Online Appendix to *Estimating and Accounting for the Output Gap with Large Bayesian Vector Autoregressions* *

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July 16, 2019

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A1 Data

IFS in the mnemonic column refers to a series being sourced from the International Financial Statistics. Otherwise, the data are sourced from the Federal Reserve Economic Data (FRED) and the FRED mnemonic is provided. The "Adjust" column refers to any data transformations: 'ln' indicates natural logarithms have been taken and 'Δ^i' indicates the variable has been differenced i times. Differencing is conducted if a Chow test for a change in mean from the first half to the second half of the sample is significant at the 10% level and/or an augmented Dicky-Fuller test rejects a unit root at the 5% level. An '**x**' in the 'BM' column indicates that a variable is included in the 23-variable benchmark BVAR.

Series	**Mnemonic**	**Adjust**	**BM**
U.S.: Commodity Price: W Texas Interm Spot Price (US$/Barrel)	IFS	ln, Δ	**x**
Real Gross Domestic Product, 3 Decimal	GDPC96	ln, Δ	**x**
Real Personal Consumption Expenditures	PCECC96	ln, Δ	**x**
Personal Consumption Expenditures: Durable Goods	PCDGx	ln, Δ	
Personal Consumption Expenditures: Services	PCESVx	ln, Δ	
Personal Consumption Expenditures: Nondurable Goods	PCNDx	ln, Δ	
Real Gross Private Domestic Investment, 3 decimal	GPDIC96	ln, Δ	
Fixed Private Investment	FPIx	ln, Δ	
Gross Private Domestic Investment: Fixed Investment: Nonresidential: Equipment	Y033RC1Q027SBEAx	ln, Δ	
Private Nonresidential Fixed Investment	PNFIx	ln, Δ	
Private Residential Fixed Investment	PRFIx	ln, Δ	
Shares of gross domestic product: Gross private domestic investment: Change in private inventories	A014RE1Q156NBEA	Δ	
Real Government Consumption Expenditures and Gross Investment	GCEC96	ln, Δ	
Real Government Consumption Expenditures and Gross Investment: Federal	A823RL1Q225SBEA	ln, Δ	
Federal Government Current Receipts	FGRECPTx	ln, Δ	
State and Local Consumption Expenditures & Gross Investment	SLCEx	ln, Δ^2	
Real Exports of Goods and Services, 3 Decimal	EXPGSC96	ln, Δ	
Real Imports of Goods and Services, 3 Decimal	IMPGSC96	ln, Δ	
Real Disposable Personal Income	DPIC96	ln, Δ	**x**
Nonfarm Business Sector: Real Output	OUTNFB	ln, Δ	
Business Sector: Real Output	OUTBS	ln, Δ	
Industrial Production Index	INDPRO	ln, Δ	**x**
Industrial Production: Final Products (Market Group)	IPFINAL	ln, Δ	
Industrial Production: Consumer Goods	IPCONGD	ln, Δ	

Industrial Production: Materials	IPMAT	ln, Δ	
Industrial Production: Durable Materials	IPDMAT	ln, Δ	
Industrial Production: Nondurable Materials	IPNMAT	ln, Δ	
Industrial Production: Durable Consumer Goods	IPDCONGD	ln, Δ	
Industrial Production: Durable Goods: Automotive products	IPB51110SQ	ln, Δ	
Industrial Production: Nondurable Consumer Goods	IPNCONGD	ln, Δ	
Industrial Production: Business Equipment	IPBUSEQ	ln, Δ	
Industrial Production: Consumer energy products	IPB51220SQ	ln, Δ	
Capacity Utilization: Manufacturing (SIC)	CUMFNS	Δ	**x**
All Employees: Total Nonfarm Payrolls	PAYEMS	ln, Δ	
All Employees: Total Private Industries	USPRIV	ln, Δ	
Civilian Employment Level	CE16OV	ln, Δ	**x**
Civilian Labor Force Participation Rate	CIVPART	Δ	
Civilian Unemployment Rate	UNRATE		**x**
Unemployment Rate: 16 to 19 years	LNS14000012	Δ	
Unemployment Rate: 20 years and over, Men	LNS14000025	Δ	
Unemployment Rate: 20 years and over, Women	LNS14000026	Δ	
Number of Civilians Unemployed for Less Than 5 Weeks	UEMPLT5	ln, Δ	
Number of Civilians Unemployed for 5 to 14 Weeks	UEMP5TO14	ln, Δ	
Number of Civilians Unemployed for 15 to 26 Weeks	UEMP15T26	ln, Δ	
Number of Civilians Unemployed for 27 Weeks and Over	UEMP27OV	ln, Δ	
Employment Level: Part-Time for Economic Reasons, All Industries	LNS12032194	ln, Δ	
Business Sector: Hours of All Persons	HOABS	ln, Δ	
Nonfarm Business Sector: Hours of All Persons	HOANBS	ln, Δ	**x**
Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing	AWHMAN	ln, Δ	
Average Weekly Overtime Hours of Production and Nonsupervisory Employees: Manufacturing	AWOTMAN	ln, Δ	
Housing Starts: Total: New Privately Owned Housing Units Started	HOUST	ln, Δ	**x**
Privately Owned Housing Starts: 5-Unit Structures or More	HOUST5F	ln, Δ	
Housing Starts in Midwest Census Region	HOUSTMW	ln, Δ	
Housing Starts in Northeast Census Region	HOUSTNE	ln, Δ	
Housing Starts in South Census Region	HOUSTS	ln, Δ	
Housing Starts in West Census Region	HOUSTW	ln, Δ	

Personal Consumption Expenditures: Chain-type Price Index	PCECTPI	ln, Δ^2	x
Personal Consumption Expenditures Excluding Food and Energy (Chain-Type Price Index)	PCEPILFE	ln, Δ^2	
Gross Domestic Product: Chain-type Price Index	GDPCTPI	ln, Δ^2	x
Gross Private Domestic Investment: Chain-type Price Index	GPDICTPI	ln, Δ	
Business Sector: Implicit Price Deflator	IPDBS	ln, Δ^2	
Personal consumption expenditures: Goods (chain-type price index)	DGDSRG3Q086SBEA	ln, Δ	
Personal consumption expenditures: Services (chain-type price index)	DSERRG3Q086SBEA	ln, Δ^2	
Consumer Price Index for All Urban Consumers: All Items	CPIAUCSL	ln, Δ	x
Producer Price Index for All Commodities	PPIACO	ln, Δ	x
Producer Price Index by Commodity Industrial Commodities	PPIIDC	ln, Δ	
Producer Price Index by Commodity for Fuels and Related Products and Power: Crude Petroleum (Domestic Production)	WPU0561	ln, Δ	
Average Hourly Earnings of Production and Nonsupervisory Employees: Construction	CES2000000008x	ln, Δ^2	
Average Hourly Earnings of Production and Nonsupervisory Employees: Manufacturing	CES3000000008x	ln, Δ^2	x
Nonfarm Business Sector: Real Compensation Per Hour	COMPRNFB	ln, Δ	
Business Sector: Real Compensation Per Hour	RCPHBS	ln, Δ	
Nonfarm Business Sector: Real Output Per Hour of All Persons	OPHNFB	ln, Δ	x
Business Sector: Real Output Per Hour of All Persons	OPHPBS	ln, Δ	
Business Sector: Unit Labor Cost	ULCBS	ln, Δ	
Nonfarm Business Sector: Unit Labor Cost	ULCNFB	ln, Δ	
Nonfarm Business Sector: Unit Nonlabor Payments	UNLPNBS	ln, Δ	
Producer Price Index by Commodity Metals and metal products: Primary nonferrous metals	PPICMM	ln, Δ	
Consumer Price Index for All Urban Consumers: Apparel	CPIAPPSL	ln, Δ	
Consumer Price Index for All Urban Consumers: Transportation	CPITRNSL	ln, Δ	
Consumer Price Index for All Urban Consumers: Medical Care	CPIMEDSL	ln, Δ	
Consumer Price Index for All Urban Consumers: Commodities	CUSR0000SAC	ln, Δ	
Consumer Price Index for All Urban Consumers: Durables	CUUR0000SAD	ln, Δ^2	
Consumer Price Index for All Urban Consumers: Services	CUSR0000SAS	ln, Δ^2	
Consumer Price Index for All Urban Consumers: All Items Less Food	CPIULFSL	ln, Δ	
Consumer Price Index for All Urban Consumers: All items less shelter	CUUR0000SA0L2	ln, Δ	
Consumer Price Index for All Urban Consumers: All items less medical care	CUSR0000SA0L5	ln, Δ	
Average Hourly Earnings of Production and Nonsupervisory Employees: Goods-Producing	CES0600000008	ln, Δ^2	
Consumer Motor Vehicle Loans Owned by Finance Companies, Outstanding	DTCOLNVHFNM	ln, Δ	

Effective Federal Funds Rate	FEDFUNDS	Δ	x
3-Month Treasury Bill: Secondary Market Rate	TB3MS	Δ	
6-Month Treasury Bill: Secondary Market Rate	TB6MS	Δ	
1-Year Treasury Constant Maturity Rate	GS1	Δ	
10-Year Treasury Constant Maturity Rate	GS10	Δ	
Moody's Seasoned Aaa Corporate Bond Yield	AAA	Δ	
Moody's Seasoned Baa Corporate Bond Yield	BAA	Δ	
Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity	BAA10YM	Δ	
6-Month Treasury Bill Minus Federal Funds Rate	TB6SMFFM	Δ	
10-Year Treasury Constant Maturity Minus Federal Funds Rate	T10YFFM	Δ	x
Real St. Louis Adjusted Monetary Base	AMBSLREALx	ln, Δ	
Real M1 Money Stock	M1REALx	ln, Δ	x
Real M2 Money Stock	M2REALx	ln, Δ	x
Real MZM Money Stock	MZMREALx	ln, Δ	
Commercial and Industrial Loans, All Commercial Banks	BUSLOANSx	ln, Δ	
Consumer Loans at All Commercial Banks	CONSUMERx	ln, Δ	
Total Nonrevolving Credit Owned and Securitized, Outstanding	NONREVSLx	ln, Δ	
Real Estate Loans, All Commercial Banks	REALLNx	ln, Δ	
Total Consumer Credit Owned and Securitized, Outstanding	TOTALSLx	ln, Δ	
Households and Nonprofit Organizations; Total Assets, Level	TABSHNOx	ln, Δ	
Households and Nonprofit Organizations; Total Liabilities, Level	TLBSHNOx	ln, Δ	
Households and Nonprofit Organizations; Credit Market Instruments; Liability, Level	CMDEBT	ln, Δ^2	
Households and Nonprofit Organizations; Net Worth, Level	TNWBSHNOx	ln, Δ	
Households and Nonprofit Organizations; Total Financial Assets, Level	TFAABSHNO	ln, Δ	
Households and nonprofit organizations; real estate at market value, Level	HNOREMQ027Sx	ln, Δ	
Households and Nonprofit Organizations; Total Financial Assets, Level	TFAABSHNOx	ln, Δ	
Shares of gross domestic product: Exports of goods and services	B020RE1Q156NBEA	Δ	
Shares of gross domestic product: Imports of goods and services	B021RE1Q156NBEA	Δ	
Industrial Production: Manufacturing (SIC)	IPMANSICS	ln, Δ	
Industrial Production: Residential utilities	IPB51222S	ln, Δ	
Industrial Production: Fuels	IPFUELS	ln, Δ	
Average (Mean) Duration of Unemployment	UEMPMEAN	ln, Δ	

Average Weekly Hours of Production and Nonsupervisory Employees: Goods-Producing	CES0600000007	ln, Δ	
Total Reserves of Depository Institutions	TOTRESNS	ln, Δ	x
Reserves of Depository Institutions, Nonborrowed	NONBORRES	ln, Δ	x
5-Year Treasury Constant Maturity Rate	GS5	Δ	
3-Month Treasury Bill Minus Federal Funds Rate	TB3SMFFM	Δ	
5-Year Treasury Constant Maturity Minus Federal Funds Rate	T5YFFM	Δ	
Moody's Seasoned Aaa Corporate Bond Minus Federal Funds Rate	AAAFFM	Δ	
Total Consumer Loans and Leases Owned and Securitized by Finance Companies, Outstanding	DTCTHFNM	ln, Δ	
Securities in Bank Credit at All Commercial Banks	INVEST	ln, Δ	
Nikkei Stock Average, Nikkei 225	NIKKEI225	ln, Δ	
Nonfinancial Corporate Business; Total Liabilities, Level	TLBSNNCBx	ln, Δ	
Nonfinancial Corporate Business; Nonfinancial Assets, Level	TTAABSNNCBx	ln, Δ	
Nonfinancial Corporate Business; Net Worth, Level	TNWMVBSNNCBx	ln, Δ	
Nonfinancial noncorporate business; total liabilities, Level	NNBTILQ027Sx	ln, Δ	
Nonfinancial noncorporate business; total assets, Level	NNBTASQ027Sx	ln, Δ	
Nonfinancial Noncorporate Business; Proprietors' Equity in Noncorporate Business (Net Worth), Level	TNWBSNNBx	ln, Δ	
Corporate Net Cash Flow with IVA	CNCFx	ln, Δ	
U.S.: Industrial Share Prices (2010=100)	IFS	ln, Δ	x

A2 Bayesian Estimation via Dummy Observations

To conduct Bayesian estimation of the model, we cast the VAR in equation (11) of the main paper into a system of multivariate regressions:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}, \tag{A1}$$

where $\mathbf{Y} = [\mathbf{Y_1}, \ldots, \mathbf{Y_T}]'$, $\mathbf{X} = [\mathbf{X_1}, \ldots, \mathbf{X_T}]'$ with $\mathbf{X_t} = [\mathbf{Y'_{t-1}}, \ldots, \mathbf{Y'_{t-p}}]'$, and $\mathbf{u} = [\mathbf{u_1}, \ldots, \mathbf{u_T}]'$. Our prior structure is a Normal-Inverse Wishart prior, which has the form

$$vec(\boldsymbol{\beta})|\Sigma \sim N(vec(\boldsymbol{\beta}_0), \Sigma \otimes \Omega_0) \quad \text{and} \quad \Sigma \sim IW(S_0, \alpha_0), \tag{A2}$$

where the prior parameters $\boldsymbol{\beta}_0$, Ω_0, S_0, and α_0 are set to be consistent with equations (12) and (13) in the main paper and the expectation of Σ being $diag(\sigma_1^2, \ldots \sigma_n^2)$. The prior from (A2) can then be implemented by choosing the following dummy observations in order to match the moments of the prior (see, e.g., Del Negro and Schorfheide, 2011; Woźniak, 2016):

$$Y_d = \begin{pmatrix} 0_{np,n} \\ diag(\sigma_1 \ldots \sigma_n) \end{pmatrix}, X_d = \begin{pmatrix} J_p \otimes diag(\sigma_1 \ldots \sigma_n)/\lambda \\ 0_{n,np} \end{pmatrix}, \tag{A3}$$

where Y_d and X_d are dummy observations, $J_p = diag(1, \ldots p)$, $S_0 = (Y_d - X_d B_0)'(Y_d - X_d B_0)$, $B_0 = (X_d' X_d)^{-1} X_d' Y_d$, $\Omega_0 = (X_d' X_d)^{-1}$, and $\alpha_0 = T_d - np$, where T_d is the number of rows for both Y_d and X_d.[1] The first block of dummy observations places the prior on all of the individual VAR slope coefficients and the second block imposes the priors on the covariance matrix.

Augmenting the regression in equation (A1) with the dummy observations gives the following:

$$\mathbf{Y}^* = \mathbf{X}^*\boldsymbol{\beta} + \mathbf{u}^*, \tag{A4}$$

where $\mathbf{Y}^* = [\mathbf{Y}', \mathbf{Y_d}']'$, $\mathbf{X}^* = [\mathbf{X}', \mathbf{X_d}']'$ and $\mathbf{u}^* = [\mathbf{u}', \mathbf{u_d}']'$. Estimating the BVAR then simply amounts to conducting least squares regression of Y^* on X^*. Therefore, the posterior distribution has the form

$$vec(\boldsymbol{\beta})|\Sigma, \mathbf{Y} \sim N(\mathbf{vec}(\tilde{\boldsymbol{\beta}}, \boldsymbol{\Sigma} \otimes (\mathbf{X}^{*\prime}\mathbf{X}^*)^{-\mathbf{1}}) \tag{A5}$$

$$\Sigma|\mathbf{Y} \sim IW(\tilde{\Sigma}, T_d + T - np + 2), \tag{A6}$$

where $\tilde{\boldsymbol{\beta}} = (\mathbf{X}^{*\prime}\mathbf{X}^*)^{-\mathbf{1}}\mathbf{X}^{*\prime}\mathbf{Y}^{*\prime}$ and $\tilde{\Sigma} = (\mathbf{Y}^* - \mathbf{X}^*\tilde{\boldsymbol{\beta}})'(\mathbf{Y}^* - \mathbf{X}^*\tilde{\boldsymbol{\beta}})$.

[1]Note that because we demean all the variables prior to estimation, we do not include a constant in our BVAR. Thus the number of parameters in each equation is np, not $n \times (p+1)$.

A3 Causal Determinants of the U.S. Output Gap and Trend Growth

The empirical application within the main paper largely abstracts from causal analysis and focuses on associating movements in the estimated output gap with different sources of information. Here, we outline and conduct a straightforward extension of the main empirical analysis to demonstrate how to conduct structural analysis by decomposing the estimated trend output and output gap into identified structural shocks.

We use standard SVAR analysis for two widely-considered structural shocks: a monetary policy shock and an oil price shock. The monetary policy shock is identified by ordering the federal funds rate after 'slow moving' variables, but before 'fast moving' ones in a Cholesky decomposition. This identification strategy is similar in spirit to work by, *inter alia*, Christiano et al. (1999) and Bernanke et al. (2005), where the idea is that financial market variables are in the fast moving block because they can respond contemporaneously to monetary policy shocks, while slow moving variables take at least a quarter to respond. The fast moving variables in our benchmark 23 variable specification are real M1 and M2, stock prices, non-borrowed reserves, total reserves, and the slope of the yield curve. The oil price shock is identified by drawing from Kilian and Vega (2011), who show that oil prices do not appear responsive to macroeconomic news and thus can be taken to be pre-determined. This in essence orders the oil price first in a Cholesky decomposition and also has precedence in the wider SVAR literature studying oil price shocks (e.g., see Edelstein and Kilian, 2009; Wong, 2015). Our system is partially identified in the sense that we only identify two out of 23 potential structural shocks in our benchmark system and we do not attempt to disentangle any of the remaining 21 unidentified shocks. However, assumed orthogonality of structural shocks makes this partial identification possible.

We consider how much a given structural shock has driven the historical BN trend and cycle by performing a variance decomposition. To set up a variance decomposition for the BN cycle, we first note that $\mathbb{E}\boldsymbol{e_t} = \mathbf{0}$. Working off equation (7) in the main paper, it can be verified that the difference between the actual h-step-ahead BN cycle and the conditional expectation of the BN cycle at time $t-1$ is

$$\mathbf{c_{t+h}} - \mathbb{E}_{\mathbf{t-1}}\mathbf{c_{t+h}} = \sum_{i=0}^{h} \boldsymbol{\Gamma}_{\boldsymbol{i+1}}\mathbf{H}\boldsymbol{e}_{\boldsymbol{t+h-i}} \tag{A7}$$

$$= \sum_{i=0}^{h} \boldsymbol{\Gamma}_{\boldsymbol{i+1}}\mathbf{HA}\boldsymbol{\varepsilon}_{\boldsymbol{t+h-i}}, \tag{A8}$$

where the second equality follows from the identification associated with the structural shocks from the SVAR. Because $\mathbb{E}(\boldsymbol{e}_t' \boldsymbol{e}_{t-i}) = 0, i > 0$, the total variance can therefore be written as

$$Var(\mathbf{c_{t+h}} - \mathbb{E}_{\mathbf{t-1}}\mathbf{c_{t+h}}) = \sum_{\mathbf{i=0}}^{\mathbf{h}} \boldsymbol{\Gamma}_{\boldsymbol{i+1}}\mathbf{H}\boldsymbol{\Sigma}\mathbf{H}'\boldsymbol{\Gamma}_{\boldsymbol{i+1}}'. \tag{A9}$$

It follows, then, that a variance decomposition of the h-step-ahead variation in the BN cycle of the l^{th}-ordered target variable can be calculated using equations (A8) and (A9):

$$FEVD_{k,h}^{c} = \frac{\left[\sum_{i=0}^{h} \mathbf{s_{np,l}}\boldsymbol{\Gamma}_{\boldsymbol{i+1}}\mathbf{s_{n,k}}\mathbf{s_{n,k}}'\mathbf{H}\mathbf{a_k}\right]^2}{\mathbf{s_{np,l}}\left[\sum_{\mathbf{i=0}}^{\mathbf{h}} \boldsymbol{\Gamma}_{\boldsymbol{i+1}}\mathbf{H}\boldsymbol{\Sigma}\mathbf{H}'\boldsymbol{\Gamma}_{\boldsymbol{i+1}}'\right]\mathbf{s}_{\mathbf{np,l}}'}, \tag{A10}$$

where $FEVD_{k,h}^{c}$ is the h-step-ahead share of the variance of the BN cycle of the target variable due to the k^{th} structural shock that is identified using the k^{th} column, $\mathbf{a_k}$, of $\mathbf{A}$. Similarly, to perform a variance decomposition of trend growth for a target variable, it is straightforward to verify from equation (9) in the main paper that the variance of the change in trend can be written as

$$Var(\boldsymbol{\Delta\tau_t} - \mathbb{E}_{\boldsymbol{t-1}}\boldsymbol{\Delta\tau_t}) = \boldsymbol{\Gamma}_{\mathbf{0}}\mathbf{H}\boldsymbol{\Sigma}\mathbf{H}'\boldsymbol{\Gamma}_{\mathbf{0}}' \tag{A11}$$

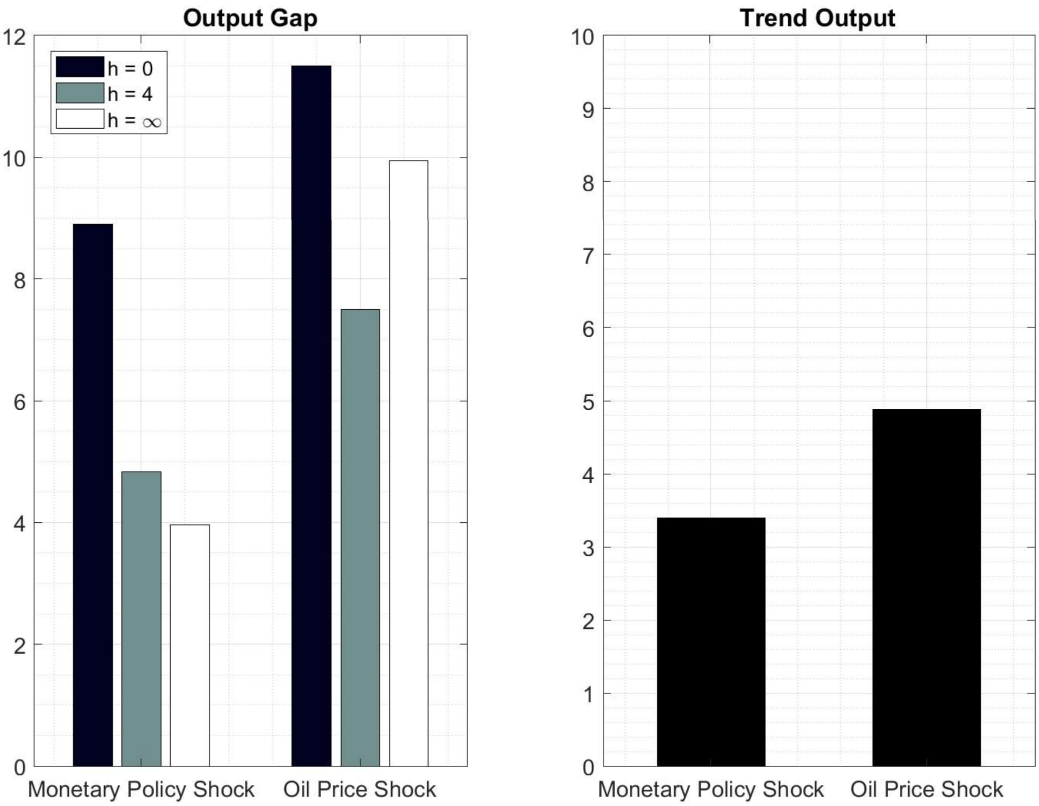
and the share of the variance can be similarly decomposed as

$$FEVD_{j}^{\tau} = \frac{\left[\sum_{i=0}^{h} \mathbf{s_{np,l}}\boldsymbol{\Gamma}_{\mathbf{0}}\mathbf{s_{n,k}}\mathbf{s_{n,k}}'\mathbf{H}\mathbf{a_k}\right]^2}{\mathbf{s_{np,l}}\left[\boldsymbol{\Gamma}_{\mathbf{0}}\mathbf{H}\boldsymbol{\Sigma}\mathbf{H}'\boldsymbol{\Gamma}_{\mathbf{0}}'\right]\mathbf{s}_{\mathbf{np,l}}'}. \tag{A12}$$

Note that due to the random walk trend, the variance of trend is unbounded as the time horizon goes to infinity. Consequently, a decomposition of the contemporaneous variance of the change in the trend is sufficient to provide insight into how much of the variation of trend growth is due to the various identified structural shocks.

Figure A1 presents a variance decomposition of the output gap and output trend growth. For the output gap, we present the share of monetary policy shocks and oil price shocks at horizons $h = 0$, $h = 4$, and $h = \infty$. Neither the monetary policy shock nor the oil price shock explain more than 10% of the variance of the output gap at any horizon. While the monetary policy shock explains about 7% of the variance of the output gap contemporaneously, its share quickly dissipates and it only explains about 4% of the unconditional variance. Therefore, it appears that the role of the monetary policy shock in driving the output gap is limited and relatively short lived. This finding is consistent with the wider SVAR literature, which often reports that monetary policy shocks explain only a small part of real economic activity. The oil price shock explains a somewhat larger share at about 10% of the variance of the output gap over all horizons. Meanwhile, consistent with traditional theories of growth that assume technology shocks are the main determinant of the long-run level of output, neither of these shocks explains much of output trend growth, with shares of about 5% for the oil price shock and less than 4% for the monetary policy shock. Notably, the latter result is reflective of the idea of long-run money neutrality, which suggests monetary policy should not have any permanent effects on the level of output.

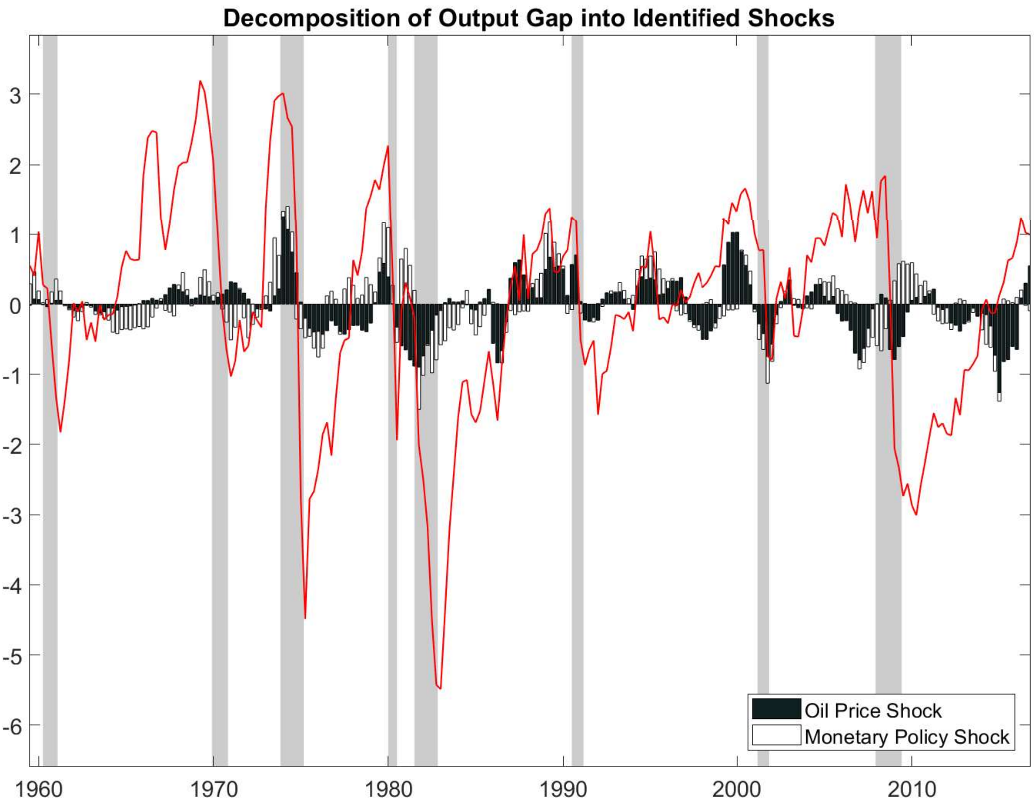
Figure A1: Variance Decompositions of Estimated U.S. Output Gap and Output Trend Growth



Notes: Results are for 23-variable benchmark BVAR. Units are percentage of total variation

Although variance decompositions are useful to gain an overall perspective on the relative importance of shocks, we can also calculate a historical contribution of shocks to the output gap to better understand specific historical episodes. This analysis is displayed in Figure A2. For realized monetary policy shocks, we can observe that they explain a large share of the positive output gap before the 1980 recession, consistent with anecdotal evidence that the Fed may have been overstimulating the economy in the 1970s. Although we can see that monetary policy shocks contributed to some of the negative output gap in the early 1980s, consistent with the Volcker disinflation, the overall output gap in the early 1980s was estimated to be large and negative, with monetary policy shocks only contributing to part of the negative gap rather than being the dominant cause. Meanwhile, a recent interpretation of the events leading to the Great Recession argues that the Fed was perhaps running the economy too hot before 2008 (e.g., see Taylor, 2012). Our historical decomposition does not support this story. We find that, while monetary policy shocks did contribute modestly to a rising positive output gap in the early 2000s, this contribution largely turned negative by 2005, while the estimated output gap continued to increase up until the advent of the Great Recession. Meanwhile, we find that realized oil price shocks tend to contribute positively to the output gap when oil prices are low and contribute negatively when oil prices are high. This can be seen from the negative contribution of oil price shocks throughout the 2000s and the positive contribution in the late 1990s. We also observe a positive contribution turning negative around 1990, consistent with the timing when the First Gulf War caused oil prices to rise from a low starting level. Furthermore, oil price shocks contributed negatively to the output gap around 1979 and 1980, consistent with the timing of the Iranian hostage crisis and the start of the Iraq-Iran War. Overall, we find that the contributions of realized monetary policy and oil price shocks line up with many well-understood historical events.

Figure A2: Historical Decomposition of the Estimated U.S. Output Gap



Notes: Units are 100 times natural log deviation from trend. Shaded bars correspond to NBER recession dates.

A4 Prior on the Signal-to-Noise Ratio

Here, we discuss how to implement a prior on the implied signal-to-noise ratio in terms of the variance of trend shocks relative to the variance of forecast errors. Typical BVAR methods would shrink a variable like log real GDP towards to a random walk process with an implied signal-to-noise ratio of 1. The underlying idea is that because a random walk provides a competitive forecast for many macroeconomic variables, shrinking towards a random walk balances overfitting, which worsens the forecasting performance of the model, with a more parsimonious and accurate forecasting model. However, a slight concern is that with larger models requiring more shrinkage, as shown by Banbura et al. (2010) and our baseline empirical analysis has shown, there is a possibility that as the number of time series relative to time series observations gets large, the model shrinks too much towards a random walk, creating an possible upward bias in the implied signal-to-noise ratio.

If one were concerned about such a possibility, it is possible to consider shrinking the target variable not towards a random walk, but towards a pre-specified signal-to-noise ratio δ, building on work by Kamber et al. (2018). To interpret this signal-to-noise ratio, $\delta = 0.01x$ implies $x\%$ of the variance of a forecast error for Δy_t is due to permanent shocks to y_t. Kamber et al. (2018) demonstrate how to perform a univariate BN decomposition with a pre-specified δ because there is a direct mapping from δ to the AR coefficients in an AR(p) model. In particular, letting ρ be the sum of AR coefficients in an AR(p) regression of output growth, the mapping between the two is $\rho = 1 - 1/\sqrt{\delta}$. In Kamber et al. (2018), the estimation of the output gap from a univariate AR(p) model of output growth treats ρ as being fixed and so can be viewed as a dogmatic prior on the signal-to-noise ratio. Here, in the multivariate environment, we place a prior on δ, but we do not make it dogmatic to allow the multivariate information to move the posterior away from the prior depending on how well the multivariate information helps to forecast Δy_t. A prior on δ amounts to placing a prior on the sum of the autoregressive coefficients in the target variable equation, which we label $\rho(\delta)$.

Recalling that Δy_t is the l^{th} variable in our BVAR and letting $\rho(\bar{\delta})$ be the sum of the autoregressive coefficients in the target variable equation consistent with a pre-specified $\bar{\delta}$. Implementing the prior on the signal-to-noise ratio implies:

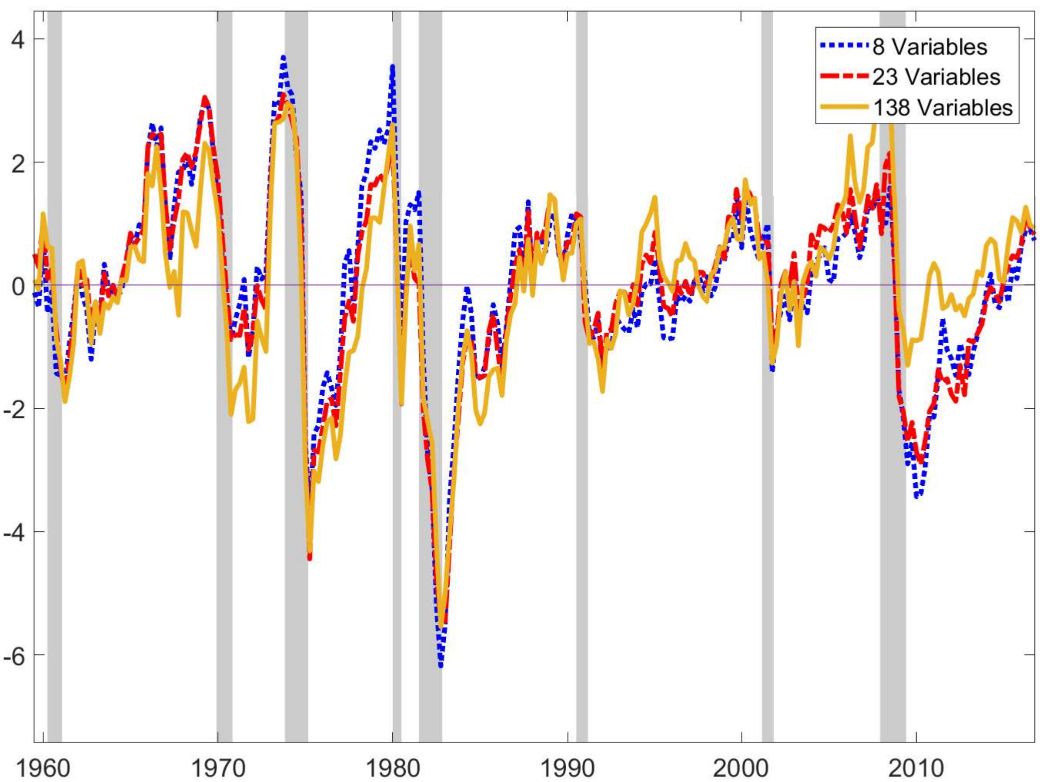
$$\mathbb{E}[\sum_{i=1}^{p} \beta_i^{ll}] = \rho(\bar{\delta}) \tag{A13}$$

$$Var[\sum_{i=1}^{p} \beta_i^{ll}] = \chi^2, \tag{A14}$$

where we set $\bar{\delta} = 0.25$ based on Kamber et al. (2018) and $\chi = \lambda/10$ to make the prior relatively informative compared to the usual Minnesota prior. The prior on the signal-to-noise ratio can be readily implemented using dummy observations. In particular, this will append the rows $\begin{bmatrix} 0_{1,l-1} & \rho/\chi & 0_{1,n-l} \end{bmatrix}$ and $\begin{bmatrix} 1_{1,n} \otimes \begin{pmatrix} 0_{1,l-1} & 1/\chi & 0_{1,n-l} \end{pmatrix} \end{bmatrix}$ to the Y_d and X_d matrices, respectively.

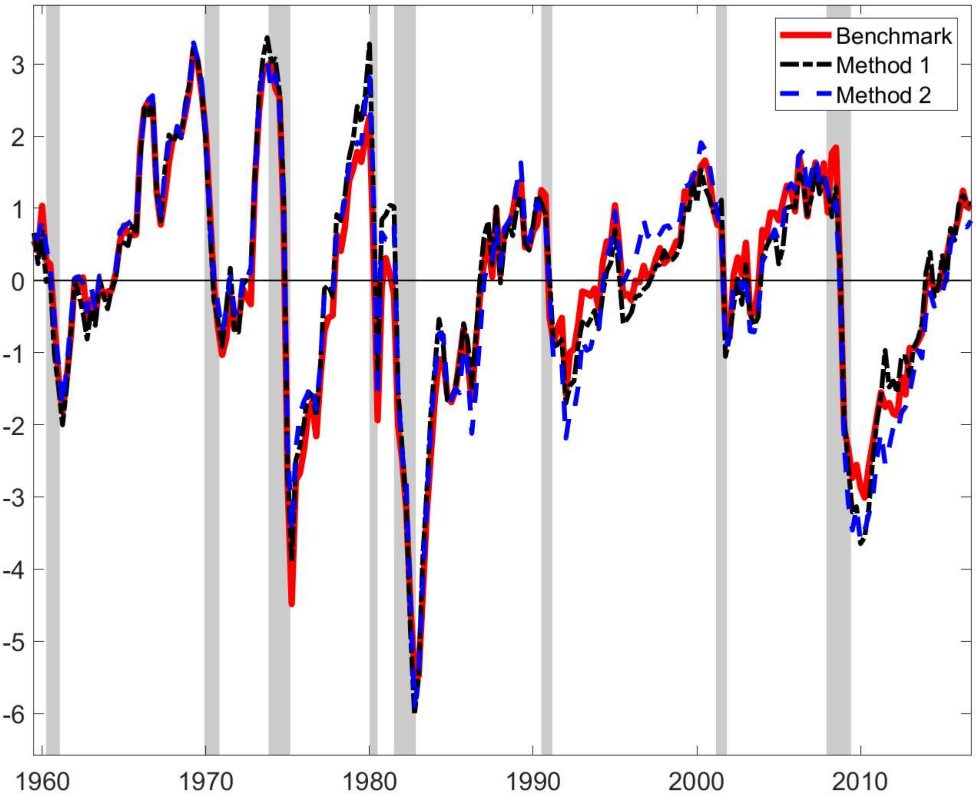
Figure A3 plots the estimated output gap for the eight-, 23- and 138-variable systems using the prior on the signal-to-noise ratio, with $\delta = 0.25$, and once again choosing the shrinkage parameter λ by optimizing on the pseudo-out-of-sample forecast performance. The results are similar to those in Figure 4 in the main text based on a Minnesota prior, suggesting that the likelihood dominates the prior on δ, at least for our empirical application. Thus, imposing such a prior would only matter for smaller samples when wanting to avoid an upward bias in the implied signal-to-noise ratio.

Figure A3: Estimated U.S. Output Gap for Various-Sized BVARs with Prior on Signal-to-Noise Ratio



Notes: Units are 100 times natural log deviation from trend. Shaded bars correspond to NBER recession dates.

Figure A4: Estimated Output Gaps for Different Methods of Variable Selection



Notes: Units are 100 times natural log deviation from trend. Shaded bars correspond to NBER recession dates. Benchmark refers to the output gap estimated from our benchmark 23-variable BVAR. Method 1 refers to our method of choosing eight variables based on jointly dropping the smallest informational shares from our benchmark BVAR. Method 2 refers to choosing eight variables by reestimating the model sequentially by the variable with the smallest informational share one at time

A5 Variable Selection for a Smaller BVAR

Here, we explore how the method of variable selection for a smaller BVAR affects our results. In addition to the method discussed in the main paper, we also consider dropping one variable at a time from our original benchmark 23-variable BVAR to determine variables for an eight-variable BVAR. That is, we first estimate the output gap first with the benchmark 23-variable BVAR, identify the variable with the smallest informational share of the 23 variables in estimating the output gap, drop that variable, and then estimated the output gap with the 22 remaining variables. We repeat this approach, always dropping the variable with smallest share, until we were left with only eight variables.

Figure A4 presents the results. We compare the output gap estimated from our benchmark 23-variable VAR together with the two methods of variable selection for an eight-variable BVAR. In the main paper, we jointly drop the 15 variables with the smallest shares. We label this

'Method 1'. We label the alternative approach of dropping one at a time to determine the eight-variable BVAR as 'Method 2'.

In general, the method of dropping variables does not matter for the estimation of the output gap. While our preferred method of jointly dropping (Method 1) appears to do a slightly better job matching the results for the 23-variable BVAR, dropping one at a time (Method 2) produces quite similar results. Table A1 reports the retained eight variables for both methods. The variables coincide in six of the eight cases, which explains why the output gaps are quite similar. Most of the relevant information for estimating the output gap is retained whichever method is used.

Table A1: Retained Variables for Different Methods of Variable Selection

Method 1	**Method 2**
GDP growth	GDP growth
PCE	PCE
Unemployment	Unemployment
CPI	CPI
Housing Starts	Housing Starts
Fed Funds Rate	Fed Funds Rate
Stock Prices	Total Reserves
Real M1	Hours

Notes: See Notes to Figure A4.

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