate and the Consumer Price Index; and regional indicators. We focus in the main paper on latest vintage or final release data (at the time of writing the latest vintage is December 2017), as they reflect the ONS's latest, and we presume best, assessment of historical economic growth. However, for our real-time nowcasting/forecasting work we use first release (nominal) data to better simulate the situation of the analyst producing nowcasts/forecasts using our model in real-time.

A.1 Nominal GVA data: first release and latest (or final) vintage

The construction of first release nominal GVA (income approach) data used in this paper follows closely that of Koop et al. (2019).[24] This earlier work provides a database of (as close as possible to) first release nominal GVA growth for 9 regions of the UK, with the smaller number of regions constructed in this work reflecting the need for a dataset of growth rates for each region on a consistent geographical basis.

In our modelling framework in this paper, in contrast, we work at the current 12 region level. These regions reflect the NUTS 1 regions of the UK, with the exception of the extra-regio (or UK Continental Shelf) region, for reasons discussed in the paper. To construct a database of first release nominal GVA growth covering the period 1967 to 2017, we therefore had to combine the information available from 1995 onwards on first release nominal GVA growth available from the ONS with the historical first release data collected in Koop et al. (2019). The nature of the changes in geography used between the statistical office regions, in place prior to 1995, and the current NUTS 1 regions of the UK, in place since 1995, mean that for five regions, which in Koop et al. (2019) were combined into two regions, we assumed that these regions shared the same growth rate in this earlier period as the aggregate, geographically consistent, region that they were part of in Koop et al. (2019).

To illustrate this in more detail, in Koop et al. (2019), which used the old Statistical Office Region classification in place prior to 1995, what is now the North East and North West of England NUTS1

[23] These data are 'balanced' in the sense of balancing the income and production approaches to measuring GVA.

[24] Available at https://www.escoe.ac.uk/download/2601/

regions comprised two (different) regions, the North and North West. The old North region comprised the whole of the current North East region, alongside a part of what is now the North West region. We have no way of separating out economic activity in the old North region between these two parts of the region. Therefore, in our database, prior to 1995 we assume that both the North East and North West of England grew at the same annual rate. The only other part of the UK affected by this change in geography is London, the South East and the East of England regions under the current statistical geography, which comprised the South East (and from 1978 was further split into Greater London and the Rest of the South East) and East Anglia (itself representing a proportion of the subsequent East of England region which also includes part of what was the South East region) under the old Statistical Office Region geography.

In order to reconcile these changing geographies in a consistent manner, we assumed that for the regions on which we have disaggregated data from 1995 onwards, but only aggregate data prior to this, the disaggregated regions grew at the same annual rate as the aggregate geographical area which they were part of on a consistent geographical basis prior to 1995.

Like Koop et al. (2019), our aim in putting together the database for the nowcasting and forecasting work in this paper was to use, as near as possible, first-release estimates of regional GVA and match these with the appropriate, similarly dated, data release for UK GVA. This strategy is in part motivated by our interest in nowcasting first release regional GVA estimates. But it also reflects the reality that final vintage data, e.g. the ONS's latest regional estimates, are not available over the whole sample period (i.e. the latest ONS data for nominal GVA(B) or GVA(I), published in December 2018, cover the period 1998-2017 or 1997-2017 only). So to get earlier data we inevitably have to look to earlier data vintages. In matching the regional data to the UK data we sought to minimize the cross-sectional aggregation error, as ideally the sum of the regional GVA data (including the UKCS) equals the annual sum of the quarterly UK data. But, we should emphasize (as is detailed in the data appendix for Koop et al. (2019)) that it was not possible to eradicate this measurement error for all years. Also, as described in the main paper, we chose to exclude the UKCS from our VAR models given its distinct time-series properties. This means that we should not expect, even absent measurement error, the cross-sectional constraint to be met exactly, as we show below.

As detailed in the data appendix to Koop et al. (2019) the first release regional nominal GVA data were matched from 1966–1996 against UK GVA data (at factor cost, seasonally adjusted (series: ABML)) again extracted from successive, similarly dated, national account data releases (obtained from the Bank of England's real-time database for nominal income; code CGCB[25]) with the secondary aim of minimizing the cross-sectional aggregation measurement error of the sum of the regional data against the quarterly UK data when aggregated to the annual frequency. From 1997 the regional data are matched against successive, similarly dated (so that again the data vintages of the regional data match that of the UK data), releases of quarterly UK GVA estimates, at basic prices, from the ONS's "Second estimate of GDP" previously known as the "UK Output, Income and Expenditure" press release/bulletins. Figure A.1 shows that the cross-sectional aggregation measurement error is time-varying and often less than zero. The average statistical discrepancy between 1966 and 1996 is -0.47%, between 1997 and 2016 it is -0.39%

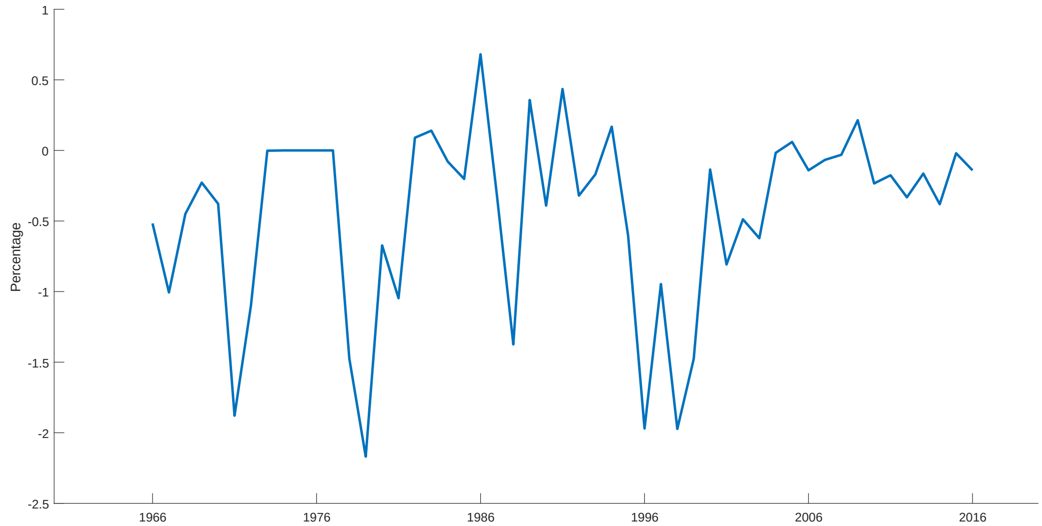
The final or latest vintage regional nominal GVA data are taken to be a combination (with the geographical reconcilition outlined above) of: (i) the historical 1966–1996 regional nominal GVA (income approach) data as released by the ONS[26] but without taking this back to first release, as described in Koop et al (2019), so that data revisions are accommodated[27]; and (ii) the December

[25] Available at http://www.bankofengland.co.uk/statistics/Documents/gdpdatabase/nominal_income.xlsx

[26] Available at https://www.ons.gov.uk/economy/regionalaccounts/grossdisposablehouseholdincome/adhocs/006226historiceconomicdataforregionsoftheuk1966to1996

[27] The ONS's historical database picks up estimates from successive yearly publications of Regional Trends. But the

Figure A.1: Discrepancy, by year, between the nominal UK Quarterly series and Regional Annual series (as % UK GVA)



2018 release of regional nominal GVA(B) data covering the period 1998–2017. The 1997 regional data are not available in balanced form, but the December 2018 data release from the ONS does provide estimates via the income approach and we use these. For the UK as a whole, the February 2019 vintage (of series AMBL) was taken as the latest vintage for quarterly nominal GVA.

A.2 Real GVA data: latest (or final) vintage

UK real quarterly GVA data on a comparable basis to the UK nominal quarterly GVA (series: ABML) data described above are produced by the ONS (series: ABMM), and can therefore be readily incorporated into our database. Again we use the February 2019 data vintage. Regional real GVA(B) data from 1998-2017 for each NUTS 1 (indeed NUTS 2 also) region of the UK are available from the ONS's December 2017 publication.[28]. But regional real GVA data are not available from this 2017 publication prior to 1998; indeed the latest release of the GVA(B) data used in this exercise is currently also the first release. However, using the database of latest release/vintage nominal GVA data for each NUTS 1 region (excl. UKCS) detailed above, it is possible to proxy the latest/final vintage estimates of real GVA growth in each of 12 NUTS 1 regions from 1966 to 1997 by deflating the nominal data using a UK aggregate-implied GDP deflator. This is a strong assumption, but without regional price data a necessary one, and assumes, in the period prior to 1998, common regional inflation. To summarize, our annual final vintage regional real GVA dataset combines the GVA(B) data produced for the first time in December 2018 (covering 1998–2017) with the final

publication lags vary, so that, for example, the 1966 GVA data come from the 1975 Regional Trends publication/vintage; while the 1970 data come from the 1976 Regional Trends publication. In general the publication lag shortens in the ONS's historical database, suggesting that more recent data have been through fewer annual rounds of revision. Our understanding, following email communication with ONS, is that this is in part because ONS chose to publish, in this historical database, the latest iteration for a given year rather than the first. When data were available, we sought to use the latest publication or data vintage for regional GVA in a given year.

[28]Data and a background methodology note are accessible here: `https://www.ons.gov.uk/economy/grossvalueaddedgva/bulletins/regionalgrossvalueaddedbalanceduk/1998to2016`

vintage, nominal regional data for the earlier period (1966–1997), deflated using a UK-wide measure of inflation.

A.3 Additional quarterly economic indicators

In addition to GVA data for the UK as a whole and for the NUTS 1 regions, we include four further quarterly macroeconomic indicators in our model. These are: the oil price (brent crude $U/BBL), the Bank Rate (Bank of England base interest rate), consumer prices (UK CPI provided by ONS), and the exchange rate between the USA and the UK ($: £). These variables are not revised and so first release and final vintages are the same. The oil price and the exchange rate enter the VAR in log differenced form. For the CPI we use the log difference relative to the same quarter in the previous year. We downloaded the Bank of England interest rate data directly from the Bank[29], and the UK consumer price index data from the ONS[30]. The oil price data were taken from Thomson Reuters Datastream[31] as the quarterly average price. The US dollar : UK pound exchange rate series was downloaded from the Bank of England's Millennium Database[32].

In our model we also make use of two additional data series relating to economic conditions in each region. The first of these is the claimant count rate measure of unemployment, accessed through `http://www.nomisweb.co.uk`. This provided claimant count rate data for NUTS1 regions of the UK back to the early 1970s. Prior to this, we assume that each region's claimant count rate evolved in line with the claimant count rate of the UK as a whole. While available monthly, we consider these data when aggregated to the quarterly frequency. The second regional indicator is the Business Optimism Indicator produced by the Confederation of British Industry (CBI). This is available on a regional basis from 1980 onward through Thomson Reuters Datastream. These data are produced for 11 regions of the UK (these reflect the NUTS 1 regional definitions with the exception of London and the South East of England where responses are combined together into a single region). Prior to 1980, we use the UK series for all regions.

B Technical Appendix

This appendix includes discussion of the state space model with state equations given by (1) and measurement equations given by (3), (4), and (5) in the main paper. In addition, we describe the stochastic volatility process given by (6), (7) and (8). We use an MCMC algorithm which draws from the full conditional posterior distributions. That is, we draw the VAR-SV model conditional on the states and the states conditional on the VAR coefficients and volatilities. Accordingly, this appendix describes econometric methods for these two parts separately. First, we describe methods for the VAR-SV, then for the states.

B.1 The VAR-SV

B.1.1 Model and Priors

We can rewrite (1), in the main paper, as a multivariate linear regression model:

$$\mathbf{y}_t = \mathbf{X}_t\beta + \epsilon_t, \epsilon_t \sim N(0, \Sigma_t), \tag{B.1}$$

[29] https://www.bankofengland.co.uk/boeapps/database/Bank-Rate.asp

[30] https://www.ons.gov.uk/economy/inflationandpriceindices/datasets/consumerpriceindices/current

[31] https://financial.thomsonreuters.com/en/products/tools-applications/trading-investment-tools/datastream-macroeconomic-analysis.html

[32] https://www.bankofengland.co.uk/statistics/research-datasets

where $\mathbf{X}_t = \mathrm{I}_n \otimes [1, \mathbf{y}_{t-1}^{'}, \ldots, \mathbf{y}_{t-p}^{'}]$ is an $n \times k$ matrix and $\beta = vec([\Phi_0, \Phi_1, \ldots, \Phi_p]^{'})$ is a $k \times 1$ vector of coefficients. We can stack (B.1) over time $t = 1, \ldots T$, to get

$$\begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_T \end{bmatrix} = \begin{bmatrix} \mathbf{X}_1 \\ \vdots \\ \mathbf{X}_T \end{bmatrix} \beta + \begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_T \end{bmatrix}, \tag{B.2}$$

$$\mathbf{y} = \mathbf{X}\beta + \epsilon, \epsilon \sim N(0, \Sigma), \tag{B.3}$$

where $\Sigma = \mathrm{diag}(\Sigma_1, \ldots, \Sigma_T)$.

The multivariate stochastic volatility specification used in this paper is given in (6), (7) and (8). The Dirichlet-Laplace priors are given in (9), (10), (11), (12). We use the same Dirichlet-Laplace priors for the $\mathbf{a}$'s and assume $i = 1, \ldots, m$

$$a_i \sim N(0, \psi_i^a \vartheta_{i,a}^2 \tau_a^2), \tag{B.4}$$

$$\psi_i^a \sim \mathrm{Exp}(\frac{1}{2}), \tag{B.5}$$

$$\vartheta_{i,a} \sim \mathrm{Dir}(\alpha_a, \ldots, \alpha_a), \tag{B.6}$$

$$\tau_a \sim \mathrm{G}(m\alpha_a, \frac{1}{2}). \tag{B.7}$$

Finally, we assume

$$\omega_{h_j}^2 \sim \mathrm{IG}(\nu_{h_j}, S_{h_j}), \quad \text{for } i = 1, \ldots, n. \tag{B.8}$$

B.1.2 The VAR-SV: MCMC Algorithm

Here we describe an MCMC algorithm for drawing from the VAR-SV parameters. In our MF-VAR-SV we draw from these conditional on the draws of the states (see below).

The conditional posterior for the VAR coefficients takes the following form:

$$\beta | \bullet \sim N(\hat{\beta}, \mathbf{K}_\beta^{-1}), \tag{B.9}$$

where

$$\mathbf{K}_\beta = \quad \mathbf{X}^{'}\Sigma^{-1}\mathbf{X} + \mathbf{S}_\beta^{-1}, \quad \hat{\beta} = \mathbf{K}_\beta^{-1}(\mathbf{X}^{'}\Sigma^{-1}\mathbf{y}), \tag{B.10}$$

where $\mathbf{S}_\beta = \mathrm{diag}(\psi_1^\beta \vartheta_{1,\beta}^2 \tau_\beta, \ldots, \psi_k^\beta \vartheta_{k,\beta}^2 \tau_\beta^2)$.

The conditional posterior for $\mathbf{a}$ takes the following form:

$$\mathbf{a} | \bullet \sim N(\hat{\mathbf{a}}, \mathbf{K}_a^{-1}), \tag{B.11}$$

where

$$\mathbf{K}_a = \quad \mathbf{E}^{'}\mathbf{D}^{-1}\mathbf{E} + \mathbf{S}_a^{-1}, \quad \hat{\mathbf{a}} = \mathbf{K}_a^{-1}(\mathbf{E}^{'}\mathbf{D}^{-1}\epsilon), \tag{B.12}$$

where $\mathbf{S}_a = \mathrm{diag}(\psi_1^a \vartheta_{1,a}^2 \tau_a, \ldots, \psi_m^a \vartheta_{m,a}^2 \tau_a^2)$, $\mathbf{D} = \mathrm{diag}(\mathbf{D}_1, \ldots, \mathbf{D}_T)'$ and, assuming $n = 3$, an example of the $\mathbf{E}$ matrix is

$$\mathbf{E}_t = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -\epsilon_{1,t} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\epsilon_{1,t} & -\epsilon_{2,t} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\epsilon_{1,t} & -\epsilon_{2,t} & -\epsilon_{3,t} \end{bmatrix}, \tag{B.13}$$

where $\mathbf{E}$ is the stacked version from $t = 1, \ldots, T$. For more information about constructing this $\mathbf{E}$ matrix; see Chan (2017, pp. 130-131).

To deal with stochastic volatility, we follow Chan and Eisenstat (2018) and apply the auxiliary mixture sampler of Kim et al. (1998) in conjunction with the precision sampler to sequentially draw each slice of $\mathbf{h}_{i,\bullet} = (h_{i,1}, \ldots, h_{i,T})'$, for $i = 1, \ldots, n$. See Chan and Hsiao (2014) and Cross and Poon (2016) for details.

To draw the initial condition $\mathbf{h}_0$, we follow Chan and Eisenstat (2018) and use

$$\mathbf{h}_0|\bullet \sim N(\hat{\mathbf{h}}_0, \mathbf{K}_{\mathbf{h}_0}^{-1}), \tag{B.14}$$

where

$$\mathbf{K}_{\mathbf{h}_0} = \quad \mathbf{V}_h^{-1} + \Sigma_h^{-1} \quad , \quad \hat{\mathbf{h}}_0 = \mathbf{K}_{\mathbf{h}_0}^{-1}(\mathbf{V}_h^{-1}\mathbf{a}_h + \Sigma_h^{-1}\mathbf{h}_1). \tag{B.15}$$

To draw Σ_h we note that $\omega_{h_i}^2$ are conditionally independent and follow

$$\omega_{h_j}^2|\bullet \sim IG(\nu_{h_j} + \tfrac{T}{2}, S_{h_j} + \tfrac{1}{2}\textstyle\sum_{t=1}^{T}(h_{j,t} - h_{j,t-1})^2), \quad \text{for } j = 1, \ldots, n. \tag{B.16}$$

As for $\psi_j^\beta, \vartheta_{j,\beta}, \tau_\beta$, following Bhattacharya et al. (2015), the conditional posterior distributions are

$$(\psi_j^\beta)^{-1}|\bullet \sim iG(\tfrac{\vartheta_{j,\beta}\tau_\beta}{|\beta_j|}, 1), \quad \text{for } j = 1, \ldots, k \tag{B.17}$$

$$\tau_\beta|\bullet \sim GIG(k(\alpha_\beta - 1), 1, 2\sum_{j=1}^{K}\frac{|\beta_j|}{\vartheta_{j,\beta}}), \tag{B.18}$$

$$R_{j,\beta}|\bullet \sim GIG(\alpha_\beta - 1, 1, 2|\beta_j|), \quad \text{for } j = 1, \ldots, k \tag{B.19}$$

and

$$\vartheta_{j,\beta} = \frac{R_{j,\beta}}{\sum_{j=1}^{k} R_{j,\beta}}. \tag{B.20}$$

We use notation where GIG is the generalized inverse Gaussian distribution; and to simulate a draw from this distribution we implement the algorithm by Devroye (2014). iG is the Inverse Gaussian distribution.

Similarly, to draw $\psi_i^a, \vartheta_{i,a}, \tau_a$ we use the following conditional posteriors:

$$(\psi_i^a)^{-1}|\bullet \sim iG(\tfrac{\vartheta_{i,a}\tau_a}{|a_i|}, 1), \quad \text{for } i = 1, \ldots, m \tag{B.21}$$

$$\tau_a|\bullet \sim GIG(m(\alpha_a - 1), 1, 2\sum_{i=1}^{m}\frac{|a_j|}{\vartheta_{i,a}}), \tag{B.22}$$

$$R_{i,a}|\bullet \sim GIG(\alpha_a - 1, 1, 2|a_i|), \quad \text{for } i = 1, \ldots, m \tag{B.23}$$

and

$$\vartheta_{i,a} = \frac{R_{i,a}}{\sum_{i=1}^{m} R_{i,a}}. \tag{B.24}$$

B.1.3 Prior Hyperparameter Choices

The hyperparameters that we choose for both the VAR and VAR-SV are $\alpha_\beta = \alpha_a = \frac{1}{2}$, $\mathbf{a}_h = \mathbf{0}$, $\mathbf{V}_h = 10 \times \mathbf{I}_n$, $\nu_i = \nu_{h_j} = 5$ and $S_i = S_{h_j} = .01$. The priors for the variances of the stochastic volatility terms are standard and similar to those made in Chan and Eisenstat (2018). The choices for the Dirichlet-Laplace hyperparameters, α_β, α_a, are the relatively noninformative default choices suggested by Bhattacharya et al. (2015). For a robustness check, we also consider the results when $\alpha_\beta = \alpha_a = 0.1$ and we find these results produce very similar results to our benchmark case of

$\alpha_\beta = \alpha_a = \frac{1}{2}$. To demonstrate, Figures B.1 through B.4 compare results using the different prior hyperparameter choices.

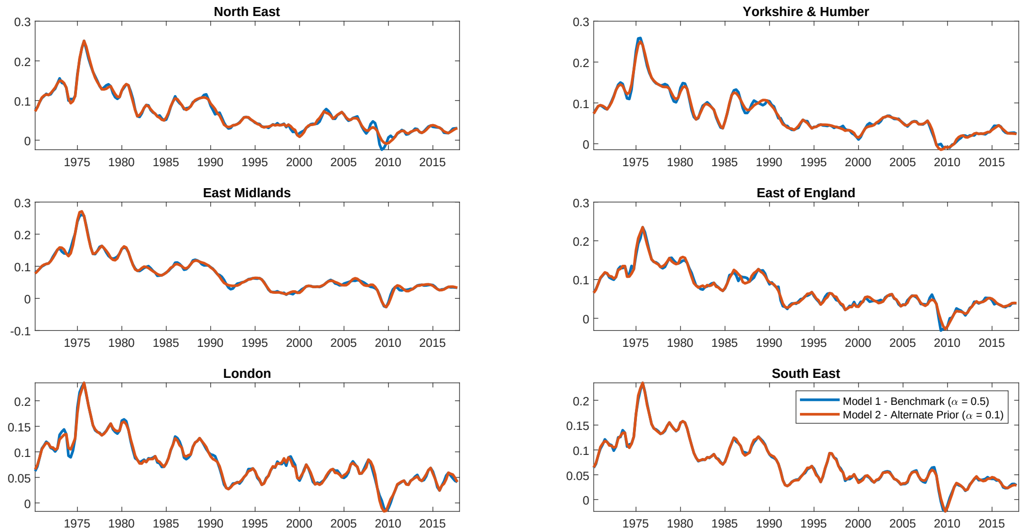


Figure B.1: Regional Nominal GVA

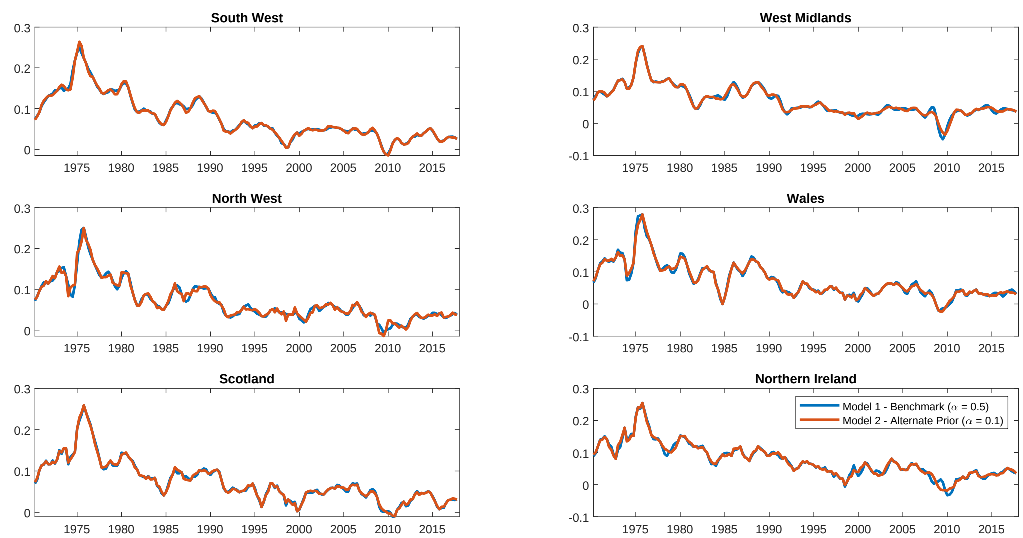


Figure B.2: Regional Nominal GVA

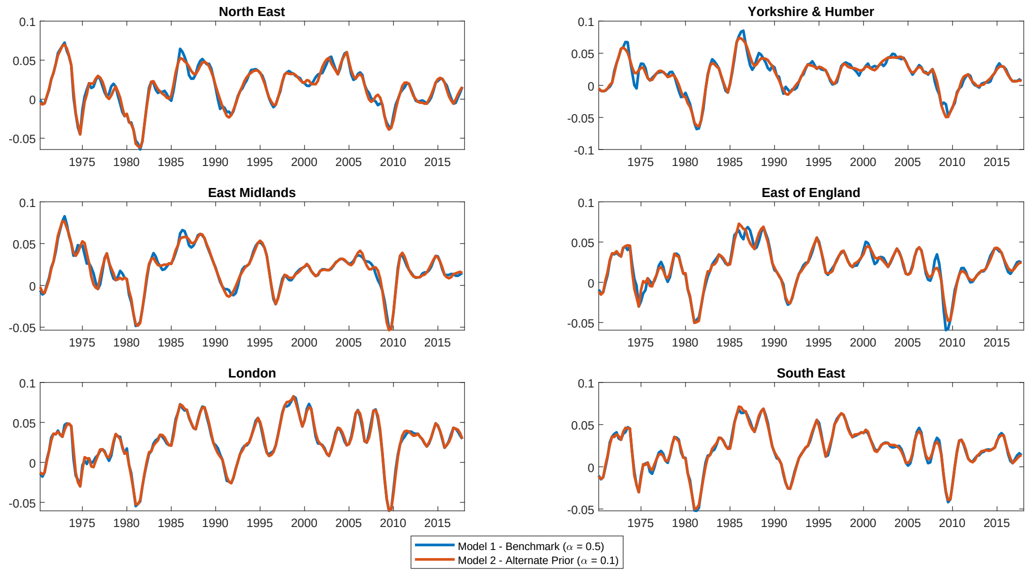


Figure B.3: Regional Real GVA

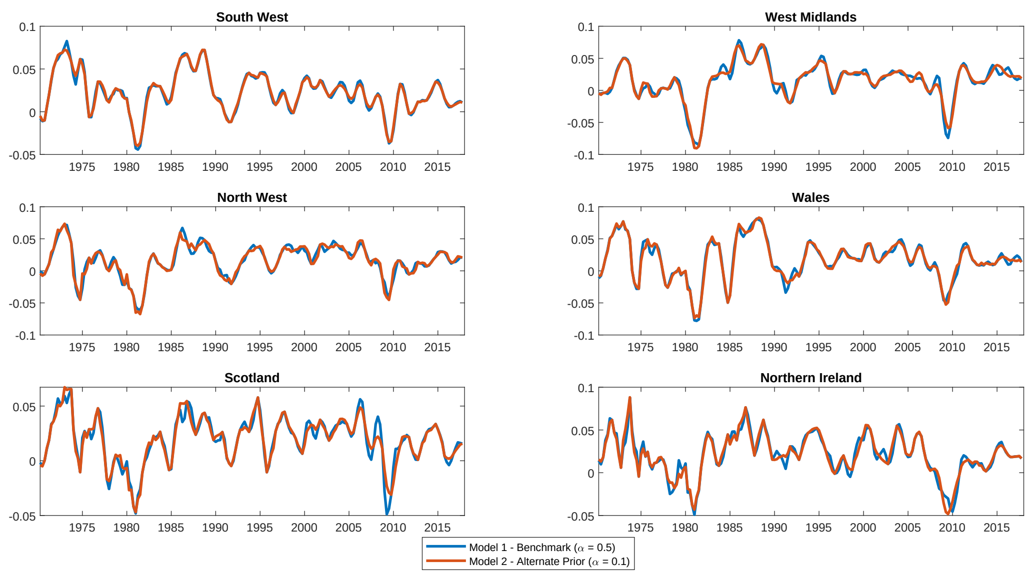


Figure B.4: Regional Real GVA

B.2 The Mixed Frequency State Space Model

To show how we add the mixed frequency aspect to the model and incorporate the cross-sectional restriction, we use a simple example where we have one quarterly frequency variable and two annual frequency variables and assume seven lags. Results extend to many regions and other lag lengths in a straightforward manner. In the context of our study, the quarterly variable is the UK GVA growth rate and the two annual frequency variables are the two regions' annual growth rates.

Our quarterly VAR can be written as:

$$\begin{bmatrix} y_t^{UK} \\ y_t^1 \\ y_t^2 \end{bmatrix} = \begin{bmatrix} \Phi_{qc} \\ \Phi_{ac} \end{bmatrix} + \begin{bmatrix} \Phi_{qq,1} & \Phi_{qa,1} \\ \Phi_{aq,1} & \Phi_{aa,1} \end{bmatrix} \begin{bmatrix} y_{t-1}^{UK} \\ y_{t-1}^1 \\ y_{t-1}^2 \end{bmatrix} + \ldots + \begin{bmatrix} \Phi_{qq,7} & \Phi_{qa,7} \\ \Phi_{aq,7} & \Phi_{aa,7} \end{bmatrix} \begin{bmatrix} y_{t-7}^{UK} \\ y_{t-7}^1 \\ y_{t-7}^2 \end{bmatrix} + \epsilon_t. \quad \text{(B.25)}$$

We can rearrange this equation into a state equation. First, we group the above VAR coefficients together as

$$\Phi_{qq} = \left[\begin{array}{cccc} \Phi_{qq,1}, & \Phi_{qq,2}, & \Phi_{qq,3} & ,\ldots,\Phi_{qq,7} \end{array} \right], \quad \text{(B.26)}$$

$$\Phi_{qa} = \left[\begin{array}{cccc} \Phi_{qa,1}, & \Phi_{qa,2}, & \Phi_{qa,3} & ,\ldots,\Phi_{qa,7} \end{array} \right], \quad \text{(B.27)}$$

$$\Phi_{aq} = \left[\begin{array}{cccc} \Phi_{aq,1} & ,\Phi_{aq,2} & ,\Phi_{aq,3} & ,\ldots,\Phi_{aq,7} \end{array} \right], \quad \text{(B.28)}$$

$$\Phi_{aa} = \left[\begin{array}{cccc} \Phi_{aa,1} & ,\Phi_{aa,2}, & \Phi_{aa,3} & ,\ldots,\Phi_{aa,7} \end{array} \right]. \quad \text{(B.29)}$$

Then our state equation is

$$\mathbf{s}_t = \Gamma_s \mathbf{s}_{t-1} + \Gamma_z \mathbf{y}_{t-p:t-1}^{UK} + \Gamma_c + \Gamma_u u_{a,t}, \quad \text{(B.30)}$$

where $\mathbf{s}_t = (y_t^1, y_t^2, y_{t-1}^1, y_{t-1}^2, y_{t-2}^1, y_{t-2}^2, y_{t-3}^1, y_{t-3}^2, \ldots, y_{t-7}^1, y_{t-7}^2)'$ is a $z \times 1$ vector containing the regional variables and their lags and $\mathbf{y}_{t-p:t-1}^{UK} = (y_{t-7}^{UK}, \ldots, y_{t-1}^{UK})'$ contains lags of the UK variables.

Using the following definitions:

$$\Gamma_s = \begin{bmatrix} \Phi_{qq} & 0 \\ \mathbf{I} & 0 \end{bmatrix}_{z\times z}, \Gamma_z = \begin{bmatrix} \Phi_{aq} \\ 0 \end{bmatrix}_{z\times p}, \Gamma_c = \begin{bmatrix} \Phi_{ac} \\ 0 \end{bmatrix}_{z\times 1}, \Gamma_u = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{z\times 2}, \quad \text{(B.31)}$$

we can obtain the measurement equation:

$$y_t^{UK} = \Lambda_{qs}\mathbf{s}_t + \Phi_{qq}\mathbf{y}_{t-p:t-1}^{UK} + \Phi_{ac} + u_{q,t}, \quad \text{(B.32)}$$

where

$$\Lambda_{qs} = \left[\begin{array}{cc} 0 & \Phi_{qa} \end{array} \right]_{1\times z}. \quad \text{(B.33)}$$

When both the quarterly and annual variables are observed at time t, the measurement equation is

$$\begin{bmatrix} y_t^{1,A} \\ y_t^{2,A} \end{bmatrix} = \Lambda_{as}\mathbf{s}_t + \Lambda_z \mathbf{y}_{t-p:t-1}^{UK} + \Phi_{qc}, \quad \text{(B.34)}$$

where

$$\Lambda_{as} = \begin{bmatrix} 0 & \Phi_{qa} \\ & M \end{bmatrix}, \Lambda_z = [\begin{array}{c} \Phi_{qq} \\ 0 \end{array}], \quad \text{(B.35)}$$

$$M = \begin{bmatrix} \frac{1}{4} & 0 & \frac{1}{2} & 0 & \frac{3}{4} & 0 & 1 & 0 & \frac{3}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{2} & 0 & \frac{3}{4} & 0 & 1 & 0 & \frac{3}{4} & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 \end{bmatrix}. \quad \text{(B.36)}$$

This incorporates the intertemporal restriction given in (2).

Finally, the cross-sectional restriction, (5), gives us an additional measurement equation. We have

$$y_t^{UK} = \mathbf{R}\mathbf{s}_t + \eta, \eta \sim N(0, \sigma_{cs}^2), \quad \text{(B.37)}$$

where

$$\mathbf{R} = \left[\begin{array}{ccc} \frac{1}{R} & \frac{1}{R} & 0 \end{array} \right]_{1\times z}. \quad \text{(B.38)}$$

We assume a tight prior for the variance of the cross-sectional restriction $\sigma_{cs}^2 \sim IG(1000, .001)$, where the prior mean of the variance is close to zero.

Thus, we have a set of state equations given by (B.30) and measurement equations given by (B.32), (B.34) and (B.37). Thus, conditional on draws of the all the other parameters of the MF-VAR-SV described earlier in this Technical Appendix, we can use standard Bayesian MCMC methods to draw the states. We use the precision sampler methods of Chan (2017) to do so.

B.2.1 The Cross-sectional Restriction Using Log-differenced Data

The proof that our cross-sectional restriction is correct and that UK GVA growth is an average of regional GVA growth rates for the R regions begins by noting you can write UK GVA growth in two ways:

$$y_t^{UK} = \ln(Y_t^{UK}) - \ln(Y_{t-1}^{UK}) \tag{B.39}$$

$$y_t^{UK} = \ln(\sum_{r=1}^{R} Y_t^r) - \ln(\sum_{r=1}^{R} Y_{t-1}^r). \tag{B.40}$$

If we take a (first order) Taylor series expansion of the log of the average in the second equation and use the fact that the geometric mean is never larger than the arithmetic mean (and the difference between the two will be small if the quarterly movements are small relative to the quarterly average) we obtain $\ln(\sum_{r=1}^{R} Y_t^r) \simeq \frac{1}{R}\sum_{r=1}^{R} \ln Y_t^r + R\ln R$. Hence,

$$y_t^{UK} \simeq \frac{1}{R}\sum_{r=1}^{R} \ln Y_t^r - \frac{1}{R}\sum_{r=1}^{R} \ln Y_{t-1}^r$$

$$y_t^{UK} \simeq \frac{1}{R}\sum_{r=1}^{R} \left(\ln Y_t^r - \ln Y_{t-1}^r\right)$$

$$y_t^{UK} \simeq \frac{1}{R}\sum_{r=1}^{R} y_t^r$$

C Empirical Appendix

C.1 Additional Connectedness Results

In the body of the paper, tables of posterior means of connectedness measures were reported. To give the reader a feeling for estimation uncertainty, Tables C.1 and C.2 present the 16th and 84th percentiles, respectively, of the posteriors of the connectedness measures. These tables are based on the nominal GVA data and are for one quarter ahead measures in 2017Q4. Results for other horizons and time periods are similar. It is worth noting that these credible intervals are fairly wide indicating a fair degree of estimation uncertainty.

For the reader interested in what the connectedness tables look like for real GVA, focusing on the posterior means, we provide Tables C.3 and C.4. Note that, just as with the nominal GVA data, the oil price has the largest impact.

We also provide a connectedness table for $h = 4$ which can be seen to lie between the results for $h = 1$ and $h = 20$ (see Table C.5).

Table C.1: Connectedness Estimates for 2017Q4, 1 quarter ahead forecast horizon, 16th percentile. Nominal GVA Data

	UK	CPI	Bank Rate	Ex. rate	Oil price	North E.	York. and H.	E. Midlands	E. of England	London	South E.	South West	West Midlands	North West	Wales	Scotland	N. Ireland	**From**
UK	72.4%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	**0.1%**
CPI	0.0%	73.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	**0.1%**
Bank Rate	0.0%	0.0%	73.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	**0.1%**
Ex. rate	0.0%	0.0%	0.0%	73.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	**0.1%**
Oil price	0.0%	0.0%	0.0%	0.0%	73.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	**0.1%**
North E.	0.0%	0.0%	0.0%	0.0%	0.0%	73.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	**0.1%**
York. and H.	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	74.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	**0.1%**
E. Midlands	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	68.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	**0.2%**
E. of England	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	69.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	**0.2%**
London	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	73.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	**0.2%**
South E.	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	72.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	**0.2%**
South West	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	73.6%	0.0%	0.0%	0.0%	0.0%	0.0%	**0.2%**
West Midlands	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	73.0%	0.0%	0.0%	0.0%	0.0%	**0.2%**
North West	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	73.2%	0.0%	0.0%	0.0%	**0.2%**
Wales	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	70.7%	0.0%	0.0%	**0.2%**
Scotland	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	71.3%	0.0%	**0.2%**
N. Ireland	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	70.5%	**0.2%**
To	**0.1%**	**0.1%**	**0.1%**	**0.1%**	**0.1%**	**0.1%**	**0.1%**	**0.2%**	**0.2%**	**0.2%**	**0.2%**	**0.2%**	**0.2%**	**0.2%**	**0.3%**	**0.3%**	**0.3%**	**0.2%**

Table C.2: Connectedness Estimates for 2017Q4, 1 quarter ahead forecast horizon, 84th percentile. Nominal GVA Data

	UK	CPI	Bank Rate	Ex. rate	Oil price	North E.	York. and H.	E. Midlands	E. of England	London	South E.	South West	West Midlands	North West	Wales	Scotland	N. Ireland	**From**
UK	91.9%	3.5%	3.6%	1.2%	1.8%	1.4%	1.3%	2.6%	2.0%	1.5%	2.7%	2.5%	1.6%	1.5%	1.5%	1.1%	2.5%	**32.4%**
CPI	3.2%	92.4%	2.0%	1.5%	1.3%	0.9%	1.2%	3.5%	2.5%	1.6%	1.3%	1.1%	1.5%	1.4%	3.3%	1.9%	2.1%	**30.3%**
Bank Rate	3.2%	2.0%	92.4%	1.2%	1.1%	1.5%	1.5%	2.2%	3.6%	2.0%	1.9%	1.5%	1.3%	1.7%	1.8%	2.0%	1.9%	**30.4%**
Ex. rate	0.8%	1.0%	0.8%	91.9%	5.3%	2.2%	2.0%	1.5%	1.3%	1.8%	1.9%	2.0%	1.9%	1.7%	3.7%	2.1%	1.9%	**31.8%**
Oil price	1.0%	0.7%	0.7%	4.5%	92.0%	1.6%	1.3%	1.7%	2.3%	2.1%	1.9%	1.6%	1.8%	3.9%	2.4%	1.9%	1.9%	**31.2%**
North E.	0.8%	0.5%	0.9%	1.9%	1.6%	92.2%	1.2%	2.0%	2.0%	1.8%	1.9%	1.9%	1.7%	2.2%	4.2%	2.6%	2.4%	**29.8%**
York. and H.	0.7%	0.7%	0.9%	1.7%	1.4%	1.3%	92.5%	1.8%	1.8%	2.4%	2.3%	2.6%	2.3%	1.9%	2.1%	2.1%	2.5%	**28.6%**
E. Midlands	1.3%	1.9%	1.2%	1.2%	1.6%	1.9%	1.6%	90.1%	5.1%	3.4%	2.5%	2.5%	1.9%	3.2%	2.5%	2.5%	3.1%	**37.3%**
E. of England	0.9%	1.3%	1.9%	1.0%	1.9%	1.7%	1.5%	4.8%	90.3%	2.2%	2.4%	2.1%	2.5%	2.4%	4.2%	2.7%	2.7%	**36.2%**
London	0.7%	0.8%	1.0%	1.4%	1.8%	1.5%	2.0%	3.2%	2.2%	92.0%	2.1%	1.5%	2.5%	2.3%	2.4%	2.4%	2.2%	**29.9%**
South E.	1.2%	0.6%	1.0%	1.4%	1.6%	1.6%	1.9%	2.4%	2.4%	2.2%	91.6%	2.1%	2.0%	2.0%	2.1%	1.7%	5.6%	**32.0%**
South West	1.2%	0.6%	0.8%	1.5%	1.4%	1.7%	2.3%	2.5%	2.2%	1.6%	2.2%	92.0%	3.4%	2.5%	2.0%	1.7%	2.3%	**30.1%**
West Midlands	0.7%	0.7%	0.7%	1.4%	1.5%	1.4%	2.0%	1.8%	2.6%	2.6%	2.0%	3.3%	91.8%	2.6%	2.8%	2.4%	2.3%	**31.0%**
North West	0.6%	0.6%	0.7%	1.1%	2.7%	1.6%	1.4%	2.6%	2.2%	2.0%	1.8%	2.2%	2.3%	92.4%	3.3%	2.4%	2.3%	**29.8%**
Wales	0.5%	1.3%	0.7%	2.2%	1.6%	2.8%	1.4%	1.9%	3.4%	2.0%	1.6%	1.5%	2.4%	3.1%	91.0%	2.2%	5.3%	**33.7%**
Scotland	0.5%	0.8%	0.9%	1.3%	1.4%	2.0%	1.5%	2.1%	2.4%	2.2%	1.5%	1.5%	2.2%	2.3%	2.6%	92.0%	7.6%	**32.7%**
N. Ireland	0.9%	0.8%	0.8%	1.1%	1.2%	1.6%	1.6%	2.3%	2.1%	1.8%	4.3%	1.8%	1.8%	2.1%	5.3%	6.6%	90.7%	**36.0%**
To	**18.1%**	**17.9%**	**18.6%**	**25.6%**	**29.1%**	**26.7%**	**25.8%**	**38.9%**	**40.1%**	**33.1%**	**34.2%**	**31.7%**	**33.2%**	**36.8%**	**46.0%**	**38.3%**	**48.8%**	**32.0%**

Table C.3: Real GVA Growth Connectedness Estimates for 2017Q4, 1 quarter ahead forecast horizon

	UK	CPI	Bank Rate	Ex. rate	Oil price	North E.	York. and H.	E. Midlands	E. of England	London	South E.	South West	West Midlands	North West	Wales	Scotland	N. Ireland	**From**
UK	81.7%	1.7%	2.0%	0.7%	1.1%	0.8%	0.7%	1.5%	1.0%	0.9%	1.9%	1.5%	0.9%	0.9%	0.8%	0.6%	1.4%	**18.3%**
CPI	1.6%	83.0%	1.1%	0.8%	0.7%	0.5%	0.7%	1.9%	1.4%	1.0%	0.8%	0.6%	0.8%	0.8%	1.9%	1.1%	1.3%	**17.0%**
Bank Rate	1.7%	1.1%	82.2%	0.7%	0.7%	0.9%	0.9%	1.5%	2.2%	1.1%	1.0%	0.8%	0.7%	1.0%	1.1%	1.2%	1.1%	**17.8%**
Ex. rate	0.5%	0.6%	0.6%	81.8%	3.1%	1.1%	1.0%	0.9%	0.7%	1.0%	1.1%	1.1%	1.0%	0.9%	2.1%	1.1%	1.1%	**18.2%**
Oil price	0.7%	0.5%	0.5%	2.6%	82.5%	0.9%	0.7%	1.0%	1.1%	1.2%	1.0%	0.9%	1.0%	1.9%	1.3%	1.1%	1.1%	**17.5%**
North E.	0.5%	0.3%	0.6%	1.0%	0.9%	83.4%	0.7%	1.0%	1.2%	1.0%	1.2%	1.0%	0.9%	1.2%	2.3%	1.3%	1.3%	**16.6%**
York. and H.	0.4%	0.4%	0.6%	1.0%	0.8%	0.8%	83.7%	1.0%	1.1%	1.3%	1.3%	1.3%	1.3%	1.1%	1.2%	1.2%	1.4%	**16.3%**
E. Midlands	0.8%	1.0%	0.9%	0.8%	1.0%	1.0%	1.0%	79.1%	2.6%	1.9%	1.3%	1.4%	1.0%	1.7%	1.4%	1.4%	1.8%	**20.9%**
E. of England	0.5%	0.8%	1.2%	0.6%	1.0%	1.1%	0.9%	2.5%	79.7%	1.2%	1.2%	1.2%	1.3%	1.4%	2.4%	1.5%	1.4%	**20.3%**
London	0.4%	0.6%	0.6%	0.8%	1.0%	0.9%	1.1%	1.8%	1.3%	82.8%	1.1%	1.0%	1.3%	1.3%	1.4%	1.3%	1.3%	**17.2%**
South E.	0.9%	0.4%	0.6%	0.9%	0.9%	1.1%	1.1%	1.3%	1.3%	1.2%	81.9%	1.1%	1.0%	1.2%	1.1%	1.0%	2.9%	**18.1%**
South West	0.8%	0.4%	0.5%	0.9%	0.8%	0.9%	1.2%	1.4%	1.4%	1.1%	1.2%	82.9%	1.7%	1.3%	1.2%	0.9%	1.4%	**17.1%**
West Midlands	0.4%	0.4%	0.4%	0.8%	0.9%	0.8%	1.2%	1.0%	1.4%	1.4%	1.1%	1.6%	83.1%	1.5%	1.5%	1.3%	1.3%	**16.9%**
North West	0.4%	0.4%	0.5%	0.7%	1.4%	1.0%	0.9%	1.4%	1.3%	1.2%	1.1%	1.2%	1.4%	82.6%	2.0%	1.3%	1.3%	**17.4%**
Wales	0.3%	0.8%	0.5%	1.3%	0.9%	1.6%	0.8%	1.1%	1.9%	1.1%	1.0%	1.0%	1.3%	1.8%	80.3%	1.2%	3.1%	**19.7%**
Scotland	0.3%	0.5%	0.6%	0.8%	0.9%	1.1%	0.9%	1.2%	1.4%	1.2%	0.9%	0.8%	1.2%	1.4%	1.4%	81.8%	3.6%	**18.2%**
N. Ireland	0.5%	0.5%	0.5%	0.7%	0.7%	1.0%	0.9%	1.4%	1.2%	1.0%	2.3%	1.1%	1.0%	1.2%	2.9%	2.9%	80.2%	**19.8%**
To	**10.7%**	**10.4%**	**11.7%**	**15.0%**	**16.8%**	**15.6%**	**14.8%**	**22.0%**	**22.5%**	**18.9%**	**19.4%**	**17.7%**	**17.9%**	**20.6%**	**25.8%**	**20.6%**	**26.8%**	18.1%

Table C.4: Real GVA Growth Connectedness Estimates for 2017Q4, 20 quarter ahead forecast horizon

	UK	CPI	Bank Rate	Ex. rate	Oil price	North E.	York. and H.	E. Midlands	E. of England	London	South E.	South West	West Midlands	North West	Wales	Scotland	N. Ireland	**From**
UK	10.2%	3.4%	3.1%	2.2%	2.2%	6.1%	5.8%	6.0%	8.3%	5.3%	6.4%	5.7%	6.6%	10.6%	6.1%	6.3%	5.7%	**89.8%**
CPI	8.4%	7.6%	4.1%	2.8%	2.6%	5.9%	5.6%	5.8%	7.9%	5.1%	5.9%	5.4%	5.8%	10.2%	5.7%	5.8%	5.3%	**92.4%**
Bank Rate	8.9%	4.8%	6.1%	2.8%	2.6%	6.0%	5.6%	5.8%	8.1%	5.1%	6.0%	5.3%	5.9%	10.2%	5.7%	5.8%	5.3%	**93.9%**
Ex. rate	8.3%	3.4%	3.1%	2.5%	2.2%	6.1%	6.0%	6.2%	8.4%	5.3%	6.3%	5.6%	7.6%	10.7%	6.2%	6.3%	5.7%	**97.5%**
Oil price	8.0%	3.1%	2.9%	2.0%	2.2%	6.3%	6.2%	6.1%	8.7%	5.4%	6.5%	5.8%	6.6%	11.5%	6.3%	6.5%	5.8%	**97.8%**
North E.	8.2%	3.0%	2.8%	2.0%	2.1%	7.0%	6.1%	6.2%	8.6%	5.4%	6.5%	5.9%	6.8%	10.7%	6.3%	6.4%	5.8%	**93.0%**
York. and H.	8.3%	2.9%	2.7%	1.9%	2.0%	6.3%	6.7%	6.3%	8.6%	5.4%	6.5%	5.9%	6.9%	10.8%	6.4%	6.5%	5.8%	**93.3%**
E. Midlands	8.3%	3.1%	2.8%	2.0%	2.1%	6.2%	6.0%	7.2%	8.5%	5.4%	6.4%	5.8%	6.8%	10.8%	6.3%	6.4%	5.8%	**92.8%**
E. of England	8.2%	2.9%	2.8%	1.9%	2.1%	6.2%	6.0%	6.3%	9.7%	5.5%	6.5%	5.9%	6.8%	10.8%	6.3%	6.5%	5.8%	**90.3%**
London	8.3%	3.0%	2.8%	2.0%	2.1%	6.2%	6.0%	6.2%	8.5%	6.2%	6.5%	5.9%	6.8%	10.8%	6.3%	6.4%	5.8%	**93.8%**
South E.	8.3%	3.0%	2.8%	1.9%	2.1%	6.1%	6.0%	6.2%	8.6%	5.5%	7.4%	5.9%	6.8%	10.8%	6.3%	6.4%	5.9%	**92.6%**
South West	8.2%	3.0%	2.8%	1.9%	2.1%	6.2%	6.1%	6.2%	8.6%	5.4%	6.5%	6.8%	6.8%	10.8%	6.4%	6.4%	5.8%	**93.2%**
West Midlands	8.2%	3.1%	2.9%	2.0%	2.1%	6.2%	6.1%	6.2%	8.5%	5.3%	6.4%	5.8%	8.0%	10.7%	6.2%	6.4%	5.8%	**92.0%**
North West	8.1%	2.8%	2.6%	1.8%	2.0%	6.3%	6.0%	6.3%	8.6%	5.4%	6.6%	5.9%	6.9%	11.9%	6.4%	6.5%	5.9%	**88.1%**
Wales	8.3%	3.1%	2.8%	2.0%	2.1%	6.3%	6.1%	6.3%	8.5%	5.4%	6.4%	5.8%	6.8%	10.8%	7.1%	6.4%	5.8%	**92.9%**
Scotland	8.2%	2.9%	2.7%	1.8%	2.0%	6.2%	6.1%	6.3%	8.6%	5.5%	6.5%	5.9%	6.9%	10.7%	6.4%	7.5%	5.9%	**92.5%**
N. Ireland	8.2%	2.8%	2.7%	1.9%	2.0%	6.3%	6.1%	6.3%	8.6%	5.5%	6.5%	5.9%	7.0%	10.7%	6.4%	6.5%	6.7%	**93.3%**
To	**132.5%**	**50.3%**	**46.3%**	**33.0%**	**34.4%**	**99.0%**	**95.9%**	**98.8%**	**135.6%**	**85.9%**	**102.5%**	**92.7%**	**107.6%**	**171.5%**	**99.7%**	**101.4%**	**92.1%**	**92.9%**

Table C.5: Real GVA Growth Connectedness Estimates for 2017Q4, 4 quarter ahead forecast horizon

	UK	CPI	Bank Rate	Ex. rate	Oil price	North E.	York. and H.	E. Midlands	E. of England	London	South E.	South West	West Midlands	North West	Wales	Scotland	N. Ireland	**From**
UK	42.2%	2.5%	2.4%	1.2%	1.3%	3.4%	3.3%	4.6%	4.9%	4.0%	4.5%	4.6%	3.7%	4.7%	4.4%	3.8%	4.3%	**57.8%**
CPI	3.9%	60.6%	3.1%	1.1%	1.0%	1.9%	1.9%	3.4%	2.9%	2.2%	2.2%	2.3%	2.4%	2.5%	3.3%	2.5%	2.7%	**39.4%**
Bank Rate	6.5%	2.8%	56.2%	1.1%	1.0%	2.3%	2.1%	3.0%	3.9%	2.5%	2.8%	2.5%	2.6%	2.7%	2.8%	2.6%	2.6%	**43.8%**
Ex. rate	5.8%	2.1%	1.8%	27.5%	2.1%	3.8%	4.6%	5.8%	5.7%	4.8%	5.5%	4.2%	4.3%	6.0%	6.9%	4.3%	4.7%	**72.5%**
Oil price	2.9%	1.1%	1.3%	1.4%	6.0%	5.8%	14.3%	3.8%	8.4%	3.6%	4.9%	3.3%	2.9%	27.2%	4.4%	4.8%	4.0%	**94.0%**
North E.	4.3%	1.6%	1.8%	1.5%	1.5%	27.2%	5.9%	5.5%	6.7%	4.3%	5.2%	5.1%	4.7%	8.0%	6.7%	5.0%	4.9%	**72.8%**
York. and H.	4.3%	1.6%	1.6%	1.4%	1.4%	6.1%	23.6%	6.5%	7.3%	4.9%	5.7%	5.3%	4.7%	9.1%	6.2%	5.2%	5.1%	**76.4%**
E. Midlands	4.6%	2.1%	1.9%	1.4%	1.5%	4.7%	4.6%	30.8%	6.8%	4.4%	4.8%	5.2%	4.8%	7.3%	5.3%	4.8%	5.1%	**69.2%**
E. of England	4.7%	1.8%	2.1%	1.2%	1.4%	4.5%	4.8%	5.5%	30.7%	5.0%	6.1%	5.0%	4.2%	7.4%	5.5%	5.4%	4.7%	**69.3%**
London	4.9%	1.7%	1.7%	1.4%	1.5%	4.6%	5.0%	5.3%	6.9%	27.4%	5.7%	5.4%	4.7%	7.7%	5.8%	5.3%	4.9%	**72.6%**
South E.	4.9%	1.6%	1.7%	1.4%	1.5%	4.4%	4.7%	5.3%	7.1%	5.3%	29.6%	5.3%	4.4%	7.4%	5.1%	4.9%	5.2%	**70.4%**
South West	4.7%	1.6%	1.7%	1.4%	1.4%	4.4%	4.7%	7.1%	6.9%	4.1%	5.1%	30.1%	4.6%	7.4%	5.3%	4.6%	4.9%	**69.9%**
West Midlands	4.4%	1.7%	1.7%	1.5%	1.4%	4.1%	4.9%	5.5%	5.7%	4.1%	4.9%	4.7%	34.5%	6.8%	5.4%	4.5%	4.4%	**65.5%**
North West	4.1%	1.5%	1.6%	1.2%	1.6%	7.2%	5.4%	5.1%	7.3%	4.5%	5.2%	4.8%	4.7%	29.1%	6.0%	5.7%	4.9%	**70.9%**
Wales	4.6%	1.9%	1.8%	1.6%	1.5%	4.8%	5.4%	5.9%	6.4%	4.4%	5.1%	5.5%	4.2%	7.5%	28.8%	4.7%	5.9%	**71.2%**
Scotland	4.3%	1.6%	1.6%	1.3%	1.4%	4.7%	4.9%	5.4%	6.6%	4.7%	5.5%	5.3%	4.2%	7.6%	5.7%	29.1%	5.9%	**70.9%**
N. Ireland	4.3%	1.6%	1.5%	1.3%	1.4%	4.9%	5.1%	5.9%	6.6%	4.7%	5.6%	5.7%	4.8%	7.2%	6.0%	5.8%	27.6%	**72.4%**
To	**73.3%**	**28.9%**	**29.4%**	**21.4%**	**22.8%**	**71.9%**	**81.5%**	**83.7%**	**100.2%**	**67.8%**	**78.8%**	**74.2%**	**65.8%**	**126.6%**	**84.8%**	**73.9%**	**74.0%**	**68.2%**

C.2 Credible Intervals for the Quarterly Regional Estimates

To convince the user that our econometric methodology is producing accurate estimates, Figures C.1 and C.2 plot quarterly estimates of annualized real regional GVA growth rates along with credible intervals which cover the 16th through 84th percentiles. Note that, for the reasons discussed in the body of the paper, these figures plot annual growth rates. Figures C.3 and C.4 present analogous results for nominal regional GVA growth.

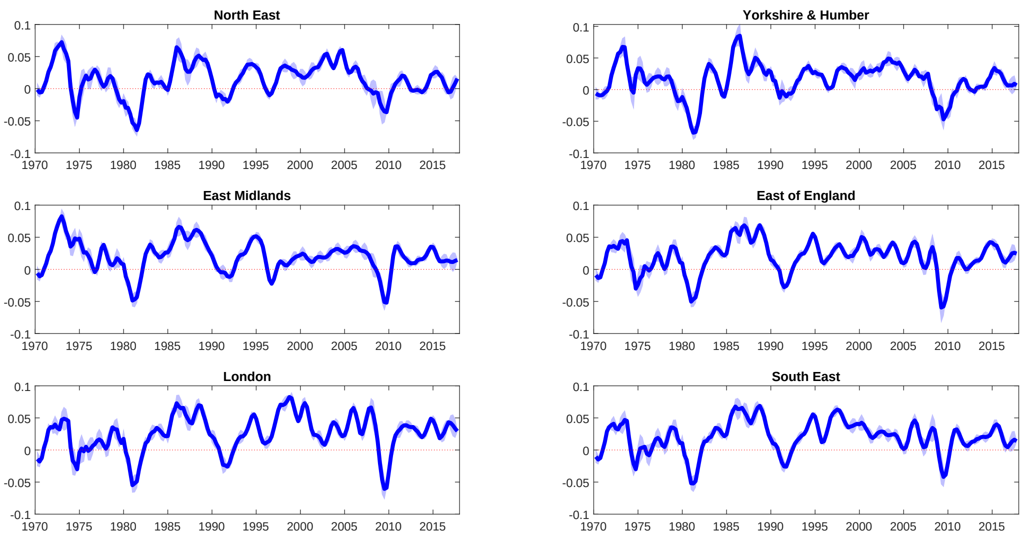


Figure C.1: Regional Real GVA Growth Rates: Estimates and Credible Intervals

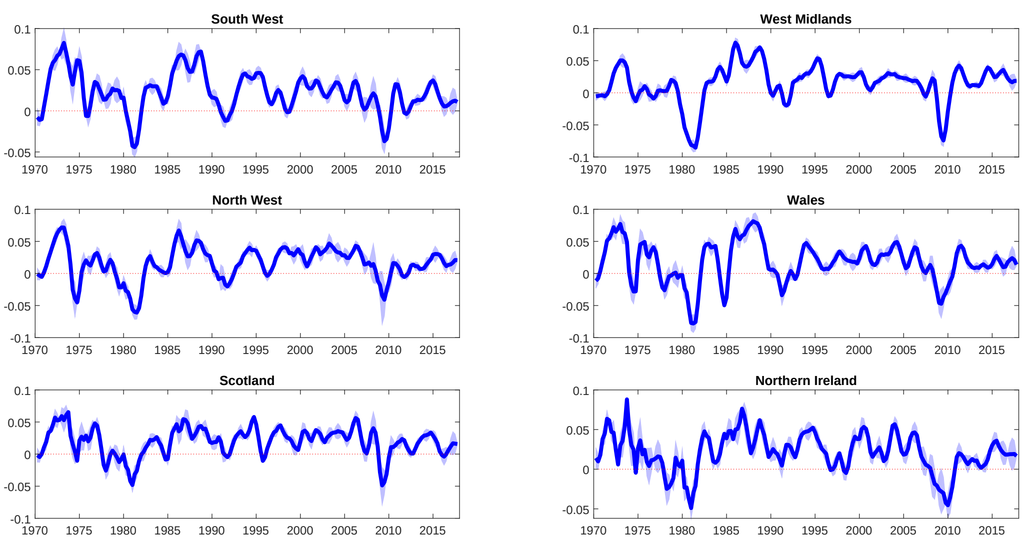


Figure C.2: Regional Real GVA Growth Rates: Estimates and Credible Intervals

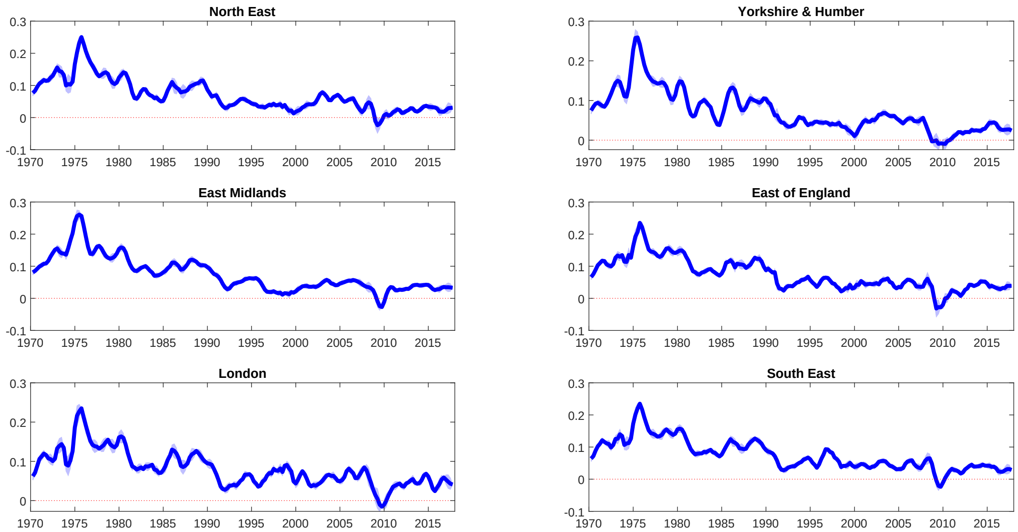


Figure C.3: Regional Nominal Growth Rates: Estimates and Credible Intervals

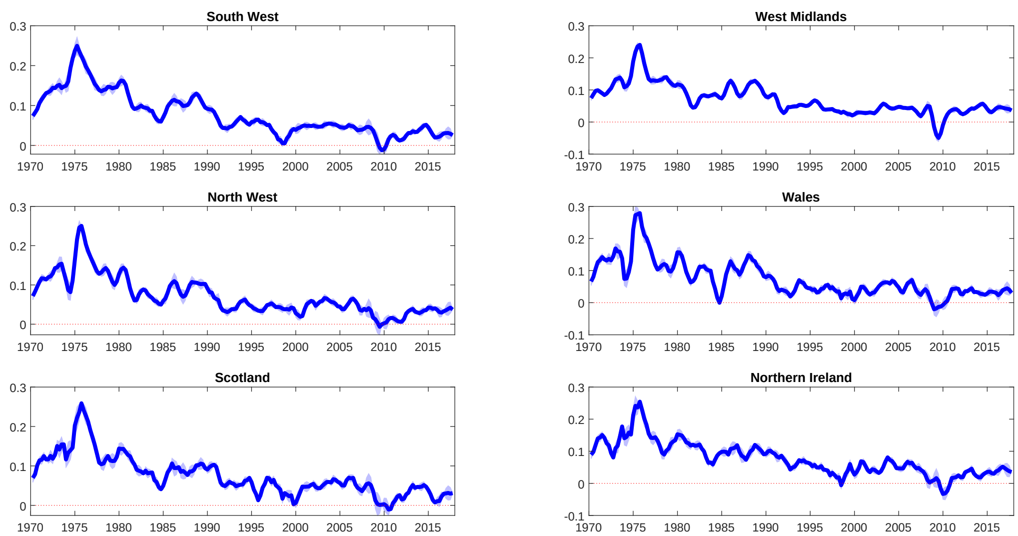


Figure C.4: Regional Nominal Growth Rates: Estimates and Credible Intervals

C.3 Results using Exact Growth Rates

Here we consider the modifications to our MF-VAR required when we model in exact growth rates rather than logarithmic differences as in the main paper. Then we explore the sensitivity of our empirical results to this choice.

We use the following notational conventions, emphasizing that here we model in exact quarter-on-quarter growth rates:

- $t = 1, .., T$ runs at the *quarterly* frequency.
- $r = 1, .., R$ denotes the R regions in the UK.
- Y_t^{UK} is GVA for the UK in quarter t.
- $y_t^{UK} = \left(\frac{Y_t^{UK} - Y_{t-1}^{UK}}{Y_{t-1}^{UK}}\right)$ is the quarterly (quarter-on-quarter) growth rate in GVA in the UK.
- Y_t^r is GVA for region r in quarter t. It is never observed.
- $Y_t^{r,A} = Y_t^r + Y_{t-1}^r + Y_{t-2}^r + Y_{t-3}^r$ is annual GVA for region r. It is observed for quarter 4 of each year, but not in other quarters.
- $y_t^{r,A} = \left(\frac{Y_t^{r,A} - Y_{t-4}^{r,A}}{Y_{t-4}^{r,A}}\right)$ is annual GVA growth in region r. It is observed, but only for quarter 4 of each year. Let $y_t^A = \left(y_t^{1,A}, .., y_t^{R,A}\right)'$ denote the vector of annual GVA growth rates for the R regions.
- $y_t^r = \left(\frac{Y_t^r - Y_{t-1}^r}{Y_{t-1}^r}\right)$ is the quarterly (quarter-on-quarter) growth rate in GVA in region r. It is never observed. Let $y_t^Q = \left(y_t^1, .., y_t^R\right)'$ denote the vector of quarterly year-on-year GVA growth rates for the R regions.

The MF-VAR is again specified in $y_t = \left(y_t^{UK}, y_t^{Q\prime}\right)'$, plus the additional macroeconomic and regional variables observed at the quarterly frequency. But the temporal and cross-sectional constraints need to be re-specified.

The temporal constraint is given as:

$$
\begin{aligned}
y_t^{r,A} = {} & \left(\frac{Y_{t-1}^r}{\sum_{j=0}^{3} Y_{t-4-j}^r}\right) y_t^r + \left(\frac{Y_{t-2}^r}{\sum_{j=0}^{3} Y_{t-4-j}^r}\right) 2y_{t-1}^r + \left(\frac{Y_{t-3}^r}{\sum_{j=0}^{3} Y_{t-4-j}^r}\right) 3y_{t-2}^r + \left(\frac{Y_{t-4}^r}{\sum_{j=0}^{3} Y_{t-4-j}^r}\right) 4y_{t-3}^r \\
& + \left(\frac{Y_{t-5}^r}{\sum_{j=0}^{3} Y_{t-4-j}^r}\right) 3y_{t-4}^r + \left(\frac{Y_{t-6}^r}{\sum_{j=0}^{3} Y_{t-4-j}^r}\right) 2y_{t-5}^r + \left(\frac{Y_{t-7}^r}{\sum_{j=0}^{3} Y_{t-4-j}^r}\right) y_{t-6}^r
\end{aligned}
\tag{C.1}
$$

where the weights, $\left(\frac{Y_{t-j}^r}{Y_{t-4}^{r,A}}\right)$, denote the share of regional output in quarter $t-j$ in annual regional output from the previous year. To avoid a nonlinear measurement equation, we proxy these weights by 1/4.

The cross-sectional restriction that UK GVA is the sum of GVA across the R regions is re-specified as:

$$y_t^{UK} = \sum_{r=1}^{R} w_t^{*r} y_t^r + \eta_t \tag{C.2}$$

where $w_t^{*r} = \left(\frac{Y_{t-1}^r}{\sum_{r=1}^{R} Y_{t-1}^r} \right)$ is the share of regional output in aggregate output in quarter t and $\eta_t \sim N(0, \sigma_{cs}^2)$. We proxy w_t^{*r} by the observed annual shares, noting that we should expect to see little within-year variation in these weights.

We re-estimate our MF-VAR-SV model, using exact growth rates with the re-specified temporal and cross-sectional restrictions, (C.1) and (C.2), on the final vintage data to produce historical quarterly estimates of both nominal and real regional growth. Figure C.5 presents the quarterly nominal and real estimates alongside the UK growth rate. To aid in comparability with the published annual regional data, our quarterly estimates are again annualized (i.e. we take our quarterly regional GVA estimates, y_t^r, and construct and plot an annual change, $y_t^{r,A}$, using (C.1)).

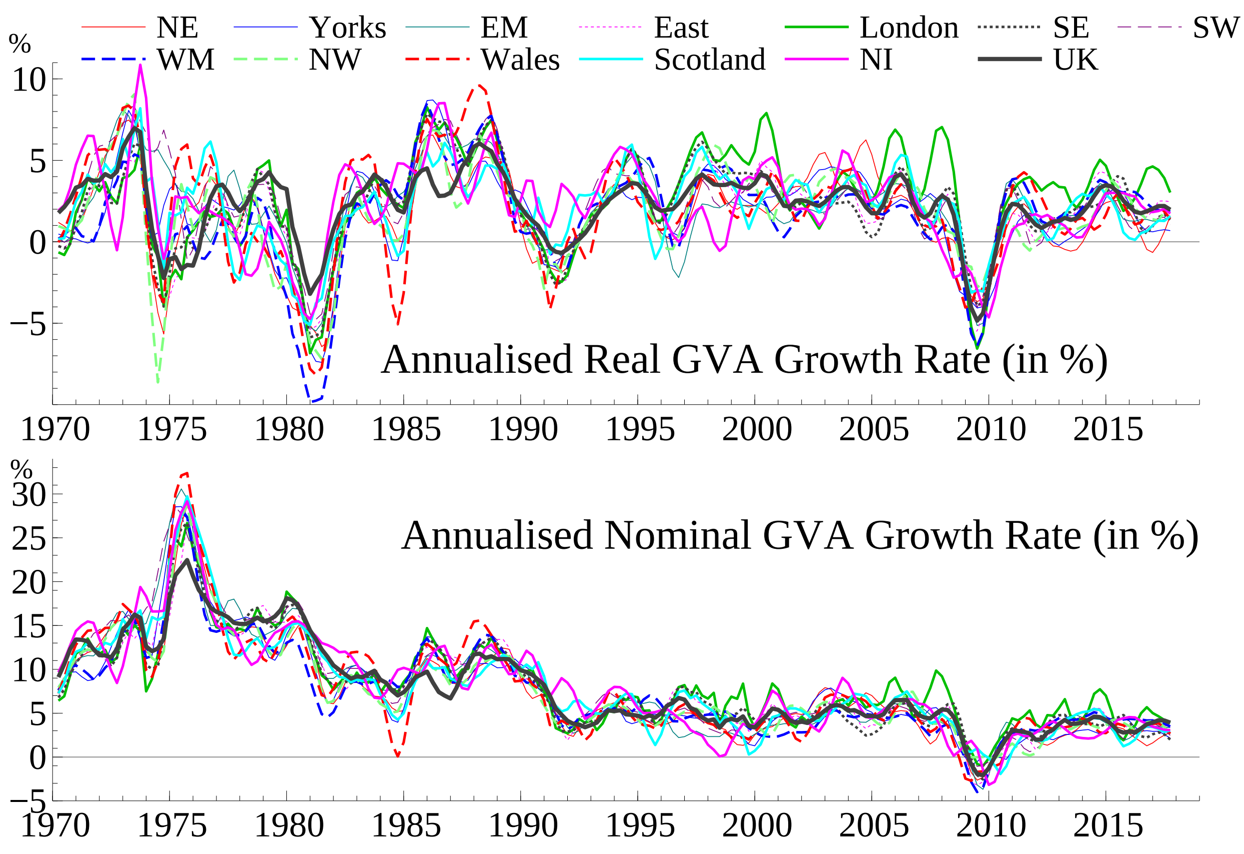


Figure C.5: Historical Estimates of Regional GVA Growth using Exact Growth Rates

Comparison of Figure C.5 with Figure 1 in the main paper, which uses logarithmic differences rather than exact growth rates, reassures that the choice of data transformation does not have a material effect on the movements of the quarterly regional estimates. The quarterly figures for the regions look very similar across Figures C.5 and 1, albeit as expected for large growth rates some differences between the scale of the two sets of estimates are seen. Table C.6 confirms how highly correlated the regional estimates in log differences are with those in exact growth rates.

Table C.6: Correlation Coefficient between log differences and exact growth rates

	Nominal GVA	Real GVA
North East	0.99	0.98
Yorkshire and The Humber	1.00	0.98
East Midlands	1.00	0.99
East of England	0.99	0.98
London	0.99	0.97
South East	0.99	0.99
South West	0.99	0.96
West Midlands	0.99	0.98
North West	0.99	0.97
Wales	0.99	0.98
Scotland	0.99	0.96
Northern Ireland	0.99	0.95

References

[1] Bhattacharya A, Pati D, Pillai NS, Dunson DB. 2015. Dirichlet–Laplace priors for optimal shrinkage. *Journal of the American Statistical Association* **110**(512): 1479-1490.

[2] Chan JCC. 2017. *Notes on Bayesian Macroeconometrics.* Manuscript available at `http://joshuachan.org/`

[3] Chan JCC, Eisenstat E. 2018. Bayesian model comparison for time-varying parameter VARs with stochastic volatility. *Journal of Applied Econometrics* **33**: 509-532.

[4] Chan JCC, Hsiao CYL. 2014. Estimation of stochastic volatility models with heavy tails and serial dependence. In *Bayesian Inference in the Social Sciences*, Jeliazkov I, Yang X-S (eds). John Wiley & Sons: New Jersey; 159-180.

[5] Cross J, Poon A. 2016. Forecasting structural change and fat-tailed events in Australian macroeconomic variables. *Economic Modelling* **58**: 34-51.

[6] Devroye L. 2014. Random variate generation for the generalized inverse Gaussian distribution. *Statistics and Computing* **24**: 239-246.

[7] Kim S, Shephard N, Chib S. 1998. Stochastic Volatility: Likelihood Inference and Comparison with ARCH models. *Review of Economic Studies* **65**(3): 361-393.

[8] Koop G, McIntyre S, Mitchell J. 2019. UK regional nowcasting using a mixed frequency Vector Autoregressive model with entropic tilting. *Journal of the Royal Statistical Society: Series A*. Forthcoming.