Appendix for
"Mismatch shocks and unemployment during the Great Recession"

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1 Roadmap

Section 2 describes the model. Section 3 deals with our empirical strategy. Section 4 provides additional results from the baseline estimation. Section 5 offers some robustness checks.

2 Model

2.1 The representative family

There is a continuum of identical households of mass one. Each household is a large family, made of a continuum of individuals of measure one. Family members are either working or searching for a job.[1] Following Merz (1995), we assume that family members pool their income before the head of the family chooses optimally per capita consumption.[2]

The representative family enters each period $t = 0, 1, 2, ...$, with B_{t-1} bonds and $\overline{K}_{t-1}$ units of physical capital. At the beginning of each period, bonds mature, providing B_{t-1} units of money. The representative family uses some of this money to purchase B_t new bonds at nominal cost B_t/R_t, where R_t denotes the gross nominal interest rate between period t and $t+1$.

The representative household owns capital and chooses the capital utilization rate, u_t, which transforms physical capital into effective capital according to

$$K_t = u_t \overline{K}_{t-1}. \tag{1}$$

The household rents $K_t(i)$ units of effective capital to intermediate-goods-producing firm $i \in [0,1]$ at the nominal rate r_t^K. The household's choice of $K_t(i)$ must satisfy

$$K_t = \int_0^1 K_t(i)\, di. \tag{2}$$

The cost of capital utilization is $a(u_t)$ per unit of physical capital. We assume the following functional form for the function a,

$$a(u_t) = \phi_{u1}(u_t - 1) + \frac{\phi_{u2}}{2}(u_t - 1)^2, \tag{3}$$

and that $u_t = 1$ in steady state.

Each period, $N_t(i)$ family members are employed at intermediate goods-producing firm $i \in [0,1]$. Each worker employed at firm i works a fixed amount of hours and earns the nominal wage $W_t(i)$. N_t denotes aggregate employment in period t and is given by

$$N_t = \int_0^1 N_t(i)\, di. \tag{4}$$

The remaining $(1 - N_t)$ family members are unemployed and and each receives nominal unemployment benefits b_t, financed through lump-sum taxes.

During period t, the representative household receives total nominal factor payments $r_t^K K_t + W_t N_t +$

[1] The model abstracts from the labor force participation decision.

[2] The use of search and matching frictions (Pissarides 2000) in business cycle models was pioneered by Merz (1995) and Andolfatto (1996) in the real business cycle (RBC) literature. More recently, the same labor market frictions have been studied in the New Keynesian model by Blanchard and Galí (2010), Christiano, Trabandt, and Walentin (2011), Christoffel, Kuester, and Linzert (2009), Gertler, Sala and Trigari (2008), Groshenny (2009 and 2013), Krause and Lubik (2007), Krause, López-Salido, and Lubik (2008), Ravenna and Walsh (2008 and 2011), Sveen and Weinke (2009), Trigari (2009), and Walsh (2005), among many others.

$(1 - N_t)\, b_t$. In addition, the household also receives nominal profits $D_t(i)$ from each firm $i \in [0,1]$, for a total of

$$D_t = \int_0^1 D_t(i)\, di. \tag{5}$$

In each period $t = 0, 1, 2, \ldots$,the family uses these resources to purchase finished goods, for both consumption and investment purposes, from the representative finished goods-producing firm at the nominal price P_t. The law of motion of physical capital is

$$\overline{K}_t \leq (1-\delta)\, \overline{K}_{t-1} + \mu_t \left[1 - F\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{6}$$

where δ denotes the depreciation rate. The function F captures the presence of adjustment costs in investment, as in Christiano, Eichenbaum and Evans (2005). We assume the following functional form for the function F,

$$F\left(\frac{I_t}{I_{t-1}}\right) = \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2, \tag{7}$$

where g_I is the steady-state growth rate of investment. Hence, along the balanced growth path, $F(g_I) = F'(g_I) = 0$ and $F''(g_I) = \phi_I > 0$. μ_t is an investment-specific technology shock affecting the efficiency with which consumption goods are transformed into capital. The investment-specific shock follows the exogenous stationary autoregressive process

$$\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \varepsilon_{\mu t}, \tag{8}$$

where $\varepsilon_{\mu t}$ is $i.i.d.N\left(0, \sigma_\mu^2\right)$.

The family's budget constraint is given by

$$\begin{aligned} P_t C_t + P_t I_t + \frac{B_t}{\epsilon_{bt} R_t} \leq\; & B_{t-1} + W_t N_t + (1 - N_t)\, b_t + r_t^K u_t \overline{K}_{t-1} \\ & - P_t a(u_t)\, \overline{K}_{t-1} - T_t + D_t \end{aligned} \tag{9}$$

for all $t = 0, 1, 2, \ldots$ As in Smets and Wouters (2007), the shock ϵ_{bt} drives a wedge between the central bank's instrument rate R_t and the return on assets held by the representative family. As noted by De Graeve, Emiris and Wouters (2009), this disturbance works as an aggregate demand shock and generates a positive comovement between consumption and investment. The risk-premium shock ϵ_{bt} follows the autoregressive process

$$\ln \epsilon_{bt} = \rho_b \ln \epsilon_{bt-1} + \varepsilon_{bt}, \tag{10}$$

where $0 < \rho_b < 1$, and ε_{bt} is $i.i.d.N\left(0, \sigma_b^2\right)$.

The family's lifetime utility is described by

$$E_t \sum_{s=0}^{\infty} \beta^s \ln\left(C_{t+s} - hC_{t+s-1}\right) \tag{11}$$

where $0 < \beta < 1$. When $h > 0$, the model allows for habit formation in consumption and consumption responds gradually to shocks.

The head of the family chooses C_t, B_t, u_t, I_t, and $\overline{K}_t$ for each $t = 0, 1, 2, \ldots$ to maximize the expected lifetime utility (10) subject to the constraints (6) and (9).

The Lagrangean reads

$$E_0 \sum_{t=0}^{\infty} \left\{ \begin{array}{c} \beta^t \ln\left(C_t - hC_{t-1}\right) + \beta^t \Lambda_t \left[\frac{B_{t-1} + W_t N_t + (1-N_t) b_t + r_t^K u_t \overline{K}_{t-1} - T_t + D_t}{P_t} - a\left(u_t\right) \overline{K}_{t-1} - C_t - I_t - \frac{B_t}{\epsilon_{bt} P_t R_t} \right] \\ + \beta^t \Upsilon_t \left[\left(1-\delta\right) \overline{K}_{t-1} + \mu_t \left(1 - \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2\right) I_t - \overline{K}_t \right] \end{array} \right\} \tag{12}$$

The first order conditions for this problem are

- C_t :

$$\Lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta h E_t \left(\frac{1}{C_{t+1} - hC_t} \right) \tag{13}$$

- B_t :

$$\Lambda_t = \epsilon_{bt} R_t \beta E_t \left(\Lambda_{t+1} \frac{P_t}{P_{t+1}} \right) \tag{14}$$

- u_t :

$$\left(\phi_{u1} - \phi_{u2}\right) + \phi_{u2} u_t = \widetilde{r}_t^K \tag{15}$$

  where $\widetilde{r}_t^K$ denotes the real rental rate of capital $\widetilde{r}_t^K = r_t^K / P_t$.

- I_t :

$$1 = \upsilon_t \mu_t \left[1 - \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2 - \phi_I \left(\frac{I_t}{I_{t-1}} - g_I\right)\left(\frac{I_t}{I_{t-1}}\right) \right] + \beta E_t \upsilon_{t+1} \mu_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \phi_I \left(\frac{I_{t+1}}{I_t} - g_I\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \tag{16}$$

  where υ_t is the marginal Tobin's Q: the Lagrange multiplier associated with the investment adjustment constraint, Υ_t, normalized by Λ_t.

- $\overline{K}_t$:

$$\upsilon_t = \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left[\left(1-\delta\right) \upsilon_{t+1} + \widetilde{r}_{t+1}^K u_{t+1} - a\left(u_{t+1}\right) \right] \right\} \tag{17}$$

- Λ_t :

$$\frac{B_{t-1} + W_t N_t + \left(1 - N_t\right) b_t + r_t^K u_t \overline{K}_{t-1} - T_t + D_t}{P_t} - a\left(u_t\right) \overline{K}_{t-1} = C_t + I_t + \frac{B_t}{\epsilon_{bt} R_t P_t} \tag{18}$$

  where Λ_t denotes the multiplier on (9) and can be interpreted as the utility to the household of an additional unit of wealth at date t.

- Υ_t :

$$\overline{K}_t = \left(1-\delta\right) \overline{K}_{t-1} + \mu_t \left[1 - \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2 \right] I_t \tag{19}$$

where Υ_t denotes the multiplier on (6) and can be interpreted as the utility to the household of an additional unit of physical capital at date t.

2.2 The representative finished goods-producing firm

During each period $t = 0, 1, 2, ...$, the representative finished goods-producing firm uses $Y_t(i)$ units of each intermediate good $i \in [0,1]$, purchased at the nominal price $P_t(i)$, to manufacture Y_t units of the finished good according to the constant-returns-to-scale technology described by

$$\left[\int_0^1 Y_t(i)^{(\theta_t-1)/\theta_t}\, di\right]^{\theta_t/(\theta_t-1)} \geq Y_t, \tag{20}$$

where θ_t translates into a random shock to the markup of price over marginal cost. This markup shock follows the autoregressive process

$$\ln\theta_t = (1-\rho_\theta)\ln\theta + \rho_\theta \ln\theta_{t-1} + \varepsilon_{\theta t}, \tag{21}$$

where $0 < \rho_\theta < 1$, $\theta > 1$, and $\varepsilon_{\theta t}$ is $i.i.d.N\left(0, \sigma_\theta^2\right)$.

Intermediate good i sells at the nominal price $P_t(i)$, while the finished good sells at the nominal price P_t. Given these prices, the finished goods-producing firm chooses Y_t and $Y_t(i)$ for all $i \in [0,1]$ to maximize its profits

$$P_t Y_t - \int_0^1 P_t(i)\, Y_t(i)\, di, \tag{22}$$

subject to the constraint (17) for each $t = 0, 1, 2, ...$ The first-order conditions for this problem are (17) with equality and

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t} Y_t \tag{23}$$

for all $i \in [0,1]$ and $t = 0, 1, 2, ...$

Competition in the market for the finished good drives the finished goods-producing firm's profits to zero in equilibrium. This zero-profit condition determines P_t as

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta_t}\, di\right]^{1/(1-\theta_t)} \tag{24}$$

for all $t = 0, 1, 2, ...$

2.3 The representative intermediate goods-producing firm

Each intermediate goods-producing firm $i \in [0,1]$ enters in period t with a stock of $N_{t-1}(i)$ employees carried from the previous period. At the beginning of period t, before production starts, $\rho N_{t-1}(i)$ jobs are destroyed, where ρ is the exogenous job destruction rate. The pool of workers ρN_{t-1} who have lost their job at the beginning of period t start searching immediately and can possibly be hired in period t. The number of employees at firm i evolves according to

$$N_t(i) = (1-\rho)\, N_{t-1}(i) + m_t(i). \tag{25}$$

$m_t(i)$ denotes the flow of new employees hired by firm i in period t, and is given by

$$m_t(i) = q_t V_t(i), \tag{26}$$

where $V_t(i)$ denotes vacancies posted by firm i in period t and q_t is the aggregate probability of filling a vacancy in period t. Workers hired in period t take part to period t production. Employment is therefore an *instantaneous* margin. However, each period some vacancies and job seekers remain unmatched. As a consequence, a firm-worker pair enjoys a joint surplus that motivates the existence of a long-run relationship between the two parties.

Aggregate employment $N_t = \int_0^1 N_t(i)\, di$ evolves over time according to

$$N_t = (1-\rho) N_{t-1} + m_t, \tag{27}$$

where $m_t = \int_0^1 m_t(i)\, di$ denotes aggregate matches in period t. Similarly, the aggregate vacancies is equal to $V_t = \int_0^1 V_t(i)\, di$. The pool of job seekers in period t, denoted by S_t, is given by

$$S_t = 1 - (1-\rho) N_{t-1}. \tag{28}$$

The matching process is described by the following aggregate CRS function

$$m_t = \zeta_t S_t^{\sigma} V_t^{1-\sigma}, \tag{29}$$

where ζ_t is an exogenous disturbance to the efficiency of the matching technology. We label this disturbance the mismatch shock and assume it follows the exogenous stationary stochastic process

$$\ln \zeta_t = (1-\rho_\zeta) \ln \zeta + \rho_\zeta \ln \zeta_{t-1} + \varepsilon_{\zeta t}, \tag{30}$$

where $\zeta > 0$ denotes the steady-state efficiency of the matching technology and $\varepsilon_{\zeta t}$ is $i.i.d.N\left(0, \sigma_\zeta^2\right)$. The probability q_t to fill a vacancy in period t is given by

$$q_t = \frac{m_t}{V_t} = \zeta_t \Theta_t^{-\sigma}, \tag{31}$$

where Θ denotes the tightness of the labor market $\Theta_t = V_t/S_t$. The probability s_t for a job seeker to find a job is

$$s_t = \frac{m_t}{S_t} = \zeta_t \Theta_t^{1-\sigma}. \tag{32}$$

Finally aggregate unemployment is defined by $U_t \equiv 1 - N_t$.

During each period $t = 0, 1, 2, ...$, the representative intermediate goods-producing firm combines $N_t(i)$ homogeneous employees with $K_t(i)$ units of efficient capital to produce $Y_t(i)$ units of intermediate good i according to the constant-returns-to-scale technology described by

$$Y_t(i) = A_t^{1-\alpha} K_t(i)^{\alpha} N_t(i)^{1-\alpha}. \tag{33}$$

A_t is an aggregate labor-augmenting technology shock whose growth rate, $z_t \equiv A_t/A_{t-1}$, follows the exogenous stationary stochastic process

$$\ln z_t = (1-\rho_z) \ln z + \rho_z \ln z_{t-1} + \varepsilon_{zt}, \tag{34}$$

where $z > 1$ denotes the steady-state growth rate of the economy and ε_{zt} is $i.i.d.N\left(0, \sigma_z^2\right)$.

The firm faces costs of hiring workers. As in Yashiv (2000 and 2006), hiring costs are a convex function of the linear combination of the number of vacancies and the number of hires. Hiring costs are measured in terms of aggregate output, and given by

$$\frac{\kappa}{2}\left(\frac{\phi_V V_t(i) + (1-\phi_V) q_t V_t(i)}{N_t(i)}\right)^2 Y_t, \tag{35}$$

where ϕ_V governs the magnitude of these costs.[3]

Intermediate goods substitute imperfectly for one another in the production function of the representative finished goods-producing firm. Hence, each intermediate goods-producing firm $i \in [0,1]$ sells its output $Y_t(i)$ in a monopolistically competitive market, setting $P_t(i)$, the price of its own product, with the commitment of satisfying the demand for good i at that price. Firms take the nominal wage as given when maximizing the discounted value of expected future profits.

Each intermediate goods-producing firm faces costs of adjusting its nominal price between periods (Rotemberg 1982), measured in terms of the finished good and given by

$$\frac{\phi_P}{2}\left(\frac{P_t(i)}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}P_{t-1}(i)} - 1\right)^2 Y_t. \tag{36}$$

ϕ_P governs the magnitude of the price adjustment cost. $\pi_t = \frac{P_t}{P_{t-1}}$ denotes the gross rate of inflation in period t. $\pi > 1$ denotes the steady-state gross rate of inflation and coincides with the central bank's target. The parameter $0 \leq \varsigma \leq 1$ governs the importance of backward-looking behavior in price setting (cf. Ireland 2007).

Following Arsenau and Chugh (2008), firms face quadratic wage-adjustment costs which are proportional to the size of their workforce and measured in terms of the finished good

$$\frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 N_t(i) Y_t, \tag{37}$$

where $\phi_W \geq 0$ governs the magnitude of the wage adjustment cost. The parameter $0 \leq \varrho \leq 1$ governs the importance of backward-looking behavior in wage setting.

Adjustment costs on the hiring rate, price and wage changes make the intermediate goods-producing firm's problem dynamic. It chooses $K_t(i)$, $N_t(i)$, $V_t(i)$ and $Y_t(i)$ and $P_t(i)$ for all $t = 0, 1, 2, \ldots$to maximize its total market value, given by

$$E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s}\left(\frac{D_{t+s}(i)}{P_{t+s}}\right) \tag{38}$$

where $\beta^t \Lambda_t / P_t$ measures the marginal utility to the representative household of an additional dollar of profits during period t and where

$$\begin{aligned} D_t(i) = {} & P_t(i) Y_t(i) - W_t(i) N_t(i) - r_t^K K_t(i) - \frac{\kappa}{2}\left(\frac{\phi_V V_t(i) + (1-\phi_V) q_t V_t(i)}{N_t(i)}\right)^2 P_t Y_t \\ & - \frac{\phi_P}{2}\left(\frac{P_t(i)}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}P_{t-1}(i)} - 1\right)^2 P_t Y_t - \frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 N_t(i) P_t Y_t, \end{aligned} \tag{39}$$

[3] Hiring costs are proportional to output and thus inherit the common stochastic trend driving productivity. This specification ensures that the unemployment rate remains stationary along the balanced steady-state growth path.

subject to the constraints

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t} Y_t, \tag{40}$$

$$Y_t(i) \leq K_t(i)^{\alpha} \left[A_t N_t(i)\right]^{1-\alpha}, \tag{41}$$

$$N_t(i) = \chi N_{t-1}(i) + q_t V_t(i), \tag{42}$$

where $\chi \equiv 1-\rho$ is the job survival rate.

This problem is equivalent to the one of choosing $K_t(i)$, $N_t(i)$, $V_t(i)$ and $P_t(i)$ to maximize (35), where

$$\begin{aligned}\frac{D_t(i)}{P_t} &= \left(\frac{P_t(i)}{P_t}\right)^{1-\theta_t} Y_t - \left(\frac{W_t(i)N_t(i) + r_t^K K_t(i)}{P_t}\right) - \frac{\kappa}{2}\left(\frac{\phi_V V_t(i) + (1-\phi_V) q_t V_t(i)}{N_t(i)}\right)^2 Y_t \\ &- \frac{\phi_P}{2}\left(\frac{P_t(i)}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}P_{t-1}(i)} - 1\right)^2 Y_t - \frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 N_t(i) Y_t,\end{aligned} \tag{43}$$

subject to the constraints

$$\left[\frac{P_t(i)}{P_t}\right]^{-\theta_t} Y_t \leq K_t(i)^{\alpha}\left[A_t N_t(i)\right]^{1-\alpha}, \tag{44}$$

$$N_t(i) = \chi N_{t-1}(i) + q_t V_t(i), \tag{45}$$

for all $t = 0, 1, 2, ...$

The Lagrangean reads

$$\begin{aligned} &E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left[\begin{array}{c} \left(\frac{P_t(i)}{P_t}\right)^{1-\theta_t} Y_t - \left(\frac{W_t(i)N_t(i)+r_t^K K_t(i)}{P_t}\right) - \frac{\kappa}{2}\left(\frac{\phi_V V_t(i)+(1-\phi_V)q_t V_t(i)}{N_t(i)}\right)^2 Y_t \\ -\frac{\phi_P}{2}\left(\frac{P_t(i)}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}P_{t-1}(i)} - 1\right)^2 Y_t - \frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 N_t(i) Y_t \end{array}\right] \\ &+ E_0 \sum_{t=0}^{\infty} \beta^t \Psi_t(i)\left[\chi N_{t-1}(i) + q_t V_t(i) - N_t(i)\right] + E_0 \sum_{t=0}^{\infty} \beta^t \Xi_t(i)\left[K_t(i)^{\alpha}\left(A_t N_t(i)\right)^{1-\alpha} - \left(\frac{P_t(i)}{P_t}\right)^{-\theta_t} Y_t\right]. \end{aligned} \tag{46}$$

The multiplier $\Psi_t(i)$ measures the value to firm i, expressed in utils, of an additional job in period t. The multiplier $\Xi_t(i)$ measures the value to firm i, expressed in utils, of an additional unit of output in period t. Hence, $\xi_t(i) = \Xi_t(i)/\Lambda_t$ represents firm i's real marginal cost in period t.

The first-order conditions for this problem are

- $K_t(i)$:

$$\widetilde{r}_t^K = \xi_t(i)\,\alpha K_t(i)^{\alpha-1}\left(A_t N_t(i)\right)^{1-\alpha} \tag{47}$$

- $N_t(i)$:

$$\begin{aligned}\frac{\Psi_t(i)}{\Lambda_t} &= \xi_t(i)(1-\alpha)\frac{Y_t(i)}{N_t(i)} - \frac{W_t(i)}{P_t} - \frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 Y_t \\ &+ \frac{\kappa}{N_t(i)}\left[\frac{\phi_V V_t(i) + (1-\phi_V) q_t V_t(i)}{N_t(i)}\right]^2 Y_t + \beta\chi\frac{\Lambda_{t+1}}{\Lambda_t}\frac{\Psi_{t+1}(i)}{\Lambda_{t+1}}\end{aligned} \tag{48}$$

This condition tells that the costs and benefits of hiring an additional worker must be equal.

- $V_t(i)$:

$$\frac{\Psi_t(i)}{\Lambda_t} = \left(\frac{\phi_V + (1-\phi_V)q_t}{N_t(i)}\right)^2 \frac{\kappa Y_t V_t(i)}{q_t} \tag{49}$$

- Vacancy posting condition :

$$\begin{aligned}\left(\frac{\phi_V + (1-\phi_V)q_t}{N_t(i)}\right)^2 \frac{\kappa Y_t V_t(i)}{q_t} &= \xi_t(i)(1-\alpha)\frac{Y_t(i)}{N_t(i)} - \frac{W_t(i)}{P_t} - \frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 Y_t \\ &+ \frac{\kappa}{N_t(i)}\left[\frac{\phi_V V_t(i) + (1-\phi_V)q_t V_t(i)}{N_t(i)}\right]^2 Y_t \\ &+ \beta\chi\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\phi_V + (1-\phi_V)q_{t+1}}{N_{t+1}(i)}\right)^2 \frac{\kappa Y_{t+1}V_{t+1}(i)}{q_{t+1}}\end{aligned} \tag{50}$$

- $P_t(i)$:

$$\begin{aligned}(1-\theta_t)\left(\frac{P_t(i)}{P_t}\right)^{-\theta_t} &= \phi_P\left(\frac{P_t(i)}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}P_{t-1}(i)} - 1\right)\left(\frac{P_t}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}P_{t-1}(i)}\right) - \theta_t\xi_t(i)\left(\frac{P_t(i)}{P_t}\right)^{-(1+\theta_t)} \\ &- \beta\phi_P E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{P_{t+1}(i)}{\pi_t^{\varsigma}\pi^{1-\varsigma}P_t(i)} - 1\right)\left(\frac{P_{t+1}(i)}{\pi_t^{\varsigma}\pi^{1-\varsigma}P_t(i)}\right)\frac{Y_{t+1}}{Y_t}\frac{P_t}{P_t(i)}\right]\end{aligned} \tag{51}$$

- $\Psi_t(i)$:

$$N_t(i) = \chi N_{t-1}(i) + q_t V_t(i) \tag{52}$$

- $\Xi_t(i)$:

$$A_t^{1-\alpha}K_t(i)^{\alpha}N_t(i)^{1-\alpha} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta_t} Y_t \tag{53}$$

2.4 Wage setting

Each period, intermediate-good producing firm i bargains with each of its employees individually over the nominal wage $W_t(i)$ to maximize the match surplus according to Nash bargaining,

$$W_t(i) = \arg\max\left[\Delta_t(i)^{\eta_t} J_t(i)^{1-\eta_t}\right]. \tag{54}$$

$\Delta_t(i)$ denotes the surplus of the representative worker while $J_t(i)$ denotes the surplus of the firm. Both $\Delta_t(i)$ and $J_t(i)$ are expressed in real terms. η_t denotes the worker's bargaining power which evolves exogenously according to

$$\ln\eta_t = (1-\rho_\eta)\ln\eta + \rho_\eta\ln\eta_{t-1} + \varepsilon_{\eta t}, \tag{55}$$

where $0 < \eta < 1$ and $\varepsilon_{\eta t}$ is $i.i.d.N\left(0, \sigma_\eta^2\right)$.

The family's value function is given by

$$
\begin{aligned}
\Omega\left(N_t\right) &= \ln\left(C_t - hC_{t-1}\right) \\
&+ \Lambda_t \left[\begin{array}{c} \int_0^1 \frac{W_t(i)N_t(i)}{P_t} di + \left(1-N_t\right)\left(\frac{b_t}{P_t}\right) \\ + \frac{B_{t-1}+r_t^K u_t \overline{K}_{t-1} - T_t + D_t}{P_t} - C_t - I_t - a\left(u_t\right)\overline{K}_{t-1} - \frac{B_t}{\epsilon_{bt} R_t P_t} \end{array} \right] \\
&+ \Upsilon_t \left[\left(1-\delta\right)\overline{K}_{t-1} + \mu_t \left(1 - \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2\right) I_t - \overline{K}_t \right] \\
&+ \beta E_t \Omega\left(N_{t+1}\right). \qquad (56)
\end{aligned}
$$

N_t evolves according to $N_t = \chi N_{t-1} + s_t\left(1-\chi N_{t-1}\right)$. The family takes the job finding rate $s_t = \frac{m_t}{S_t}$ as given. To ensure that the model is consistent with balanced growth, unemployment benefits b_t are proportional to the value of the nominal wage along the balanced growth path $b_t = \tau W_{ss,t}$, where τ is the replacement ratio. Following Trigari (2009) and Ravenna and Walsh (2008), $\Delta_t\left(i\right)$ is defined as the change in the family's value function $\Omega\left(N_t\right)$ from having one additional member employed. Thus, the surplus of an employee at firm i, expressed in utils, is given by

$$
\begin{aligned}
\widetilde{\Delta}_t\left(i\right) &= \frac{\partial\Omega\left(N_t\right)}{\partial N_t\left(i\right)}, \\
&= \Lambda_t\left(\frac{W_t\left(i\right)}{P_t} - \frac{b_t}{P_t}\right) + \beta E_t\left[\chi\left(1-s_{t+1}\right)\right]\widetilde{\Delta}_{t+1}\left(i\right). \qquad (57)
\end{aligned}
$$

The worker's surplus from the match, expressed in consumption goods, is given by

$$
\Delta_t\left(i\right) = \frac{W_t\left(i\right)}{P_t} - \frac{b_t}{P_t} + \beta E_t\left[\chi\left(1-s_{t+1}\right)\right]\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\Delta_{t+1}\left(i\right). \qquad (58)
$$

The employer's surplus from the match, expressed in real terms, is given by $J_t\left(i\right) = \frac{\Psi_t(i)}{\Lambda_t}$

$$
\begin{aligned}
J_t\left(i\right) &= \xi_t\left(i\right)\left(1-\alpha\right)\frac{Y_t\left(i\right)}{N_t\left(i\right)} - \frac{W_t\left(i\right)}{P_t} - \frac{\phi_W}{2}\left(\frac{W_t\left(i\right)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}\left(i\right)} - 1\right)^2 Y_t \\
&+ \frac{\kappa}{N_t\left(i\right)}\left[\frac{\phi_V V_t\left(i\right) + \left(1-\phi_V\right)q_t V_t\left(i\right)}{N_t\left(i\right)}\right]^2 Y_t + \beta\chi E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}J_{t+1}\left(i\right)\right). \qquad (59)
\end{aligned}
$$

Nash bargaining over the nominal wage yields the following first-order condition

$$
\eta_t J_t\left(i\right)\frac{\partial\Delta_t\left(i\right)}{\partial W_t\left(i\right)} = -\left(1-\eta_t\right)\Delta_t\left(i\right)\frac{\partial J_t\left(i\right)}{\partial W_t\left(i\right)}, \qquad (60)
$$

where

$$
\frac{\partial\Delta_t\left(i\right)}{\partial W_t\left(i\right)} = \frac{1}{P_t}, \qquad (61)
$$

$$
-\frac{\partial J_t\left(i\right)}{\partial W_t\left(i\right)} = \left\{ \begin{array}{c} \frac{1}{P_t} + \phi_W Y_t\left(\frac{1}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)}\right)\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right) \\ -\beta\chi\phi_W E_t\left[\frac{\Lambda_{t+1}Y_{t+1}}{\Lambda_t W_t(i)}\left(\frac{W_{t+1}(i)}{z\pi_t^{\varrho}\pi^{1-\varrho}W_t(i)}\right)\left(\frac{W_{t+1}(i)}{z\pi_t^{\varrho}\pi^{1-\varrho}W_t(i)} - 1\right)\right] \end{array} \right\}. \qquad (62)
$$

When $\phi_W = 0$, adjusting nominal wages is costless for the firm. In that case, the effects of a marginal increase in the nominal wage on the worker's surplus and on the firm's surplus have the same magnitude (with opposite signs):

$$\text{if } \phi_W = 0, \text{ then } \frac{\partial \Delta_t(i)}{\partial W_t(i)} = -\frac{\partial J_t(i)}{\partial W_t(i)} = \frac{1}{P_t}. \tag{63}$$

In the absence of nominal wage-adjustment costs, Nash bargaining over the nominal wage implies the usual first-order condition

$$\Delta_t(i) = \left(\frac{\eta_t}{1-\eta_t}\right) J_t(i). \tag{64}$$

Thus, as pointed out by Arsenau and Chugh (2008), Nash bargaining over the nominal wage when there are no nominal wage adjustment costs is equivalent to Nash bargaining over the real wage. The presence of nominal wage-adjustment costs (beared by the firm) affects the *effective* bargaining powers of the firm and the worker respectively. In the presence of nominal wage adjustment costs, the first-order condition from Nash bargaining is given by

$$\Delta_t(i) = \frac{\eta_t}{(1-\eta_t)} \frac{[\partial \Delta_t(i)/\partial W_t(i)]}{[-\partial J_t(i)/\partial W_t(i)]} J_t(i), \tag{65}$$

$$\Delta_t(i) = \text{Ŋ}_{it} J_t(i), \tag{66}$$

where we have introduced the notation

$$\text{Ŋ}_{it} \equiv \frac{\left(\frac{\eta_t}{1-\eta_t}\right)\left(\frac{\partial \Delta_t(i)}{\partial W_t(i)}\right)}{\left(-\frac{\partial J_t(i)}{\partial W_t(i)}\right)}. \tag{67}$$

Substituting the expressions of the two partial derivatives into the first-order condition, we obtain

$$\begin{aligned}
&\text{Ŋ}_{it}\left[\xi_t(i)(1-\alpha)\frac{Y_t(i)}{N_t(i)} - \frac{W_t(i)}{P_t} - \frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 Y_t\right] \\
&+ \text{Ŋ}_{it}\left[\frac{\kappa}{N_t(i)}\left(\frac{\phi_V V_t(i) + (1-\phi_V) q_t V_t(i)}{N_t(i)}\right)^2 Y_t\right] \\
&+ \text{Ŋ}_{it}\beta\chi E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t} J_{t+1}(i)\right] \\
&= \frac{W_t(i)}{P_t} - \frac{b_t}{P_t} + \beta\chi E_t\left[(1-s_{t+1})\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\Delta_{t+1}(i)\right],
\end{aligned} \tag{68}$$

Using the fact that $\Delta_{t+1}(i) = \text{Ŋ}_{it+1} J_{t+1}(i)$ in the above equation, we obtain

$$\begin{aligned}
&\text{Ŋ}_{it}\left[\xi_t(i)(1-\alpha)\frac{Y_t(i)}{N_t(i)} - \frac{W_t(i)}{P_t} - \frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 Y_t\right] \\
&+ \text{Ŋ}_{it}\left[\frac{\kappa}{N_t(i)}\left(\frac{\phi_V V_t(i) + (1-\phi_V) q_t V_t(i)}{N_t(i)}\right)^2 Y_t\right] \\
&+ \text{Ŋ}_{it}\beta\chi E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t} J_{t+1}(i)\right] \\
&= \frac{W_t(i)}{P_t} - \frac{b_t}{P_t} + \beta\chi E_t\left[(1-s_{t+1})\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\text{Ŋ}_{it+1} J_{t+1}(i)\right],
\end{aligned} \tag{69}$$

Now, let us recall the definition of the firm's surplus

$$J_t(i) = \frac{\Psi_t(i)}{\Lambda_t} = \left(\frac{\phi_V + (1-\phi_V)\, q_t}{N_t(i)}\right)^2 \frac{\kappa Y_t V_t(i)}{q_t}. \tag{70}$$

Using this expression of $J_{t+1}(i)$, the real-wage equation becomes

$$\begin{aligned}
&\frac{W_t(i)}{P_t} - \eta_{it}\left[\xi_t(i)(1-\alpha)\frac{Y_t(i)}{N_t(i)} - \frac{W_t(i)}{P_t} - \frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 Y_t\right] \\
&- \eta_{it}\left[\frac{\kappa}{N_t(i)}\left(\frac{\phi_V V_t(i) + (1-\phi_V)\, q_t V_t(i)}{N_t(i)}\right)^2 Y_t\right] \\
&= \eta_{it}\beta\chi E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\phi_V + (1-\phi_V)\, q_{t+1}}{N_{t+1}(i)}\right)^2 \frac{\kappa Y_{t+1}V_{t+1}(i)}{q_{t+1}}\right] \\
&+ \frac{b_t}{P_t} - \beta\chi E_t\left[(1-s_{t+1})\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\eta_{it+1}\left(\frac{\phi_V + (1-\phi_V)\, q_{t+1}}{N_{t+1}(i)}\right)^2 \frac{\kappa Y_{t+1}V_{t+1}(i)}{q_{t+1}}\right].
\end{aligned} \tag{71}$$

Finally, the equation governing the dynamics of the real wage at firm i is given by

$$\begin{aligned}
\frac{W_t(i)}{P_t} = &\left(\frac{\eta_{it}}{1+\eta_{it}}\right)\left[\begin{array}{c}
\xi_t(i)(1-\alpha)\frac{Y_t(i)}{N_t(i)} - \frac{\phi_W}{2}\left(\frac{W_t(i)}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}(i)} - 1\right)^2 Y_t \\
+\frac{\kappa}{N_t(i)}\left(\frac{\phi_V V_t(i)+(1-\phi_V)q_t V_t(i)}{N_t(i)}\right)^2 Y_t \\
+\beta\chi E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\left(\frac{\phi_V+(1-\phi_V)q_{t+1}}{N_{t+1}(i)}\right)^2 \frac{\kappa Y_{t+1}V_{t+1}(i)}{q_{t+1}}
\end{array}\right] \\
&+ \frac{1}{(1+\eta_{it})}\left[\frac{b_t}{P_t} - \beta\chi E_t \eta_{it+1}(1-s_{t+1})\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\left(\frac{\phi_V + (1-\phi_V)\, q_{t+1}}{N_{t+1}(i)}\right)^2 \frac{\kappa Y_{t+1}V_{t+1}(i)}{q_{t+1}}\right].
\end{aligned} \tag{72}$$

2.5 Government

The central bank adjusts the short-term nominal gross interest rate R_t by following a Taylor-type rule

$$\ln\left(\frac{R_t}{R}\right) = \rho_r \ln\left(\frac{R_{t-1}}{R}\right) + (1-\rho_r)\left\{\rho_\pi \ln\left[\left(\frac{P_t/P_{t-4}}{\Pi}\right)^{1/4}\right] + \rho_y \ln\left[\left(\frac{Y_t/Y_{t-4}}{G_y}\right)^{1/4}\right]\right\} + \ln\epsilon_{mpt}, \tag{73}$$

where $\Pi_t = P_t/P_{t-4}$ and $G_{yt} = Y_t/Y_{t-4}$ and Π and G_y denote the steady state values of Π_t and G_{yt} respectively. The degree of interest-rate smoothing ρ_r and the reaction coefficients ρ_π, ρ_y are positive. The monetary policy shock ϵ_{mpt} follows an AR(1) process

$$\ln\epsilon_{mpt} = \rho_{mp}\ln\epsilon_{mpt-1} + \varepsilon_{mpt}, \tag{74}$$

with $0 \leq \rho_{mp} < 1$ and $\varepsilon_{mpt} \sim i.i.d.N\left(0, \sigma_{mp}^2\right)$.

The government budget constraint is of the form

$$P_t G_t + (1-N_t)\, b_t = \left(\frac{B_t}{R_t} - B_{t-1}\right) + T_t, \tag{75}$$

where T_t denotes total nominal lump-sum transfers. Public spending is an exogenous time-varying fraction of GDP

$$G_t = \left(1 - \frac{1}{\epsilon_{gt}}\right) Y_t, \tag{76}$$

where ϵ_{gt} evolves according to

$$\ln \epsilon_{gt} = (1-\rho_g)\ln \epsilon_g + \rho_g \ln \epsilon_{gt-1} + \varepsilon_{gt}, \tag{77}$$

with $\varepsilon_{gt} \sim i.i.d.N\left(0,\sigma_g^2\right)$.

2.6 The aggregate resource constraint

In a symmetric equilibrium, all intermediate goods-producing firms make identical decisions, so that $Y_t(i) = Y_t$, $P_t(i) = P_t$, $N_t(i) = N_t$, $V_t(i) = V_t$, $K_t(i) = K_t$ for all $i \in [0,1]$ and $t = 0,1,2,.....$ Moreover, workers are homogeneous and all workers at a given firm i receive the same nominal wage $W_t(i)$, so that $W_t(i) = W_t$ for all $i \in [0,1]$ and $t = 0,1,2,....$ The aggregate resource constraint is obtained by aggregating the household budget constraint over all intermediate sectors $i \in [0,1]$,

$$\left[\begin{array}{c} \frac{1}{\epsilon_{gt}} - \frac{\kappa}{2}\left(\frac{\phi_V V_t + (1-\phi_V) q_t V_t}{N_t}\right)^2 - \frac{\phi_P}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}} - 1\right)^2 - \\ \frac{\phi_W}{2}\left(\frac{W_t}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}} - 1\right)^2 N_t \end{array}\right] Y_t = C_t + I_t + a(u_t)\overline{K}_{t-1}. \tag{78}$$

$$\aleph_t = \frac{\phi_V V_t + (1-\phi_V) m_t}{N_t} \tag{79}$$

$$\left[\begin{array}{c} \frac{1}{\epsilon_{gt}} - \frac{\kappa}{2}\aleph_t^2 - \frac{\phi_P}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}} - 1\right)^2 \\ -\frac{\phi_W}{2}\left(\frac{W_t}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}} - 1\right)^2 N_t \end{array}\right] Y_t = C_t + I_t + \left[\phi_{u1}(u_t - 1) + \frac{\phi_{u2}}{2}(u_t-1)^2\right]\overline{K}_{t-1} \tag{80}$$

2.7 The symmetric equilibrium

In a symmetric equilibrium, $Y_t(i) = Y_t$, $P_t(i) = P_t$, $N_t(i) = N_t$, $V_t(i) = V_t$, $K_t(i) = K_t$, $W_t(i) = W_t$ for all $i \in [0,1]$ and $t = 0,1,2,....$Defining the real wage $\widetilde{W}_t = W_t/P_t$, the gross rate of price inflation $\pi_t = P_t/P_{t-1}$, the system of equilibrium conditions becomes

1. Y_t

$$\left[\begin{array}{c} \frac{1}{\epsilon_{gt}} - \frac{\kappa}{2}\aleph_t^2 - \frac{\phi_P}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}} - 1\right)^2 \\ -\frac{\phi_W}{2}\left(\frac{W_t}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}W_{t-1}} - 1\right)^2 N_t \end{array}\right] Y_t = C_t + I_t + \left[\phi_{u1}(u_t - 1) + \frac{\phi_{u2}}{2}(u_t-1)^2\right]\overline{K}_{t-1}$$

2. $\aleph_t$

$$\aleph_t = \frac{\phi_V V_t + (1-\phi_V) m_t}{N_t}$$

3. m_t

$$m_t = q_t V_t$$

4. x_t

$$x_t = \frac{m_t}{N_t}$$

5. K_t

$$K_t = u_t \overline{K}_{t-1}$$

6. $\overline{K}_t$

$$\overline{K}_t = (1-\delta)\,\overline{K}_{t-1} + \mu_t \left[1 - \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2\right] I_t$$

7. μ_t

$$\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \varepsilon_{\mu t}$$

8. ϵ_{bt}

$$\ln \epsilon_{bt} = \rho_b \ln \epsilon_{bt-1} + \varepsilon_{bt}$$

9. Λ_t

$$\Lambda_t = \beta \epsilon_{bt} R_t E_t \left(\frac{\Lambda_{t+1}}{\pi_{t+1}}\right)$$

10. C_t

$$\Lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta h E_t \left(\frac{1}{C_{t+1} - hC_t}\right)$$

11. $\widetilde{r}_t^K = \frac{r_t^K}{P_t}$

$$(\phi_{u1} - \phi_{u2}) + \phi_{u2} u_t = \widetilde{r}_t^K$$

12. I_t

$$\begin{aligned} 1 = {} & \upsilon_t \mu_t \left[1 - \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2 - \phi_I \left(\frac{I_t}{I_{t-1}} - g_I\right)\left(\frac{I_t}{I_{t-1}}\right)\right] \\ & + \beta E_t \upsilon_{t+1} \mu_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \phi_I \left(\frac{I_{t+1}}{I_t} - g_I\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \end{aligned}$$

13. $\upsilon_t = \frac{\Upsilon_t}{\Lambda_t}$

$$\upsilon_t = \beta E_t \left\{\frac{\Lambda_{t+1}}{\Lambda_t}\left[(1-\delta)\,\upsilon_{t+1} + \widetilde{r}_{t+1}^K u_{t+1} - a\,(u_{t+1})\right]\right\}$$

14. θ_t

$$\ln \theta_t = (1 - \rho_\theta) \ln \theta + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta t}$$

15. N_t

$$N_t = \chi N_{t-1} + q_t V_t$$

16. S_t

$$S_t = 1 - \chi N_{t-1}$$

17. U_t

$$U_t = 1 - N_t$$

18. $\Theta_t = \frac{V_t}{S_t}$

$$\Theta_t = \frac{V_t}{S_t}$$

19. q_t

$$q_t = \zeta_t \left(\frac{S_t}{V_t}\right)^{\sigma}$$
$$q_t = \zeta_t \left(\frac{V_t}{S_t}\right)^{-\sigma}$$
$$q_t = \zeta_t \Theta_t^{-\sigma}$$

20. s_t

$$s_t = \zeta_t \left(\frac{V_t}{S_t}\right)^{1-\sigma}$$
$$s_t = \zeta_t \Theta_t^{1-\sigma}$$

21. ζ_t

$$\ln \zeta_t = (1 - \rho_\zeta) \ln \zeta + \rho_\zeta \ln \zeta_{t-1} + \varepsilon_{\zeta t}$$

22. V_t

$$\aleph_t^2 \frac{\kappa Y_t}{m_t} = \xi_t (1-\alpha) \frac{Y_t}{N_t} - \widetilde{W}_t - \frac{\phi_W}{2} \left(\frac{W_t}{z \pi_{t-1}^{\varrho} \pi^{1-\varrho} W_{t-1}} - 1\right)^2 Y_t$$
$$+ \frac{\kappa}{N_t} \aleph_t^2 Y_t + \beta \chi \frac{\Lambda_{t+1}}{\Lambda_t} \aleph_{t+1}^2 \frac{\kappa Y_{t+1}}{m_{t+1}}$$

23. u_t

$$Y_t = A_t^{1-\alpha} {K_t}^{\alpha} {N_t}^{1-\alpha}$$

24. A_t

$$z_t = \frac{A_t}{A_{t-1}}$$

25. $z_t = \frac{A_t}{A_{t-1}}$

$$\ln (z_t) = (1 - \rho_z) \ln (z) + \rho_z \ln (z_{t-1}) + \varepsilon_{zt}$$

26. $\xi_t = \frac{\Xi_t}{\Lambda_t}$

$$\widetilde{r}_t^K = \left(\alpha \frac{Y_t}{K_t}\right) \xi_t$$

27. π_t

$$\phi_P \left(\frac{\pi_t}{\pi_{t-1}^{\varsigma} \pi^{1-\varsigma}} - 1\right) \left(\frac{\pi_t}{\pi_{t-1}^{\varsigma} \pi^{1-\varsigma}}\right) = (1-\theta_t) + \theta_t \xi_t + \beta \phi_P E_t \left[\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) \left(\frac{\pi_{t+1}}{\pi_t^{\varsigma} \pi^{1-\varsigma}} - 1\right) \left(\frac{\pi_{t+1}}{\pi_t^{\varsigma} \pi^{1-\varsigma}}\right) \left(\frac{Y_{t+1}}{Y_t}\right)\right]$$

28. $\widetilde{b}_t = \frac{b_t}{P_t}$

$$\widetilde{b}_t = \tau \widetilde{W}_{ss,t}$$

29. $\widetilde{W}_t = \frac{W_t}{P_t}$

$$\begin{aligned} \widetilde{W}_t = &\left(\frac{\gimel_t}{1+\gimel_t}\right) \left[\xi_t (1-\alpha) \frac{Y_t}{N_t} - \frac{\phi_W}{2} \left(\frac{W_t}{z \pi_{t-1}^{\varrho} \pi^{1-\varrho} W_{t-1}} - 1\right)^2 Y_t + \frac{\kappa}{N_t} \aleph_t^2 Y_t\right] \\ &+ \left(\frac{\gimel_t}{1+\gimel_t}\right) \left[\beta \chi E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) \aleph_{t+1}^2 \frac{\kappa Y_{t+1}}{m_{t+1}}\right] \\ &+ \frac{1}{(1+\gimel_t)} \left[\widetilde{b}_t - \beta \chi E_t \gimel_{t+1} (1 - s_{t+1}) \left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) \aleph_{t+1}^2 \frac{\kappa Y_{t+1}}{m_{t+1}}\right] \end{aligned}$$

30. $\gimel_t$

$$\gimel_t = \frac{\left(\frac{\eta_t}{1-\eta_t}\right) \left(\frac{\widetilde{W}_t}{Y_t}\right)}{\frac{\widetilde{W}_t}{Y_t} + \phi_W \left(\frac{W_t}{z \pi_{t-1}^{\varrho} \pi^{1-\varrho} W_{t-1}} - 1\right) \left(\frac{W_t}{z \pi_{t-1}^{\varrho} \pi^{1-\varrho} W_{t-1}}\right) - \beta \chi \phi_W E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \left(\frac{W_{t+1}}{z \pi_t^{\varrho} \pi^{1-\varrho} W_t} - 1\right) \left(\frac{W_{t+1}}{z \pi_t^{\varrho} \pi^{1-\varrho} W_t}\right) \frac{Y_{t+1}}{Y_t}\right]}$$

31. η_t

$$\ln \eta_t = (1-\rho_\eta) \ln \eta + \rho_\eta \ln \eta_{t-1} + \varepsilon_{\eta t}$$

32. R_t

$$\ln\left(\frac{R_t}{R}\right) = \rho_r \ln\left(\frac{R_{t-1}}{R}\right) + (1-\rho_r) \left\{\rho_\pi \ln\left[\left(\frac{P_t/P_{t-4}}{\Pi}\right)^{1/4}\right] + \rho_y \ln\left[\left(\frac{Y_t/Y_{t-4}}{G_y}\right)^{1/4}\right]\right\} + \ln \epsilon_{mpt}$$

33. ϵ_{mpt}

$$\ln \epsilon_{mpt} = \rho_{mp} \ln \epsilon_{mpt-1} + \varepsilon_{mpt}$$

34. G_t

$$G_t = \left(1 - \frac{1}{\epsilon_{gt}}\right) Y_t$$

35. ϵ_{gt}

$$\ln \epsilon_{gt} = (1-\rho_g)\ln \epsilon_g + \rho_g \ln \epsilon_{gt-1} + \varepsilon_{gt}$$

36. gy_t : Quarterly gross rate of output growth

$$gy_t = Y_t / Y_{t-1}$$

37. gc_t : Quarterly gross rate of consumption growth

$$gc_t = C_t / C_{t-1}$$

38. gi_t : Quarterly gross rate of investment growth

$$gi_t = I_t / I_{t-1}$$

39. gw_t : Quarterly gross rate of real wage growth

$$gw_t = \widetilde{W}_t / \widetilde{W}_{t-1}$$

These 39 equations determine equilibrium values for the 39 variables Y_t, K_t, $\overline{K}_t$, u_t, C_t, Λ_t, R_t, G_t, I_t, υ_t, $\widetilde{r}_t^K$, ξ_t, N_t, S_t, U_t, V_t, $\aleph_t$, m_t, x_t, Θ_t, q_t, s_t, $\widetilde{W}_t$, $\daleth_t$, $\widetilde{b}_t$, π_t, μ_t, ϵ_{bt}, A_t, z_t, ζ_t, θ_t, η_t, ϵ_{mpt}, ϵ_{gt}, gy_t, gc_t, gi_t, gw_t.

2.8 The stationary transformed economy

Output, consumption, investment, capital and the real wage share the stochastic trend induced by the unit root process of neutral technological progress. We first rewrite the model in terms of stationary variables, and then loglinearize this transformed model economy around its steady state. This approximate model can then be solved using standard methods. The following variables are stationary and need not to be transformed: u_t, R_t, $\widetilde{r}_t^K$, $\upsilon_t = \frac{\Upsilon_t}{\Lambda_t}$, ξ_t, N_t, S_t, U_t, V_t, $\aleph_t$, m_t, x_t, q_t, s_t, $\pi_t = \frac{P_t}{P_{t-1}}$, μ_t, a_t, z_t, ζ_t, θ_t, η_t, ϵ_{mpt}, ϵ_{gt} and $\daleth_t$. we define the transformed variables $y_t = Y_t/A_t$, $k_t = K_t/A_t$, $\overline{k}_t = \overline{K}_t/A_t$, $c_t = C_t/A_t$, $\lambda_t = A_t\Lambda_t$, $i_t = I_t/A_t$, $\widetilde{w}_t = \widetilde{W}_t/A_t$, $\widetilde{\mathbf{b}}_t = \widetilde{b}_t/A_t$, $g_t = G_t/A_t$. The stationarized economy contains only 38 equations in 38 variables because the level of the non-stationary productivity shock A_t is not included.

1. $y_t = Y_t/A_t$

$$\left[\frac{1}{\epsilon_{gt}} - \frac{\kappa}{2}\aleph_t^2 - \frac{\phi_P}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\varsigma}\pi^{1-\varsigma}} - 1\right)^2 - \frac{\phi_W}{2}\left(\frac{z_t\pi_t\widetilde{w}_t}{z\pi_{t-1}^{\varrho}\pi^{1-\varrho}\widetilde{w}_{t-1}} - 1\right)^2 N_t\right] y_t$$
$$= c_t + i_t + \left[\phi_{u1}(u_t - 1) + \frac{\phi_{u2}}{2}(u_t-1)^2\right]\overline{k}_{t-1}\frac{1}{z_t}$$

2. $\aleph_t$

$$\aleph_t = \frac{\phi_V V_t + (1-\phi_V)\, m_t}{N_t}$$

3. m_t

$$m_t = q_t V_t$$

4. x_t

$$x_t = \frac{m_t}{N_t}$$

5. $k_t = K_t/A_t$

$$k_t = u_t \overline{k}_{t-1} \frac{1}{z_t}$$

6. $\overline{k}_t = \overline{K}_t/A_t$

$$\overline{k}_t = (1-\delta)\,\overline{k}_{t-1}\frac{1}{z_t} + \mu_t \left[1 - \frac{\phi_I}{2}\left(\frac{i_t}{i_{t-1}} z_t - g_I\right)^2\right] i_t$$

7. μ_t

$$\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \varepsilon_{\mu t}$$

8. ϵ_{bt}

$$\ln \epsilon_{bt} = \rho_b \ln \epsilon_{bt-1} + \varepsilon_{bt}$$

9. $\lambda_t = A_t \Lambda_t$

$$\lambda_t = \beta \epsilon_{bt} R_t E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \frac{1}{z_{t+1}}\right)$$

10. $c_t = C_t/A_t$

$$\lambda_t = \frac{z_t}{z_t c_t - h c_{t-1}} - \beta h E_t \left(\frac{1}{c_{t+1} z_{t+1} - h c_t}\right)$$

11. $\widetilde{r}_t^K = \frac{r_t^K}{P_t}$

$$(\phi_{u1} - \phi_{u2}) + \phi_{u2} u_t = \widetilde{r}_t^K$$

12. $i_t = I_t/A_t$

$$\begin{aligned} 1 = {} & \upsilon_t \mu_t \left[1 - \frac{\phi_I}{2}\left(\frac{i_t}{i_{t-1}} z_t - g_I\right)^2 - \phi_I \left(\frac{i_t}{i_{t-1}} z_t - g_I\right)\left(\frac{i_t}{i_{t-1}} z_t\right)\right] \\ & + \beta E_t \upsilon_{t+1} \mu_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{z_{t+1}} \phi_I \left(\frac{i_{t+1}}{i_t} z_{t+1} - g_I\right)\left(\frac{i_{t+1}}{i_t} z_{t+1}\right)^2 \end{aligned}$$

13. $\upsilon_t = \frac{\Upsilon_t}{\Lambda_t}$

$$\upsilon_t = \beta E_t \left\{\frac{\lambda_{t+1}}{\lambda_t} \frac{1}{z_{t+1}} \left[(1-\delta)\,\upsilon_{t+1} + \widetilde{r}_{t+1}^K u_{t+1} - \phi_{u1}\,(u_{t+1} - 1) - \frac{\phi_{u2}}{2}\,(u_{t+1} - 1)^2\right]\right\}$$

14. u_t

$$y_t = k_t^{\alpha} N_t^{1-\alpha}$$

15. $z_t = \frac{A_t}{A_{t-1}}$

$$\ln z_t = (1-\rho_z) \ln z + \rho_z \ln z_{t-1} + \varepsilon_{zt}$$

16. $\xi_t \equiv \frac{\Xi_t}{\Lambda_t}$

$$\widetilde{r}_t^K = \alpha \frac{y_t}{k_t} \xi_t$$

17. N_t

$$N_t = \chi N_{t-1} + q_t V_t$$

18. S_t

$$S_t = 1 - \chi N_{t-1}$$

19. U_t

$$U_t = 1 - N_t$$

20. $\Theta_t = \frac{V_t}{S_t}$

$$\Theta_t = \frac{V_t}{S_t}$$

21. q_t

$$q_t = \zeta_t \Theta_t^{-\sigma}$$

22. s_t

$$s_t = \zeta_t \Theta_t^{1-\sigma}$$

23. ζ_t

$$\ln \zeta_t = (1-\rho_\zeta) \ln \zeta + \rho_\zeta \ln \zeta_{t-1} + \varepsilon_{\zeta t}$$

24. V_t

$$\frac{\kappa \aleph_t^2 y_t}{m_t} = \xi_t (1-\alpha) \frac{y_t}{N_t} - \widetilde{w}_t - \frac{\phi_W}{2} \left(\frac{z_t \pi_t \widetilde{w}_t}{z \pi_{t-1}^{\varrho} \pi^{1-\varrho} \widetilde{w}_{t-1}} - 1 \right)^2 y_t + \frac{\kappa \aleph_t^2 y_t}{N_t} + \beta \chi \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa \aleph_{t+1}^2 y_{t+1}}{m_{t+1}}$$

25. θ_t

$$\ln \theta_t = (1-\rho_\theta) \ln \theta + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta t}$$

26. $\pi_t = \frac{P_t}{P_{t-1}}$

$$0 = (1-\theta_t) + \theta_t \xi_t - \phi_P \left(\frac{\pi_t}{\pi_{t-1}^{\varsigma} \pi^{1-\varsigma}} - 1 \right) \left(\frac{\pi_t}{\pi_{t-1}^{\varsigma} \pi^{1-\varsigma}} \right)$$
$$+ \beta \phi_P E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_{t+1}}{\pi_t^{\varsigma} \pi^{1-\varsigma}} - 1 \right) \left(\frac{\pi_{t+1}}{\pi_t^{\varsigma} \pi^{1-\varsigma}} \right) \frac{y_{t+1}}{y_t} \right]$$

27. $\widetilde{\mathbf{b}}_t = \widetilde{b}_t / A_t$

$$\widetilde{\mathbf{b}}_t = \widetilde{\mathbf{b}} = \tau \widetilde{w}$$

28. $\widetilde{w}_t = \widetilde{W}_t / A_t$

$$\widetilde{w}_t = \left(\frac{\beth_t}{1+\beth_t} \right) \left[\xi_t (1-\alpha) \frac{y_t}{N_t} - \frac{\phi_W}{2} \left(\frac{z_t \pi_t \widetilde{w}_t}{z \pi_{t-1}^{\varrho} \pi^{1-\varrho} \widetilde{w}_{t-1}} - 1 \right)^2 y_t + \frac{\kappa \aleph_t^2 y_t}{N_t} + \beta \chi E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \frac{\kappa \aleph_{t+1}^2 y_{t+1}}{m_{t+1}} \right]$$
$$+ \frac{1}{1+\beth_t} \left[\widetilde{\mathbf{b}} - \beta \chi E_t \beth_{t+1} (1 - s_{t+1}) \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa \aleph_{t+1}^2 y_{t+1}}{m_{t+1}} \right]$$

29. $\beth_t$

$$\beth_t = \left[\left(\frac{\eta_t}{1-\eta_t} \right) \frac{\widetilde{w}_t}{y_t} \right] / \left\{ \begin{array}{c} \frac{\widetilde{w}_t}{y_t} + \phi_W \left(\frac{z_t \pi_t \widetilde{w}_t}{z \pi_{t-1}^{\varrho} \pi^{1-\varrho} \widetilde{w}_{t-1}} - 1 \right) \left(\frac{z_t \pi_t \widetilde{w}_t}{z \pi_{t-1}^{\varrho} \pi^{1-\varrho} \widetilde{w}_{t-1}} \right) \\ -\beta \chi \phi_W E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\frac{z_{t+1} \pi_{t+1} \widetilde{w}_{t+1}}{z \pi_t^{\varrho} \pi^{1-\varrho} \widetilde{w}_t} - 1 \right) \left(\frac{z_{t+1} \pi_{t+1} \widetilde{w}_{t+1}}{z \pi_t^{\varrho} \pi^{1-\varrho} \widetilde{w}_t} \right) \frac{y_{t+1}}{y_t} \right] \end{array} \right\}$$

30. η_t

$$\ln \eta_t = (1-\rho_\eta) \ln \eta + \rho_\eta \ln \eta_{t-1} + \varepsilon_{\eta t}$$

31. R_t

$$\ln \left(\frac{R_t}{R} \right) = \rho_r \ln \left(\frac{R_{t-1}}{R} \right) + (1-\rho_r) \rho_\pi \ln \left[\frac{(\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3})^{1/4}}{\pi} \right]$$
$$+ (1-\rho_r) \rho_y \ln \left[\frac{(gy_t gy_{t-1} gy_{t-2} gy_{t-3})^{1/4}}{z} \right] + \ln \epsilon_{mpt}$$

32. ϵ_{mpt}

$$\ln \epsilon_{mpt} = \rho_{mp} \ln \epsilon_{mpt-1} + \varepsilon_{mpt}$$

33. $g_t = G_t / A_t$

$$g_t = \left(1 - \frac{1}{\epsilon_{gt}} \right) y_t$$

34. ϵ_{gt}

$$\ln \epsilon_{gt} = (1-\rho_g) \ln \epsilon_g + \rho_g \ln \epsilon_{gt-1} + \varepsilon_{gt}$$

35. $gy_t = Y_t / Y_{t-1}$

$$gy_t = \frac{y_t}{y_{t-1}} z_t$$

36. $gc_t = C_t / C_{t-1}$

$$gc_t = \left(\frac{c_t}{c_{t-1}} \right) z_t$$

37. $gi_t = I_t / I_{t-1}$

$$gi_t = \left(\frac{i_t}{i_{t-1}} \right) z_t$$

38. $gw_t = \widetilde{W}_t / \widetilde{W}_{t-1}$

$$gw_t = \left(\frac{\widetilde{w}_t}{\widetilde{w}_{t-1}} \right) z_t$$

2.9 The steady state of the transformed economy

In the absence of shocks, the economy converges to a steady-state growth path in which all stationary variables are constant: for all t, $y_t = y$, $k_t = k$, $\overline{k}_t = \overline{k}$, $u_t = u = 1$, $\lambda_t = \lambda$, $\upsilon_t = \upsilon$, $\xi_t = \xi$, $c_t = c$, $\widetilde{r}^K_t = \widetilde{r}^K$, $i_t = i$, $g_t = g$, $N_t = N$, $S_t = S$, $U_t = U$, $V_t = V$, $\aleph_t = \aleph$, $m_t = m$, $x_t = x$, $q_t = q$, $s_t = s$, $Ŋ_t = Ŋ$, $\widetilde{w}_t = \widetilde{w}$, $\widetilde{\mathbf{b}} = \widetilde{\mathbf{b}}$, $R_t = R$, $\pi_t = \pi$, $\mu_t = \mu = 1$, $\epsilon_{bt} = \epsilon_b = 1$, $z_t = z$, $\zeta_t = \zeta$, $\theta_t = \theta$, $\eta_t = \eta$, $\epsilon_{gt} = \epsilon_g$, $\epsilon_{rt} = \epsilon_r$, $g_{yt} = g_{ct} = g_{It} = g_{At} = z$. Notice that the steady-state values μ, u and ϵ_b are normalized to 1.

1. μ_t

$$\mu = 1$$

2. ϵ_{bt}

$$\epsilon_b = 1$$

3. u_t

$$u = 1$$

4. z_t

z : calibrated at sample mean of gross quarterly growth rate of per-capita real output

5. gy_t

$$gy = z$$

6. gc_t

$$gc = z$$

7. gi_t

$$gi = z$$

8. gw_t

$$gw = z$$

9. g_t

$$\frac{g}{y} = \left(1 - \frac{1}{\epsilon_g}\right) := 0.20 \text{ (calibrated)}$$

10. ϵ_{gt}

$$\frac{1}{\epsilon_g} - \frac{\kappa}{2}\aleph_t^2 = \frac{c+i}{y}$$

11. $\aleph_t$

$$\aleph = \frac{\phi_V V + (1-\phi_V)\, m}{N}$$

12. m_t

$$m = qV$$

13. x_t

$$x = \frac{m}{N}$$

14. k_t

$$zk = \overline{k}$$

15. $\overline{k}_t$

$$(z - 1 + \delta)\,\overline{k} = zi$$

16. λ_t

$$\beta = \frac{\pi z}{R}$$

17. c_t

$$\lambda c = \frac{z - \beta h}{z - h}$$

18. $\widetilde{r}_t^K$

$$\phi_{u1} = \widetilde{r}^K$$

19. i_t

$$1 = \upsilon$$

20. υ_t

$$\frac{z}{\beta} = 1 - \delta + \widetilde{r}^K$$

21. N_t

$$\rho N = qV \quad \text{where} \quad \rho \equiv 1 - \chi$$

22. S_t

$$S = 1 - \chi N$$

23. U_t

U : calibrated at sample mean of unemployment rate

24. $\Theta_t = \frac{V_t}{S_t}$

$$\Theta = \frac{V}{S}$$

25. q_t

$q = \zeta\Theta^{-\sigma} := 0.7$ (calibrated. just a normalization)

26. s_t

$$s = \zeta\Theta^{1-\sigma}$$

27. ζ_t

ζ : backed out from the steady state condition $\zeta = q\left(\frac{V}{S}\right)^{\sigma}$

28. y_t

$$y = k^{\alpha}N^{1-\alpha}$$

29. ξ_t

$$\widetilde{r}^K = \alpha\frac{y}{k}\xi$$

30. V_t

$$\frac{(1-\beta)\,\chi}{\rho}\kappa\aleph^2 = \xi\,(1-\alpha) - \frac{\widetilde{w}N}{y}$$

31. θ_t

$$\xi = \frac{\theta - 1}{\theta}$$

32. π_t

π : calibrated at sample mean of gross quarterly growth rate GDP deflator

33. $\widetilde{\mathbf{b}}_t$

$$\widetilde{\mathbf{b}} = \tau \widetilde{w}$$

34. $\widetilde{w}_t$

$$\widetilde{w} = \eta \left[(1-\alpha)\,\xi \frac{y}{N} + \left(\frac{1}{N} + \chi\beta \frac{s}{m} \right) \kappa \aleph^2 y \right] + (1-\eta)\,\widetilde{\mathbf{b}}$$
$$\Leftrightarrow \frac{1-(1-\eta)\,\tau}{\eta} \frac{\widetilde{w}N}{y} = \xi\,(1-\alpha) + \left(1 + \beta\chi \frac{s}{\rho} \right) \kappa \aleph^2$$

35. Ŋ_t

$$\text{Ŋ} = \frac{\eta}{1-\eta}$$

36. η_t

η : backed out from steady state conditions (see Table 4 below)

37. ϵ_{rt}

$$\epsilon_{mp} = 1$$

38. R_t

R : calibrated at sample mean of gross quarterly nominal rate of interest

2.10 The loglinear model with rescaled shocks

Two disturbances are normalized prior to estimation: the price-markup shock $\widehat{\theta}_t$ and the wage bargaining shock $\widehat{\eta}_t$. Rescalling these two shocks only affects the New Keynesian Phillips Curve and the equation for the evolution of the effective bargaining power.

$$\widehat{\theta}_t^* = \left[\frac{1}{(1+\beta\varsigma)\,\phi_P} \right] \widehat{\theta}_t$$
$$\widehat{\theta}_t^* = \rho_{\theta^*} \widehat{\theta}_{t-1}^* - \varepsilon_{\theta^* t}$$
$$\rho_{\theta^*} = \rho_\theta$$
$$\varepsilon_{\theta^* t} \sim i.i.d.N\left(0, \sigma_{\theta^*}^2\right)$$
$$\sigma_{\theta^*} = \left[\frac{1}{(1+\beta\varsigma)\,\phi_P} \right] \sigma_\theta$$

$$\begin{aligned}
\widehat{\eta}_t^* &= \left(\frac{1}{1-\eta}\right)\widehat{\eta}_t \\
\widehat{\eta}_t^* &= \rho_{\eta^*}\widehat{\eta}_{t-1}^* + \varepsilon_{\eta^* t} \\
\rho_{\eta^*} &= \rho_\eta \\
\varepsilon_{\eta^* t} &\sim i.i.d.N\left(0, \sigma_{\eta^*}^2\right) \\
\sigma_{\eta^*} &= \left(\frac{1}{1-\eta}\right)\sigma_\eta
\end{aligned}$$

1. y_t

$$\frac{c+i}{y}\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t + \phi_{u1}\frac{k}{y}\widehat{u}_t + \frac{1}{\epsilon_g}\widehat{\epsilon}_{gt} + \kappa\aleph^2\widehat{\aleph}_t$$

2. k_t

$$\widehat{k}_t = \widehat{u}_t + \widehat{\overline{k}}_{t-1} - \widehat{z}_t$$

3. $\overline{k}_t$

$$z\widehat{\overline{k}}_t = (1-\delta)\widehat{\overline{k}}_{t-1} - (1-\delta)\widehat{z}_t + (z-1+\delta)\widehat{\mu}_t + (z-1+\delta)\widehat{i}_t$$

4. λ_t

$$\widehat{\lambda}_t = \widehat{\epsilon}_{bt} + \widehat{R}_t + \widehat{\lambda}_{t+1} - \widehat{\pi}_{t+1} - \widehat{z}_{t+1}$$

5. c_t

$$\begin{aligned}
\widehat{\lambda}_t = {} & \frac{\beta hz}{(z-\beta h)(z-h)}\widehat{c}_{t+1} - \frac{z^2+\beta h^2}{(z-\beta h)(z-h)}\widehat{c}_t + \frac{hz}{(z-\beta h)(z-h)}\widehat{c}_{t-1} \\
& + \frac{\beta hz}{(z-\beta h)(z-h)}\widehat{z}_{t+1} - \frac{hz}{(z-\beta h)(z-h)}\widehat{z}_t
\end{aligned}$$

6. $\widetilde{r}_t^K$

$$\widehat{\widetilde{r}}_t^K = \left(\frac{\phi_{u2}}{\phi_{u1}}\right)\widehat{u}_t$$

7. i_t

$$\widehat{\upsilon}_t = \left[(1+\beta)\left(\phi_I z^2\right)\right]\widehat{i}_t + \left(\phi_I z^2\right)\widehat{z}_t - \left(\phi_I z^2\right)\widehat{i}_{t-1} - \widehat{\mu}_t - \left(\beta\phi_I z^2\right)\widehat{i}_{t+1} - \left(\beta\phi_I z^2\right)\widehat{z}_{t+1}$$

8. υ_t

$$\widehat{\upsilon}_t = \widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \widehat{z}_{t+1} + \left[(1-\delta)\beta z^{-1}\right]\widehat{\upsilon}_{t+1} + \left(\beta z^{-1}\widetilde{r}^K\right)\widehat{\widetilde{r}}_{t+1}^K$$

9. u_t

$$\widehat{y}_t = \alpha\widehat{k}_t + (1-\alpha)\widehat{N}_t$$

10. ξ_t

$$\widehat{\xi}_t = \widehat{\widetilde{r}}_t^K - \widehat{y}_t + \widehat{k}_t$$

11. π_t

$$\widehat{\pi}_t = \left(\frac{\varsigma}{1+\beta\varsigma}\right)\widehat{\pi}_{t-1} + \left(\frac{\beta}{1+\beta\varsigma}\right)\widehat{\pi}_{t+1} + \left(\frac{1}{1+\beta\varsigma}\right)\left(\frac{\theta-1}{\phi_P}\right)\widehat{\xi}_t - \widehat{\theta}_t^*$$

12. N_t

$$\widehat{N}_t = \chi\widehat{N}_{t-1} + (1-\chi)\,\widehat{q}_t + (1-\chi)\,\widehat{V}_t$$

13. U_t

$$\widehat{U}_t = -\left(\frac{N}{1-N}\right)\widehat{N}_t$$

14. Θ_t

$$\widehat{\Theta}_t = \widehat{V}_t + \left(\frac{\chi N}{S}\right)\widehat{N}_{t-1}$$

15. q_t

$$\widehat{q}_t = \widehat{\zeta}_t - \sigma\widehat{\Theta}_t$$

16. s_t

$$\widehat{s}_t = \widehat{\zeta}_t + (1-\sigma)\,\widehat{\Theta}_t$$

17. $\aleph_t$:

$$\widehat{\aleph}_t = \left[\frac{\phi_V V}{\phi_V V + (1-\phi_V)\,m}\right]\widehat{V}_t + \left[\frac{(1-\phi_V)\,m}{\phi_V V + (1-\phi_V)\,m}\right]\widehat{m}_t - \widehat{N}_t$$

18. m_t :

$$\widehat{m}_t = \widehat{q}_t + \widehat{V}_t$$

19. x_t :

$$\widehat{x}_t = \widehat{m}_t - \widehat{N}_t$$

20. V_t :

$$\begin{aligned}2\frac{\chi}{\rho}\kappa\aleph^2\widehat{\aleph}_t = {} & \frac{\kappa\aleph^2}{\rho}\widehat{x}_t + (1-\alpha)\,\xi\widehat{\xi}_t + \left(\frac{\widetilde{w}N}{y} - \frac{\beta\chi}{\rho}\kappa\aleph^2\right)\left(\widehat{y}_t - \widehat{N}_t\right) \\ & - \frac{\widetilde{w}N}{y}\widehat{\widetilde{w}}_t + \frac{\beta\chi}{\rho}\kappa\aleph^2\left(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{y}_{t+1} - \widehat{N}_{t+1} + 2\widehat{\aleph}_{t+1} - \widehat{x}_{t+1}\right)\end{aligned}$$

21. $\widetilde{w}_t$:

$$
\begin{aligned}
\frac{1}{\eta}\frac{\widetilde{w}N}{y}\widehat{\widetilde{w}}_t = {} & (1-\alpha)\,\xi\widehat{\xi}_t + \left[(1-\alpha)\,\xi + \kappa\aleph^2\right]\left(\widehat{y}_t - \widehat{N}_t\right) + 2\kappa\aleph^2\widehat{\aleph}_t \\
& + \beta\chi\frac{s}{\rho}\kappa\aleph^2\left(\widehat{s}_{t+1} + \widehat{\lambda}_{t+1} - \widehat{\lambda}_t + 2\widehat{\aleph}_{t+1} + \widehat{y}_{t+1} - \widehat{m}_{t+1}\right) \\
& - \left[\frac{\widetilde{w}N}{y} - (1-\alpha)\,\xi - \left(1 + \frac{\beta\chi}{\rho}\right)\kappa\aleph^2\right]\widehat{\text{Ŋ}}_t - \beta\chi\frac{(1-s)}{\rho}\kappa\aleph^2\widehat{\text{Ŋ}}_{t+1}
\end{aligned}
$$

22. Ŋ_t

$$
\begin{aligned}
\widehat{\text{Ŋ}}_t = {} & \widehat{\eta}_t^* + \left(\beta\chi\phi_W\frac{y}{\widetilde{w}}\right)\widehat{z}_{t+1} + \left(\beta\chi\phi_W\frac{y}{\widetilde{w}}\right)\widehat{\pi}_{t+1} + \left(\beta\chi\phi_W\frac{y}{\widetilde{w}}\right)\widehat{\widetilde{w}}_{t+1} - \left[\left(\phi_W\frac{y}{\widetilde{w}}\right)(1+\beta\chi)\right]\widehat{\widetilde{w}}_t \\
& - \left[\left(\phi_W\frac{y}{\widetilde{w}}\right)(1+\beta\chi\varrho)\right]\widehat{\pi}_t - \left(\phi_W\frac{y}{\widetilde{w}}\right)\widehat{z}_t + \left(\phi_W\frac{y}{\widetilde{w}}\right)\widehat{\widetilde{w}}_{t-1} + \left(\phi_W\frac{y}{\widetilde{w}}\right)\varrho\widehat{\pi}_{t-1}
\end{aligned}
$$

23. R_t

$$
\begin{aligned}
\widehat{R}_t = {} & \rho_r\widehat{R}_{t-1} + \frac{(1-\rho_r)\,\rho_\pi}{4}\left(\widehat{\pi}_t + \widehat{\pi}_{t-1} + \widehat{\pi}_{t-2} + \widehat{\pi}_{t-3}\right) \\
& + \frac{(1-\rho_r)\,\rho_y}{4}\left(\widehat{gy}_t + \widehat{gy}_{t-1} + \widehat{gy}_{t-2} + \widehat{gy}_{t-3}\right) + \widehat{\epsilon}_{mpt}
\end{aligned}
$$

24. $gy_t = Y_t/Y_{t-1}$

$$
\widehat{gy}_t = \widehat{y}_t - \widehat{y}_{t-1} + \widehat{z}_t
$$

25. $gc_t = C_t/C_{t-1}$

$$
\widehat{gc}_t = \widehat{c}_t - \widehat{c}_{t-1} + \widehat{z}_t
$$

26. $gi_t = I_t/I_{t-1}$

$$
\widehat{gi}_t = \widehat{i}_t - \widehat{i}_{t-1} + \widehat{z}_t
$$

27. $gw_t = \widetilde{W}_t/\widetilde{W}_{t-1}$

$$
\widehat{gw}_t = \widehat{\widetilde{w}}_t - \widehat{\widetilde{w}}_{t-1} + \widehat{z}_t
$$

28. μ_t

$$
\widehat{\mu}_t = \rho_\mu\widehat{\mu}_{t-1} + \varepsilon_{\mu t}
$$

29. ϵ_{bt}

$$
\widehat{\epsilon}_{bt} = \rho_b\widehat{\epsilon}_{bt-1} + \varepsilon_{bt}
$$

30. z_t

$$
\widehat{z}_t = \rho_z\widehat{z}_{t-1} + \varepsilon_{zt}
$$

31. ζ_t

$$\widehat{\zeta}_t = \rho_\zeta \widehat{\zeta}_{t-1} + \varepsilon_{\zeta t}$$

32. θ_t

$$\widehat{\theta}_t = \rho_\theta \widehat{\theta}_{t-1} + \varepsilon_{\theta t}$$

33. η_t

$$\widehat{\eta}_t = \rho_\eta \widehat{\eta}_{t-1} + \varepsilon_{\eta t}$$

34. ϵ_{gt}

$$\widehat{\epsilon}_{gt} = \rho_g \widehat{\epsilon}_{gt-1} + \varepsilon_{gt}$$

35. ϵ_{mt}

$$\widehat{\epsilon}_{mpt} = \rho_{mp} \widehat{\epsilon}_{mpt-1} + \varepsilon_{mpt}$$

2.11 Natural equilibrium: no nominal rigidities, no markup shocks and no bargaining power shocks

We compute the natural equilibrium by setting equal to zero the two parameters ϕ_P and ϕ_W that governs the degree of nominal rigidities in prices and wages respectively and by turning off the price-markup shock θ_t and the bargaining-power shock η_t.

1. c_t^p

$$\frac{c+i}{y}\widehat{y}_t^p = \frac{c}{y}\widehat{c}_t^p + \frac{i}{y}\widehat{i}_t^p + \phi_{u1}\frac{k}{y}\widehat{u}_t^p + \kappa \aleph^2 \widehat{\aleph}_t^p + \frac{1}{\epsilon_g}\widehat{\epsilon}_{gt}$$

2. k_t^p

$$\widehat{k}_t^p = \widehat{u}_t^p + \widehat{\overline{k}}_{t-1}^p - \widehat{z}_t$$

3. $\overline{k}_t^p$

$$z\widehat{\overline{k}}_t^p = (1-\delta)\,\widehat{\overline{k}}_{t-1}^p - (1-\delta)\,\widehat{z}_t + (z-1+\delta)\,\widehat{\mu}_t + (z-1+\delta)\,\widehat{i}_t^p$$

4. $\widetilde{R}_t^p$

$$\widehat{\lambda}_t^p = \widehat{\epsilon}_{bt} + \widehat{\widetilde{R}}_t^p + \widehat{\lambda}_{t+1}^p - \widehat{z}_{t+1}$$

5. λ_t^p

$$\widehat{\lambda}_t^p = \frac{\beta hz}{(z-\beta h)(z-h)}\widehat{c}_{t+1}^p - \frac{z^2+\beta h^2}{(z-\beta h)(z-h)}\widehat{c}_t^p + \frac{hz}{(z-\beta h)(z-h)}\widehat{c}_{t-1}^p + \frac{\beta hz}{(z-\beta h)(z-h)}\widehat{z}_{t+1} - \frac{hz}{(z-\beta h)(z-h)}\widehat{z}_t$$

6. u_t^p

$$\widehat{\widetilde{r}}_t^{K,p} = \left(\frac{\phi_{u2}}{\phi_{u1}}\right)\widehat{u}_t^p$$

7. i_t^p

$$\widehat{\upsilon}_t^p = \left[(1+\beta)\left(\phi_I z^2\right)\right]\widehat{i}_t^p + \left(\phi_I z^2\right)\widehat{z}_t - \left(\phi_I z^2\right)\widehat{i}_{t-1}^p - \widehat{\mu}_t - \left(\beta\phi_I z^2\right)\widehat{i}_{t+1}^p - \left(\beta\phi_I z^2\right)\widehat{z}_{t+1}$$

8. υ_t^p

$$\widehat{\upsilon}_t^p = \widehat{\lambda}_{t+1}^p - \widehat{\lambda}_t^p - \widehat{z}_{t+1} + \left[(1-\delta)\beta z^{-1}\right]\widehat{\upsilon}_{t+1}^p + \left(\beta z^{-1}\widetilde{r}^K\right)\widehat{\widetilde{r}}_{t+1}^{K,p}$$

9. y_t^p

$$\widehat{y}_t^p = \alpha\widehat{k}_t^p + (1-\alpha)\widehat{N}_t^p$$

10. $\widetilde{r}_t^{K,p}$

$$\widehat{\widetilde{r}}_t^{K,p} = \widehat{y}_t^p - \widehat{k}_t^p$$

11. N_t^p

$$\widehat{N}_t^p = \chi\widehat{N}_{t-1}^p + (1-\chi)\widehat{q}_t^p + (1-\chi)\widehat{V}_t^p$$

12. U_t^p

$$\widehat{U}_t^p = -\left(\frac{N}{1-N}\right)\widehat{N}_t^p$$

13. Θ_t^p

$$\widehat{\Theta}_t^p = \widehat{V}_t^p + \left(\frac{\chi N}{S}\right)\widehat{N}_{t-1}^p$$

14. q_t^p

$$\widehat{q}_t^p = \widehat{\zeta}_t - \sigma\widehat{\Theta}_t^p$$

15. s_t^p

$$\widehat{s}_t^p = \widehat{\zeta}_t + (1-\sigma)\widehat{\Theta}_t^p$$

16. $\aleph_t^p$

$$\widehat{\aleph}_t^p = \left[\frac{\phi_V V}{\phi_V V + (1-\phi_V)\, m}\right] \widehat{V}_t^p + \left[\frac{(1-\phi_V)\, m}{\phi_V V + (1-\phi_V)\, m}\right] \widehat{m}_t^p - \widehat{N}_t^p$$

17. m_t^p

$$\widehat{m}_t^p = \widehat{q}_t^p + \widehat{V}_t^p$$

18. x_t^p

$$\widehat{x}_t^p = \widehat{m}_t^p - \widehat{N}_t^p$$

19. V_t^p

$$\begin{aligned} 2\frac{\chi}{\rho}\kappa\aleph^2\widehat{\aleph}_t^p &= \frac{\kappa\aleph^2}{\rho}\widehat{x}_t^p + \left(\frac{\widetilde{w}N}{y} - \frac{\beta\chi}{\rho}\kappa\aleph^2\right)\left(\widehat{y}_t^p - \widehat{N}_t^p\right) - \frac{\widetilde{w}N}{y}\widehat{\widetilde{w}}_t^p \\ &+ \frac{\beta\chi}{\rho}\kappa\aleph^2\left(\widehat{\lambda}_{t+1}^p - \widehat{\lambda}_t^p + \widehat{y}_{t+1}^p - \widehat{N}_{t+1}^p + 2\widehat{\aleph}_{t+1}^p - \widehat{x}_{t+1}^p\right) \end{aligned}$$

20. $\widetilde{w}_t^p$

$$\begin{aligned} \frac{1}{\eta}\frac{\widetilde{w}N}{y}\widehat{\widetilde{w}}_t^p &= \left[(1-\alpha)\,\xi + \kappa\aleph^2\right]\left(\widehat{y}_t^p - \widehat{N}_t^p\right) + 2\kappa\aleph^2\widehat{\aleph}_t^p \\ &+ \beta\chi\frac{s}{\rho}\kappa\aleph^2\left(\widehat{s}_{t+1}^p + \widehat{\lambda}_{t+1}^p - \widehat{\lambda}_t^p + 2\widehat{\aleph}_{t+1}^p + \widehat{y}_{t+1}^p - \widehat{m}_{t+1}^p\right) \end{aligned}$$

3 Empirical analysis

3.1 Data

The model includes as many shocks as observables. The estimation uses quarterly data on eight key macro variables. X_t is the vector of observables at time t. X_t is expressed in logarithmic deviations from sample mean. X_t contains eight variables: the quarterly growth rate of output, the quarterly growth rate of consumption, the quarterly growth rate of investment, the quarterly growth rate of real wages, the vacancy rate, the unemployment rate, the quarterly inflation rate and the quarterly gross nominal interest rate

$$X_t = \begin{bmatrix} \ln(Y_t) - \ln(Y_{t-1}) - \ln(g_y) \\ \ln(C_t) - \ln(C_{t-1}) - \ln(g_c) \\ \ln(I_t) - \ln(I_{t-1}) - \ln(g_i) \\ \ln(W_t) - \ln(W_{t-1}) - \ln(g_w) \\ \ln(V_t) - \ln(V) \\ \ln(U_t) - \ln(U) \\ \ln(P_t) - \ln(P_{t-1}) - \ln(g_p) \\ \ln(R_t) - \ln(R) \end{bmatrix}.$$

Y_t is the level of real GDP per capita, C_t is the level of real consumption per capita, I_t is the level of real investment per-capita, W_t is the real wage, U_t is the unemployment rate, V_t is the vacancy rate, P_t is the level of the GDP deflator and R_t is the gross effective federal funds rate, expressed on a quarterly basis.

Except for the vacancy rate, we construct all other series using data downloadable from the FREDII database. In particular, we measure nominal consumption using data on nominal personal consumption expenditures of nondurables and services. Nominal investment corresponds to the sum of personal consumption expenditures of durables and gross private domestic investment. Nominal output is measured by nominal GDP. Per capita real GDP, consumption, and investment are obtained by dividing the nominal series by the GDP deflator and population. Real wages correspond to nominal compensation per hour in the nonfarm business sector, divided by the GDP deflator. Consistently with the model, we measure population by the labor force. The unemployment rate is the number of unemployed persons (16 years of age and older) divided by the labor force. Inflation is the first difference of the log of the GDP deflator. The nominal interest rate is measured by the effective federal funds rate.

We measure vacancies using the series constructed by Barnichon (2010). We then construct the vacancy rate as the ratio of vacancies to the sum of vacancies and the number of employed people (cf. Justiniano and Michelacci, 2011). Following the arguments in Shimer (2005), we detrend the vacancy rate using the HP filter with a smoothing weight equal to 10^6 to remove the secular trend in the series (cf. also Justiniano and Michelacci 2011 and Davis, Faberman and Haltiwanger 2013).

3.2 Calibrated parameters

We calibrate 13 parameters. The steady-state values of output growth, inflation, the interest rate and the unemployment rate are set equal to their respective sample average over the period 1957Q1-2008Q3 in the baseline estimation (or, in the sensitivity analysis: 1957Q1-2013Q2 and 1985Q1-2008Q3). The value for the elasticity of the matching function with respect to unemployment is based on the recent estimates obtained by Barnichon and Figura (2014), Justiniano and Michelacci (2011), Lubik (2013), Shimer (2005) and Sedlacek (2014). The calibration of the job destruction rate is based on Yashiv (2006). The calibration of the replacement rate is a conservative value advocated by Shimer (2005). These choices avoid indeterminacy issues that are widespread in this kind of model, as shown by Kurozumi and Van Zandweghe (2010) among others. In preliminary estimation rounds, the estimate of the parameter governing the degree of indexation to past inflation was systematically driven towards zero. This phenomenon is consistent with the findings reported by Ireland (2007). It is also in line with the microevidence on price-setting behavior. Hence we calibrate that parameter to 0.01. The quarterly depreciation rate is set equal to 0.025. The capital share of output is calibrated at 0.33. The elasticity of substitution between intermediate goods is set equal to 6, implying a steady-state markup of 20 percent as in Rotemberg and Woodford (1995). The vacancy-filling rate is set equal to 0.70, which is just a normalization. The steady-state government spending/output ratio is set equal to 0.20. Table A1 reports the calibrated parameters.

3.3 Bayesian estimation

Our priors are standard (Smets and Wouters 2007; Gertler, Sala and Trigari 2008). We normalize the price-markup shock and the wage-markup shock, so that these enter with a unit coefficient in the model's equations. Such procedure facilitates the identification of the standard deviations of these two disturbances. We use the random walk Metropolis-Hasting algorithm to generate 500,000 draws from the posterior distribution. The algorithm is tuned to achieve an acceptance ratio between 25 and 30 percent. We discard the first 250,000 draws. Tables A2 and A3 summarize the priors and posteriors.

Table A1: Calibrated parameters

		$57Q1-08Q3$	$57Q1-13Q2$	$85Q1-08Q3$
Capital depreciation rate	δ	0.025	0.025	0.025
Capital share	α	0.33	0.33	0.33
Elasticity of substitution btw goods	θ	6	6	6
Backward-looking price setting	ς	0.01	0.01	0.01
Replacement rate	τ	0.4	0.4	0.4
Job destruction rate	ρ	0.085	0.085	0.085
Elasticity of matches to unemp.	σ	0.65	0.65	0.65
Probability to fill a vacancy within a quarter	q	0.70	0.70	0.70
Exogenous spending/output ratio	g/y	0.20	0.20	0.20
Unemployment rate	U	0.0578	0.0602	0.0564
Quarterly gross growth rate	z	1.0039	1.0038	1.0043
Quarterly gross inflation rate	π	1.0088	1.0083	1.0061
Quarterly gross nominal interest rate	R	1.0139	1.0128	1.0125

Table A2: Priors and Posteriors of Structural Parameters

		Priors	Posteriors 5%	Median	95%
Weight of pre-match hiring cost	ϕ_V	Beta (0.5,0.2)	0.20	0.35	0.52
Hiring cost/output ratio	$1000\frac{\kappa}{2}\aleph^2$	IGamma (5,1)	2.17	2.46	2.79
Habit in consump.	h	Beta (0.7,0.1)	0.59	0.65	0.69
Invest. adj. cost	ϕ_I	IGamma (5,1)	2.60	3.10	3.85
Capital ut. cost	ϕ_{u2}	IGamma (0.5,0.1)	0.47	0.63	0.88
Price adjust. cost	ϕ_P	IGamma (60,10)	52.40	60.50	69.30
Wage adjust. cost	ϕ_W	IGamma (150,25)	111.50	135.90	171.60
Wage indexation	ϱ	Beta (0.5,0.2)	0.87	0.94	0.98
Interest smoothing	ρ_r	Beta (0.7,0.1)	0.25	0.34	0.43
Resp. to inflation	ρ_π	IGamma (1.5,0.1)	1.73	1.94	2.21
Resp. to growth	ρ_y	IGamma (0.5,0.1)	0.39	0.48	0.58

Table A3: Priors and Posteriors of Shock Parameters

		Priors	Posteriors 5%	Median	95%
Technology growth	ρ_z	Beta (0.3,0.1)	0.22	0.29	0.36
	$100\sigma_z$	IGamma (0.1,3)	1.18	1.27	1.37
Monetary policy	ρ_{mp}	Beta (0.5,0.2)	0.69	0.79	0.86
	$100\sigma_{mp}$	IGamma (0.1,3)	0.19	0.21	0.23
Investment	ρ_μ	Beta (0.5,0.2)	0.72	0.77	0.82
	$100\sigma_\mu$	IGamma (0.1,3)	5.28	6.17	7.43
Risk premium	ρ_b	Beta (0.5,0.2)	0.58	0.67	0.75
	$100\sigma_b$	IGamma (0.1,3)	0.58	0.82	1.12
Matching efficiency	ρ_ζ	Beta (0.5,0.2)	0.88	0.92	0.96
	$100\sigma_\zeta$	IGamma (0.1,3)	2.16	2.31	2.50
Price markup (rescaled)	ρ_{θ^*}	Beta (0.5,0.2)	0.90	0.93	0.96
	$100\sigma_{\theta^*}$	IGamma (0.1,3)	0.09	0.10	0.11
Bargaining power (rescaled)	ρ_{η^*}	Beta (0.5,0.2)	0.09	0.18	0.27
	$100\sigma_{\eta^*}$	IGamma (0.1,3)	130.60	161.35	204.15
Government spending	ρ_g	Beta (0.7,0.1)	0.91	0.93	0.95
	$100\sigma_g$	IGamma (0.1,3)	0.44	0.47	0.51

Table A4: Parameters derived from steady-state conditions

Employment rate	N	$N = 1 - U$
Vacancy	V	$V = \frac{\rho N}{q}$
Matches	m	$m = qV$
Discount factor	β	$\beta = \frac{z\pi}{R}$
Job survival rate	χ	$\chi = 1 - \rho$
Mean of exogenous spending shock	ϵ_g	$\epsilon_g = \frac{1}{1-g/y}$
Real marginal cost	ξ	$\xi = \frac{\theta-1}{\theta}$
Quarterly net real rental rate of capital	$\widetilde{r}^K$	$\widetilde{r}^K = \frac{z}{\beta} - 1 + \delta$
Capital utilization cost first parameter	ϕ_{u1}	$\phi_{u1} = \widetilde{r}^K$
Capital/output ratio	k/y	$\frac{k}{y} = \frac{\alpha\xi}{\widetilde{r}^K}$
Investment/capital ratio	i/k	$\frac{i}{k} = z - 1 + \delta$
Investment/output ratio	i/y	$\frac{i}{y} = \frac{i}{k}\frac{k}{y}$
Consumption/output ratio	c/y	$\frac{c}{y} = \frac{1}{\epsilon_g} - \frac{\kappa}{2}\aleph^2 - \frac{i}{y}$
Pool of job seekers	S	$S = 1 - \chi N$
Matching function efficiency	ζ	$\zeta = q\left(\frac{V}{S}\right)^{\sigma}$
Job finding rate	s	$s = \zeta\left(\frac{V}{S}\right)^{1-\sigma}$
Employees' share of output	$\widetilde{w}N/y$	$\frac{\widetilde{w}N}{y} = \xi(1-\alpha) - \frac{(1-\beta)\chi}{\rho}2\left(\frac{\kappa}{2}\aleph^2\right)$
Bargaining power	η	$\eta = \frac{1-\tau}{\vartheta-\tau}$ where $\vartheta \equiv \frac{\xi(1-\alpha)+\left(1+\beta\chi\frac{s}{\rho}\right)2\left(\frac{\kappa}{2}\aleph^2\right)}{\frac{\widetilde{w}N}{y}}$
Effective bargaining power	Ŋ	$Ŋ = \frac{\eta}{1-\eta}$

4 Additional details on the propagation of shocks

In the main text we have concentrated our attention on the transmission mechanism for matching efficiency shocks. In this section we comment on the dynamics induced by the other shocks that are relatively standard. In Figure A0 we plot the responses of the actual and natural rates of unemployment to the six shocks that affect the natural rate. The natural rate of unemployment is defined as the counterfactual rate of unemployment that emerges in the presence of flexible prices and wages and thus corresponds to the concept of unemployment in Real Business Cycle models (Shimer 2005).

The responses of the actual rate are in line with the previous literature. Unemployment is countercyclical in response to all shocks. A partial exception is the case of the neutral technology shocks: on impact (and only on impact) an expansionary technology shock increases unemployment. This is a standard result in New Keynesian models due to the presence of nominal and real rigidities (cf. Galí 1999).

The natural rate does not react to monetary policy and risk premium shocks. It is well known that these shocks propagate only in the presence of nominal rigidities. The natural rate of unemployment reacts little also to technology and investment specific shocks. This result is also well known in the literature since Shimer (2005) and the following literature on the unemployment volatility puzzle. Notice that the nominal rigidities deliver a substantial propagation to these disturbances, thus meaning that the actual rate of unemployment is immune to the unemployment volatility puzzle. In contrast, the natural rate reacts little to technology and investment specific shocks, in line with the measures of natural rates obtained with statistical methods. In the absence of nominal rigidities, an exogenous increase in government spending leads to a very small rise in the unemployment rate. The negative wealth effect triggered by the fiscal impulse generates a fall in consumption and a rise in the real interest rate. Higher real interest rates provide firms with an incentive to raise the rate of capacity utilization, thereby substituting capital services for labor. This channel is amplified by the inelasticity of labor supply in the search and matching model.

As discussed in the main text, the matching efficiency shock has a larger effect on the natural rate than on the actual rate, unlike all the other shocks. This explains why the natural rate is driven almost exclusively by the matching efficiency shock.

In Figure A1 we plot simulated data on vacancies and unemployment conditional on each kind of disturbances. In each panel, the vertical and the horizontal axis correspond respectively to the vacancy rate and the unemployment rate, both expressed in percentage deviations from steady state. Each panel plots pseudo-data points simulated from the model calibrated at the posterior mode and drawing the i.i.d. innovations from normal distributions with mean zero and standard deviation set equal to the corresponding posterior mode estimate. We remark that only the mismatch shock generates a positive conditional correlation between unemployment and vacancies. This point is discussed in detail in the main text and is related to the presence of sticky prices and a pre-match component in total hiring costs. In the data unemployment and vacancies are strongly negatively correlated and, therefore, the other shocks have a better chance to explain aggregate dynamics. Nevertheless, mismatch shocks may play a role in periods when unemployment and vacancies move together.

In Figure A2 we plot the contribution of each shock to the Beveridge curve dynamics. The grey dots represents the dynamics induced by all the eight shocks together. The black dots show how each shock in isolation has moved the Beveridge curve over the period 2008:Q1-2013:Q2. Mismatch shocks have shifted the Beveridge curve to the right. Notice, however, that also other shocks explain part of the shift. All shocks are able to generate the loop typical of Beveridge curve dynamics in recent years and do not generate trajectories along a line. This point has been emphasized by Christiano, Eichenbaum and Trabandt (2014) in a recent paper. However, mismatch shocks are very important to match the shift to the right from a quantitative point of view and more so in recent years. Notice the large effects induced (in opposite directions) by risk premium shocks and fiscal shocks.

5 Sensitivity analysis

In this section we provide additional details on the sensitivity analysis that we conduct to investigate the robustness of our results. We modify the model along four dimensions: i) the sample period for estimation, ii) the calibration for the elasticity of the matching function to unemployment, iii) the calibration for the replacement rate, iv) the role of a time-varying separation rate. The results from these different experiments are summarized in Figure A45 where we plot a counterfactual historical decomposition for unemployment over the period 2008:Q1-2013Q2 in the absence of mismatch shocks. We compare these extensions to our baseline model (thin-solid line) and to the data (bold-solid line). We now describe each experiment in turn.

5.1 Sample period

In our baseline model the sample period used for estimation is 1957:Q1-2008:Q3. We now want to investigate the robustness of our results when we consider a longer sample (thus including the Great Recession) and a shorter sample (only the Great Moderation period).

In the first experiment we extend our sample period until 2013:Q2 to exploit the information on the recent shift of the Beveridge curve for estimation purposes. In Figures A3 to A8 we present our results for the extended sample. Matching efficiency is slightly more volatile (Figure A3) than in our baseline estimates but all in all these figures are almost identical to the ones for the baseline case.

In the second extension we focus on a shorter but more homogenous period as the Great Moderation (1985:Q1-2008:Q3). Our baseline sample period is long and may be subject to structural breaks. In contrast, the Great Moderation period is a period of relative stability that may be useful as a cross-check. In Figures A9 to A15 we present the results related to this experiment. Once again all our results on the role of matching efficiency shocks are confirmed. The only difference that we can identify with respect to the baseline case is that the relative importance of the other shocks change slightly, in particular for the risk premium shock. This point can be seen when comparing Figure A2 to Figure A12. However, even from a quantitative point of view these differences are minor. To sum up we conclude that the choice of the sample period for estimation purposes is largely inconsequential.

5.2 Alternative calibration of the matching function elasticity

A key parameter that affects directly the estimated series for matching efficiency shocks is the elasticity of the matching function to unemployment (σ).[4] In our baseline model we calibrate it at 0.65, a value in the middle of the range (0.55-0.75) found in a series of recent studies (Barnichon and Figura 2014; Justiniano and Michelacci 2011; Shimer 2005; Sedlacek 2014). These values are slightly higher than the ones advocated by Petrongolo and Pissarides (2001) and much higher than the value of 0.4 used by Blanchard and Diamond (1989). Given the importance of this parameter, we reestimate our model with σ equal to 0.55 (almost at the bottom of the Petrongolo and Pissarides' range) and with σ equal to 0.75 as in Justiniano and Michelacci (2011).

We plot the estimated series for matching efficiency shocks with σ calibrated at 0.55 in Figure A16. In our baseline case (Figure 3 in the main text) matching efficiency increases during some Recessions and declines in others. With σ equal to 0.55 matching efficiency becomes more countercyclical: it now often increases during Recessions with the clear exception of the Great Recession when we still identify a substantial decline,

[4] In our model this parmeter should be called elasticity of the matching function to searchers since the pool of searchers is not equivalent to unempoyment.

followed by a partial rebound and a new and even more pronounced decline. A different series for matching efficiency translates into a different estimate for the natural rate of unemployment given the prominent role of mismatch shocks in its dynamics. In Figure A17 we see that the low frequency dynamics of the natural rate are not affected. However, at high frequencies the correlation between the actual rate and the natural rate is now lower. The natural rate still increases during the Great Recession and keeps increasing in the aftermath as in our baseline case. Mismatch shocks are now less important to explain the shift in the Beveridge curve during the Great Recession (cf. Figure A19 and Figure 5 in the main text) but they are still crucial to explain why unemployment was so high in recent years. Mismatch shocks are still the dominant drivers of the natural rate as it can be seen in Figure A21. We conclude that our main results are confirmed but a low value of σ impacts the estimate of matching efficiency and the behavior of natural rate at high frequencies.

Not surprisingly we find the opposite results with a high value of σ. When σ is calibrated at 0.75, matching efficiency declines in almost all Recessions (thus becoming very procyclical, cf. Figure A22) and the natural rate of unemployment becomes more correlated with the actual rate at high frequencies (cf. Figure A23). Matching efficiency shocks are now crucial to explain the Beveridge curve dynamics both during the Great Recession and in its aftermath (cf. Figure A26).

5.3 Alternative calibration of the replacement rate

In our baseline model we use a conservative value for the replacement rate ($\tau = 0.4$) based on Shimer (2005). The replacement rate determines the value of the outside option for workers and is a contentious parameter in the literature. Higher values for the replacement rate, in combination with a low bargaining power for workers, have been used by Hagedorn and Manovskii (2008) among others to generate higher unemployment volatility in response to technology shocks in models with flexible prices and wages. Therefore, we may suspect that the dominant role of mismatch shocks in driving the natural rate in our baseline model may rely on a too limited propagation of the other real shocks. To investigate this issue, we set the replacement rate at 0.7 and we re-estimate the model over the same sample period. Figures A28 to A33 summarize the outcome of this experiment. All the main results described in our baseline model are confirmed under this alternative calibration. The only noticeable difference is that now mismatch shocks play a slightly lower role in the historical decomposition of the natural rate (Figure A33): now technology and investment-specific shocks propagate more under flexible prices and wages and thus play a larger role. Nevertheless, mismatch shocks are still the main drivers of the natural rate. We conclude that our results are robust to a different parameterization of the replacement rate.

5.4 Time-varying separation rate

In this last set of experiments we consider exogenous shocks to the separation rate. Hosios (1994) and Shimer (2005) among others have shown that shocks to the separation rate are also able to move unemployment and vacancies in the same direction.

Separation rate correlated with the state of the economy. In a first experiment we assume that the separation rate is negatively related to the state economy (i.e. the separation rate is low in good times) where the state of the economy is summarized by the technology and the investment-specific shocks, the two main drivers of business cycle fluctuations in our model. We assume the following specification:

$$\ln \rho_t = (1 - \rho_\rho) \ln \rho + \rho_\rho \ln \rho_{t-1} - \delta_z \varepsilon_{zt} - \delta_\mu \varepsilon_{\mu t} + \varepsilon_{\rho t}$$

where we impose in the estimation that δ_z and δ_μ have to be positive and $\varepsilon_{\rho t}$ represents an exogenous separation shock. The priors on the new parameters δ_z and δ_μ are Uniform. In this specification we extend the baseline model by including an additional shock (the separation shock) and by using an additional observable variable (the separation rate). More specifically, we use the transition probability from employment to unemployment corrected for margin error based on CPS data computed by Elsby, Hobijn and Sahin (2015). The sample period is 1968 Q1-2008Q3.

In Figures A34 to A38 we present graphically our results. In this version of our model matching efficiency increases during the Great Recession and declines only in the aftermath (Figure A34). The estimated series for the natural rate of unemployment is similar to the one derived in our baseline model (Figure A35). The increase in the separation rate during the Great Recession is mainly due to negative investment-specific shocks. Exogenous separation shocks play a role in the pre-Great Recession period and tend to lower the separation rate in recent years to compensate the effect of negative investment shocks (Figure A36). The historical decomposition for unemployment in Figure A37 reveals that matching efficiency shocks are less important in this specification of the model. Nevertheless, they still play a non-negligible role in slowing down the recovery in recent years. The natural rate of unemployment is now driven also by separation and investment-specific shocks. The role of technology and fiscal shocks is limited. The time-varying separation rate seems to be a powerful propagator for investment-specific shocks, as it can be seen also from Figure A39 where we see that they play a large role in generating the Beveridge curve dynamics observed in recent years. In contrast, exogenous separation shocks have shifted the Beveridge curve in the opposite direction.

Exogenous separation rate. In the last experiment we consider the case of a purely exogenous separation rate. The separation rate follows now the following process:

$$\ln \rho_t = (1 - \rho_\rho) \ln \rho + \rho_\rho \ln \rho_{t-1} + \varepsilon_{\rho t}$$

The results for this version of the model with nine observables and nine shocks are presented in Figures A40 to A44. Not surprisingly, exogenous separation shocks become more important in this case and are now the main drivers of the natural rate of unemployment. Nevertheless, the estimate of the natural rate is surprisingly stable across the different experiments. The decline in matching efficiency is again a feature of the post-Great Recession period when mismatch shock still contribute to slowing down the recovery and to increasing the natural rate of unemployment.

5.5 Summary

We conclude that when we change the sample period, the calibration for the elasticity of the matching function to unemployment or the calibration of the replacement rate, all our main results are confirmed. Matching efficiency shocks are not important drivers of the business cycle but they may play a role in selected periods and they are the most important driver of the natural rate. According to our analysis, they contribute substantially to explain the shift of the Beveridge curve and the weak recovery in the aftermath of the Great Recession. When we include separation shocks, the results change in some dimensions. On the one hand, mismatch shocks are not anymore the main drivers of the natural rate, although they still play a relevant role in recent years. On the other hand, the estimate of the natural rate is similar to the one obtained in our baseline model.

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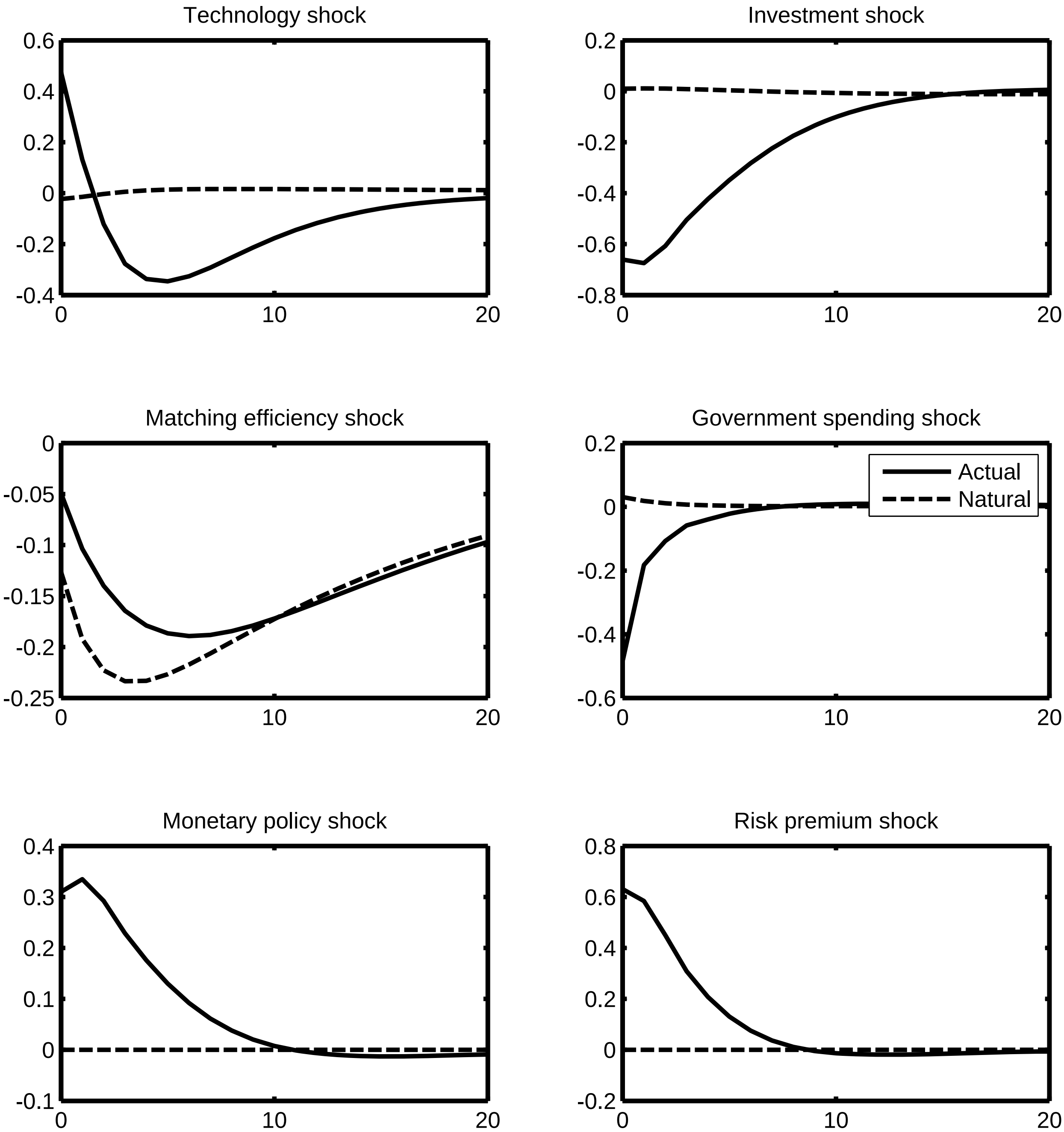


Fig. A0: Impulse responses of the actual and natural unemployment rates, expressed in percentage points. The responses are computed at the posterior mode. The size of each shock is one standard deviation.

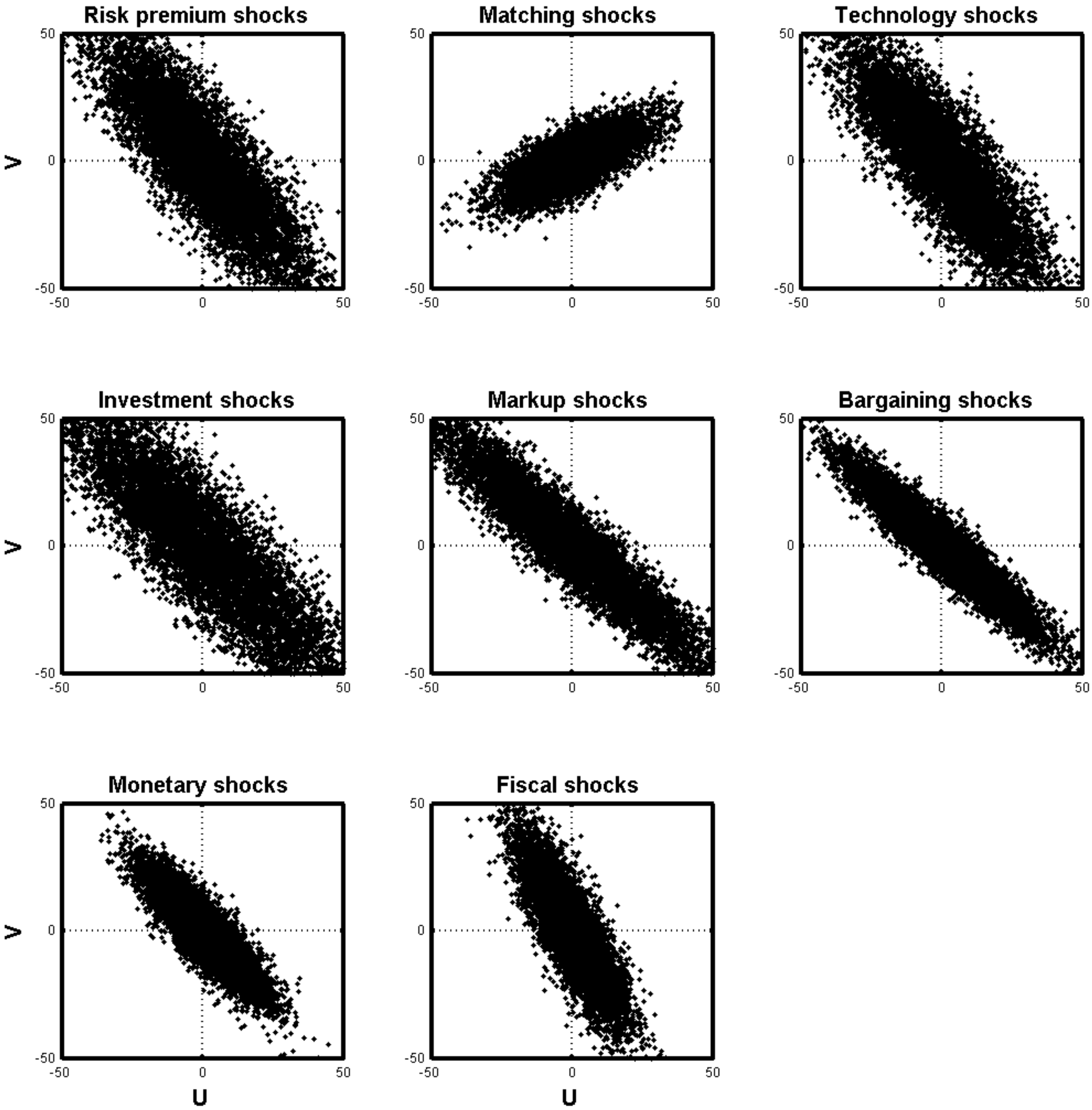


Fig A1: Simulated conditional Beveridge curves

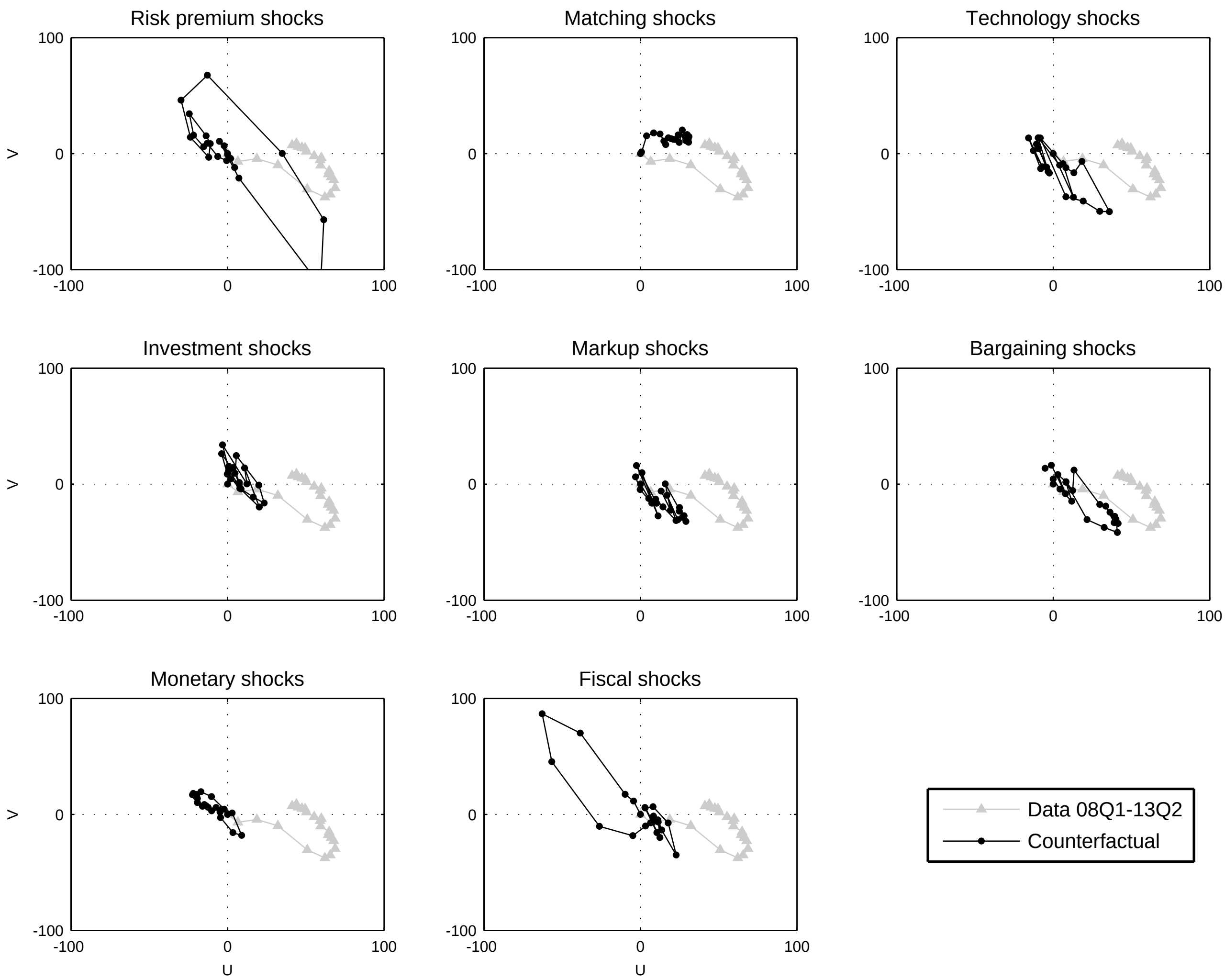


Fig A2: Contribution of each shock to the Beveridge curve 2008Q1-2013Q2 (% dev. from 2008Q1).

Robustness Check #1 - Estimation Period: 1957:Q1 - 2013:Q2

Calibrated Parameters: Check #1 [57:Q1-13:Q2]

Capital depreciation rate	δ	0.0250
Capital share	α	0.33
Elasticity of substitution btw goods	θ	6
Backward-looking price setting	ς	0.01
Replacement rate	τ	0.40
Job destruction rate	ρ	0.085
Elasticity of matches to unemp.	σ	0.65
Probability to fill a vacancy within a quarter	q	0.70
Exogenous spending/output ratio	g/y	0.20
Unemployment rate	U	0.0602
Quarterly gross growth rate	z	1.0038
Quarterly gross inflation rate	π	1.0083
Quarterly gross nominal interest rate	R	1.0128

Priors and Posteriors: Check #1 [57:Q1-13:Q2]

		Priors	Post. Mode
Weight of pre-match cost in total hiring cost	ϕ_V	Beta (0.5,0.2)	0.34
Hiring cost/output ratio	$1000\frac{\kappa}{2}\aleph^2$	IGamma (5,1)	2.40
Habit in consump.	h	Beta (0.7,0.1)	0.64
Invest. adj. cost	ϕ_I	IGamma (5,1)	2.90
Capital ut. cost	ϕ_{u2}	IGamma (0.5,0.1)	0.63
Price adjust. cost	ϕ_P	IGamma (60,10)	60.94
Wage adjust. cost	ϕ_W	IGamma (150,25)	131.24
Wage indexation	ϱ	Beta (0.5,0.2)	0.96
Interest smoothing	ρ_r	Beta (0.7,0.1)	0.34
Resp. to inflation	ρ_π	IGamma (1.5,0.1)	1.97
Resp. to growth	ρ_y	IGamma (0.5,0.1)	0.36

Fig A3: Matching Efficiency (estim. period: 57Q1-13Q2)

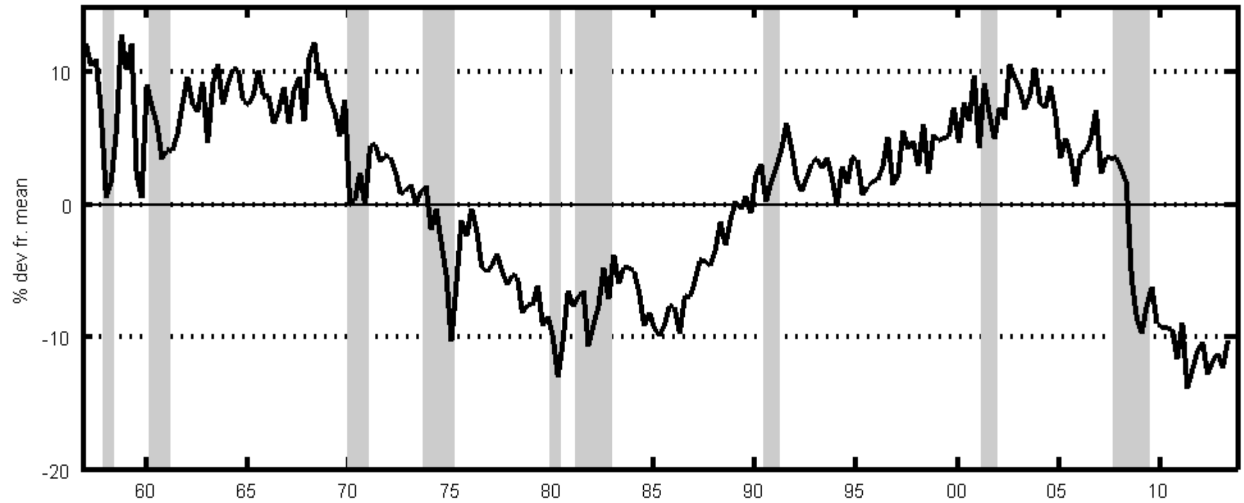


Fig A4: Unemployment Rates: Actual vs Natural (estim. period: 57Q1-13Q2)

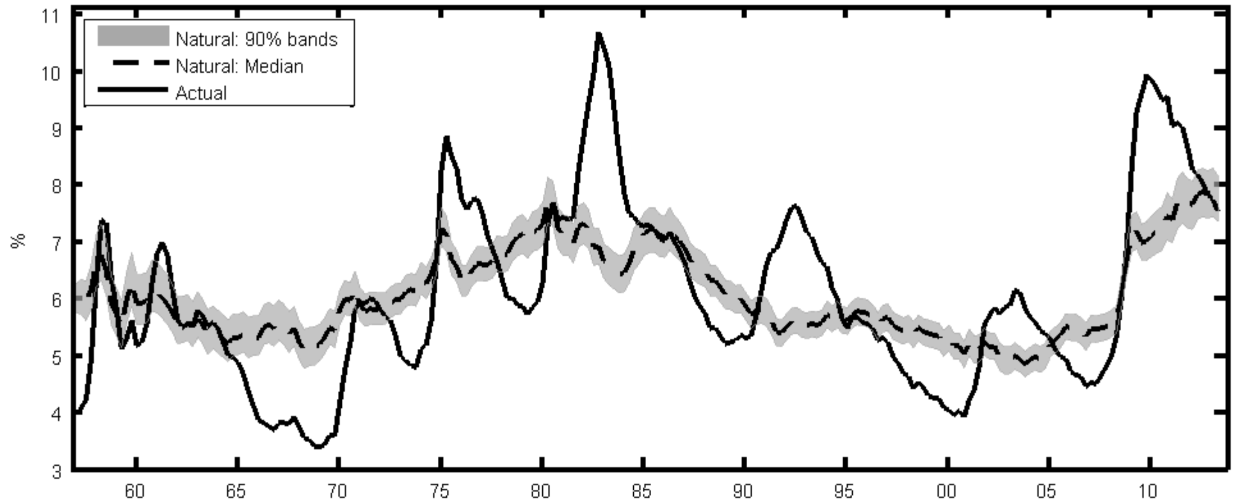
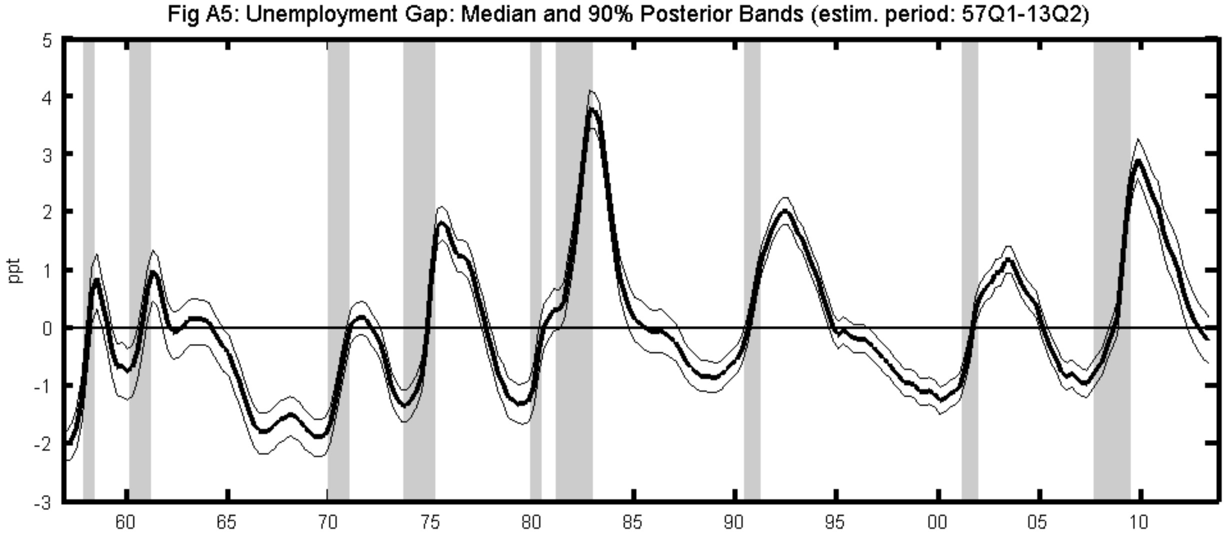


Fig A5: Unemployment Gap: Median and 90% Posterior Bands (estim. period: 57Q1-13Q2)



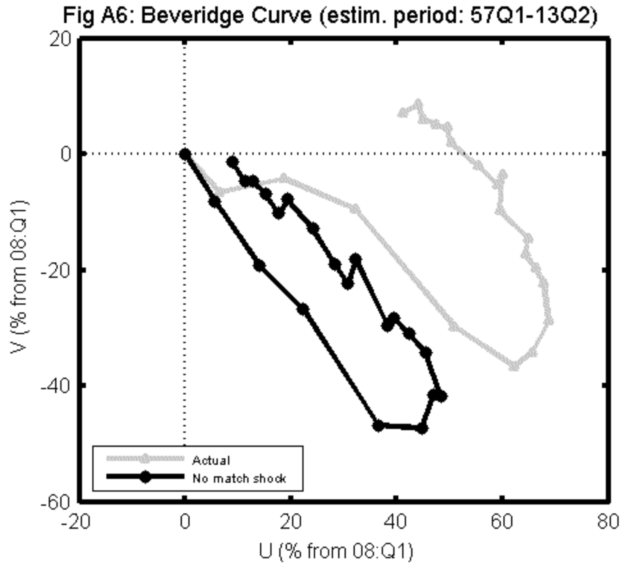


Fig A6: Beveridge Curve (estim. period: 57Q1-13Q2)
V (% from 08:Q1)
U (% from 08:Q1)
Actual
No match shock

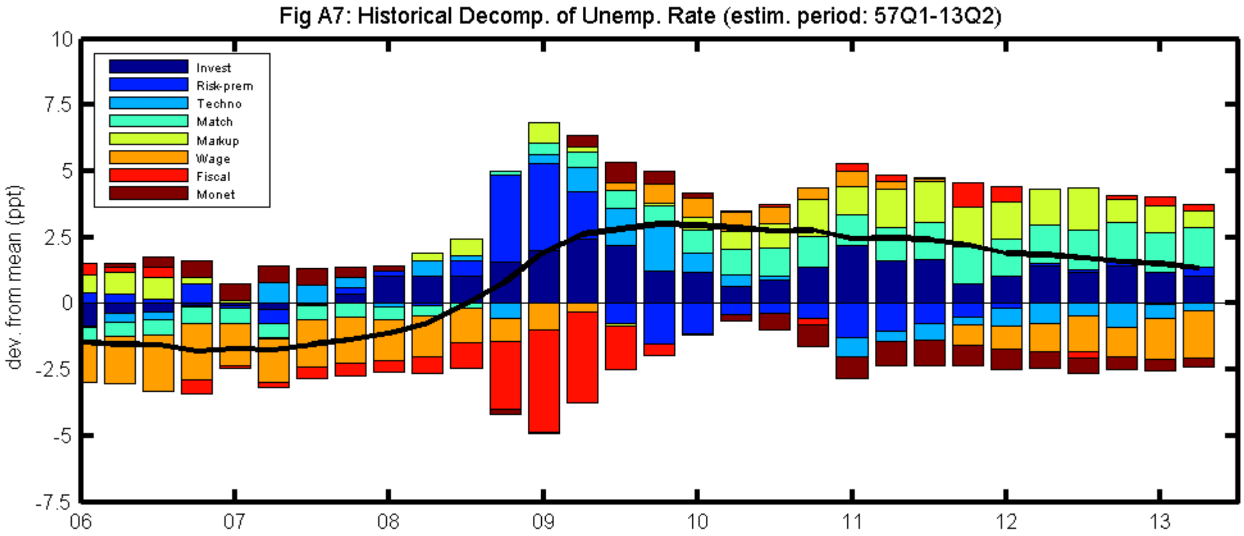


Fig A7: Historical Decomp. of Unemp. Rate (estim. period: 57Q1-13Q2)
dev. from mean (ppt)
Invest
Risk-prem
Techno
Match
Markup
Wage
Fiscal
Monet

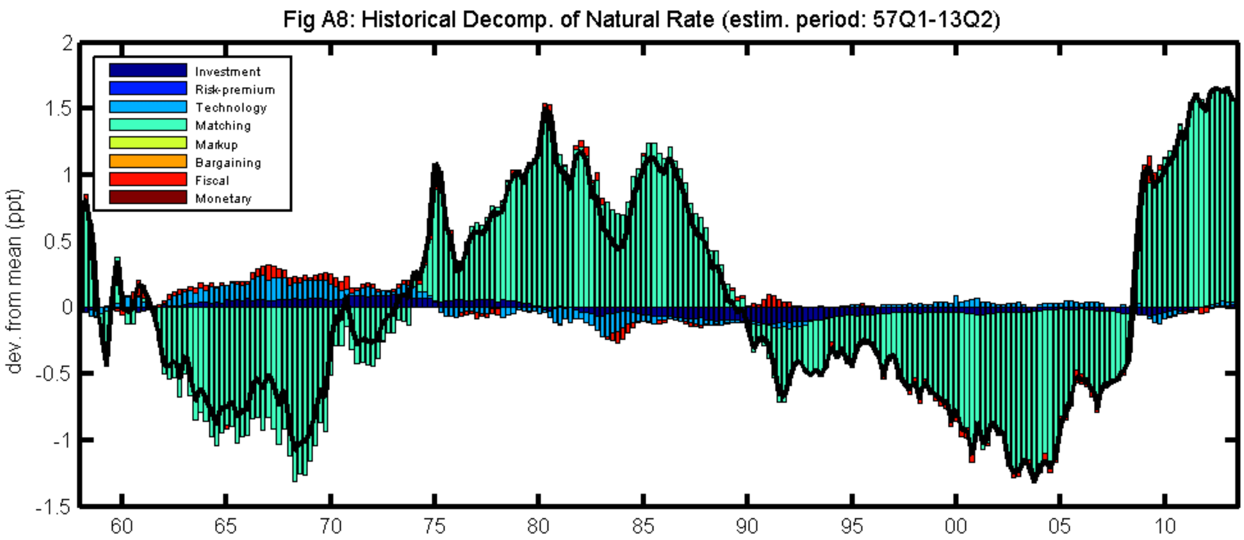


Fig A8: Historical Decomp. of Natural Rate (estim. period: 57Q1-13Q2)
dev. from mean (ppt)
Investment
Risk-premium
Technology
Matching
Markup
Bargaining
Fiscal
Monetary

Robustness Check #2 - Estimation Period: 1985:Q1 - 2008:Q3

Calibrated Parameters: Check #2 [85:Q1-08:Q3]

Capital depreciation rate	δ	0.0250
Capital share	α	0.33
Elasticity of substitution btw goods	θ	6
Backward-looking price setting	ς	0.01
Replacement rate	τ	0.40
Job destruction rate	ρ	0.085
Elasticity of matches to unemp.	σ	0.65
Probability to fill a vacancy within a quarter	q	0.70
Exogenous spending/output ratio	g/y	0.20
Unemployment rate	U	0.0564
Quarterly gross growth rate	z	1.0043
Quarterly gross inflation rate	π	1.0061
Quarterly gross nominal interest rate	R	1.0125

Priors and Posteriors: Check #2 [85:Q1-08:Q3]

		Priors	Post. Mode
Weight of pre-match cost in total hiring cost	ϕ_V	Beta (0.5,0.2)	0.25
Hiring cost/output ratio	$1000\frac{\kappa}{2}\aleph^2$	IGamma (5,1)	3.00
Habit in consump.	h	Beta (0.7,0.1)	0.54
Invest. adj. cost	ϕ_I	IGamma (5,1)	3.7
Capital ut. cost	ϕ_{u2}	IGamma (0.5,0.1)	0.54
Price adjust. cost	ϕ_P	IGamma (60,10)	81.65
Wage adjust. cost	ϕ_W	IGamma (150,25)	112.82
Wage indexation	ϱ	Beta (0.5,0.2)	0.71
Interest smoothing	ρ_r	Beta (0.7,0.1)	0.59
Resp. to inflation	ρ_π	IGamma (1.5,0.1)	2.33
Resp. to growth	ρ_y	IGamma (0.5,0.1)	0.41

Fig A9: Matching Efficiency (estim period: 85Q1-08Q3)

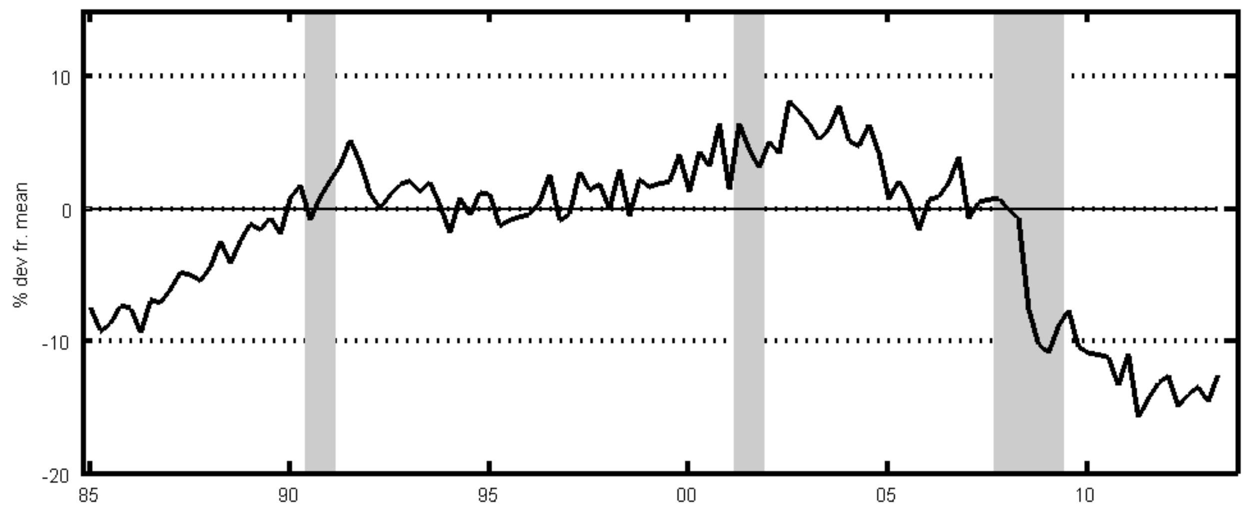


Fig A10: Unemployment Rates: Actual vs Natural (estim period: 85Q1-08Q3)

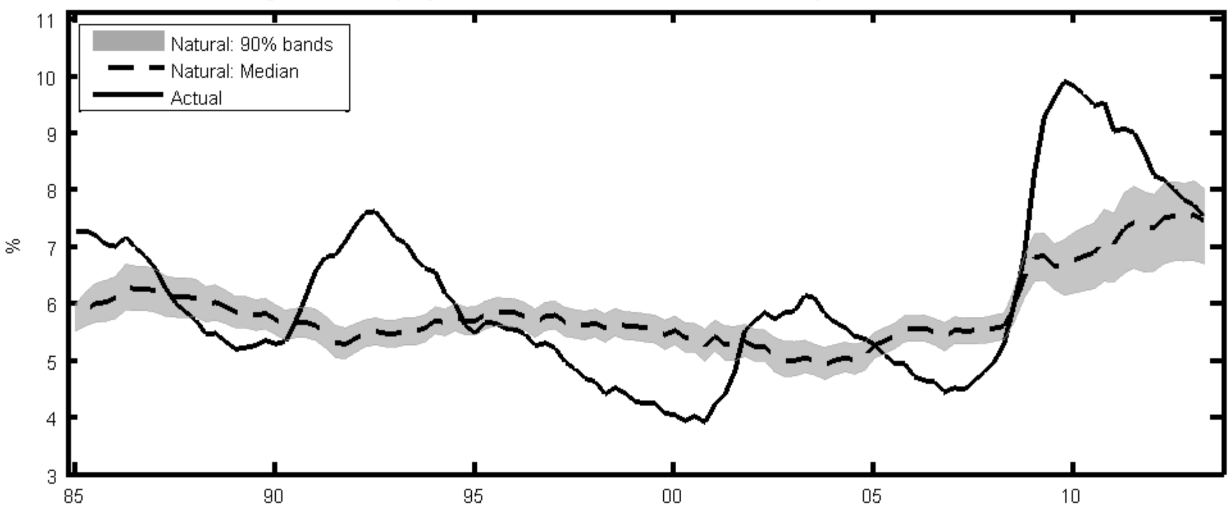


Fig A11: Unemployment Gap: Median and 90% Posterior Bands (estim period: 85Q1-08Q3)

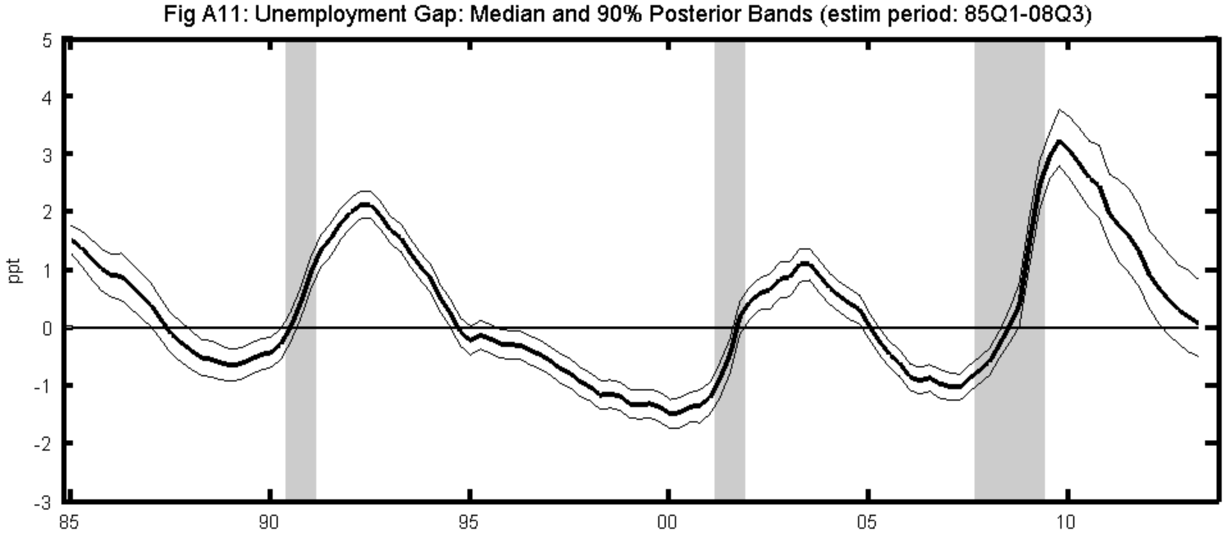


Fig A13: Beveridge Curve (estim. period: 85Q1-08Q3)

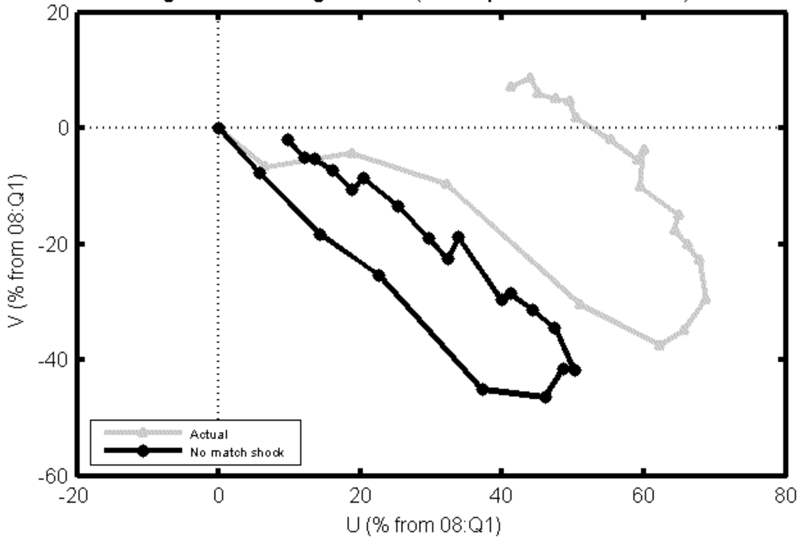


Fig A14: Historical Decomp. of Unemp. Rate (estim. period: 85Q1-08Q3)

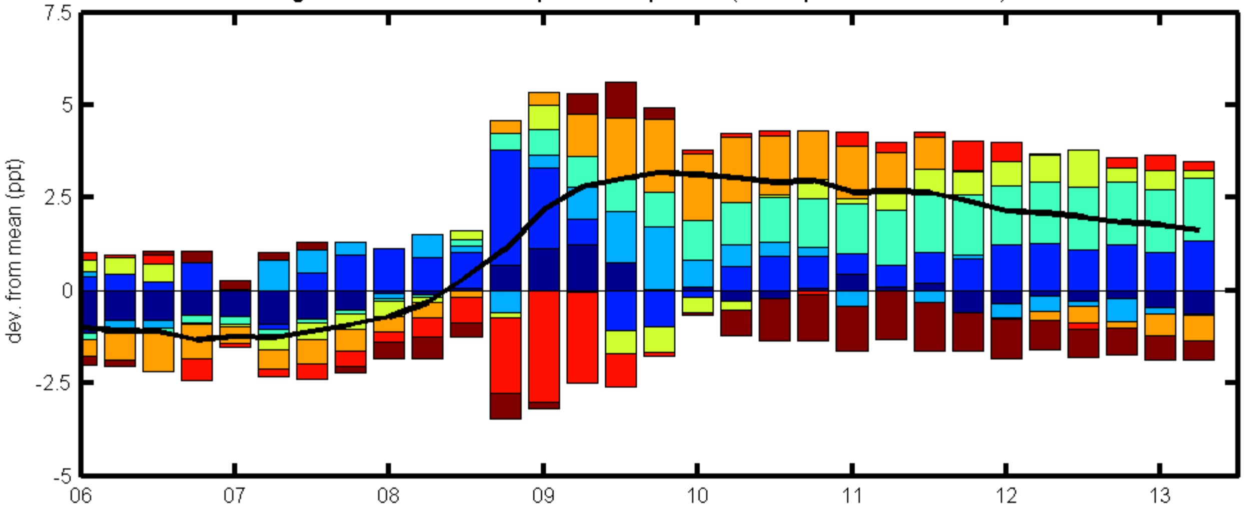
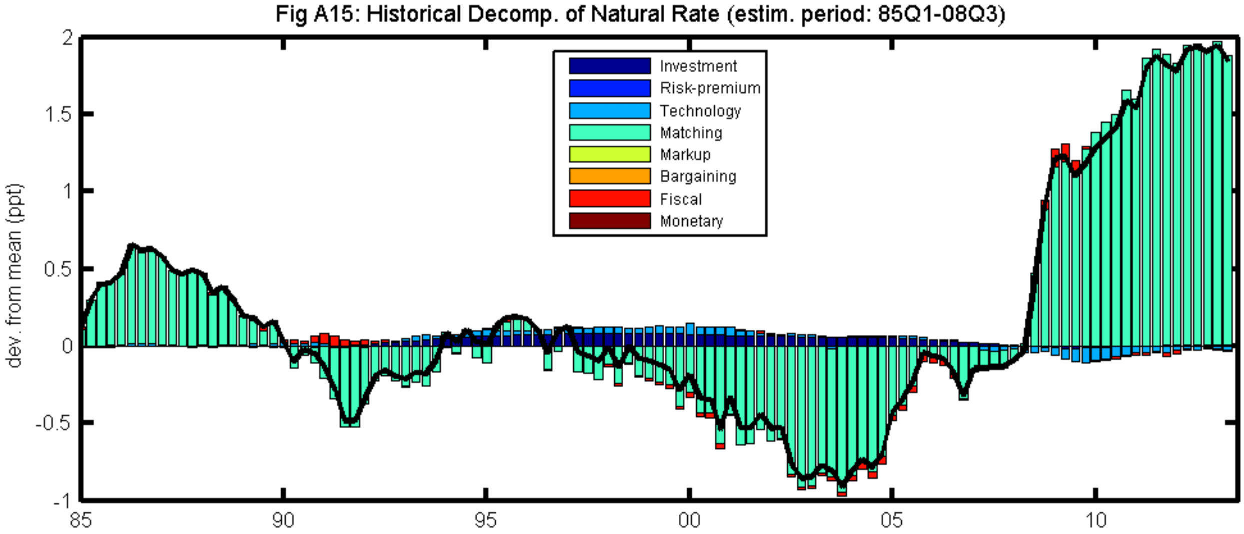


Fig A15: Historical Decomp. of Natural Rate (estim. period: 85Q1-08Q3)



Robustness Check #3 - $\sigma = 0.55$ (Est. Per.: 57:Q1 - 08:Q3)

Calibrated Parameters: Check #3 Low Sigma

Capital depreciation rate	δ	0.0250
Capital share	α	0.33
Elasticity of substitution btw goods	θ	6
Backward-looking price setting	ς	0.01
Replacement rate	τ	0.40
Job destruction rate	ρ	0.085
Elasticity of matches to unemp.	σ	$\mathbf{0.55}$
Probability to fill a vacancy within a quarter	q	0.70
Exogenous spending/output ratio	g/y	0.20
Unemployment rate	U	0.0578
Quarterly gross growth rate	z	1.0039
Quarterly gross inflation rate	π	1.0088
Quarterly gross nominal interest rate	R	1.0139

Priors and Posteriors: Check #3 Low Sigma

		Priors	Post. Mode
Weight of pre-match cost in total hiring cost	ϕ_V	Beta (0.5,0.2)	0.34
Hiring cost/output ratio	$1000\frac{\kappa}{2}\aleph^2$	IGamma (5,1)	2.56
Habit in consump.	h	Beta (0.7,0.1)	0.65
Invest. adj. cost	ϕ_I	IGamma (5,1)	3.07
Capital ut. cost	ϕ_{u2}	IGamma (0.5,0.1)	0.58
Price adjust. cost	ϕ_P	IGamma (60,10)	59.35
Wage adjust. cost	ϕ_W	IGamma (150,25)	141.29
Wage indexation	ϱ	Beta (0.5,0.2)	0.95
Interest smoothing	ρ_r	Beta (0.7,0.1)	0.33
Resp. to inflation	ρ_π	IGamma (1.5,0.1)	1.92
Resp. to growth	ρ_y	IGamma (0.5,0.1)	0.37

Fig A16: Matching Efficiency (σ= 0.55; estim period: 57-08Q3)

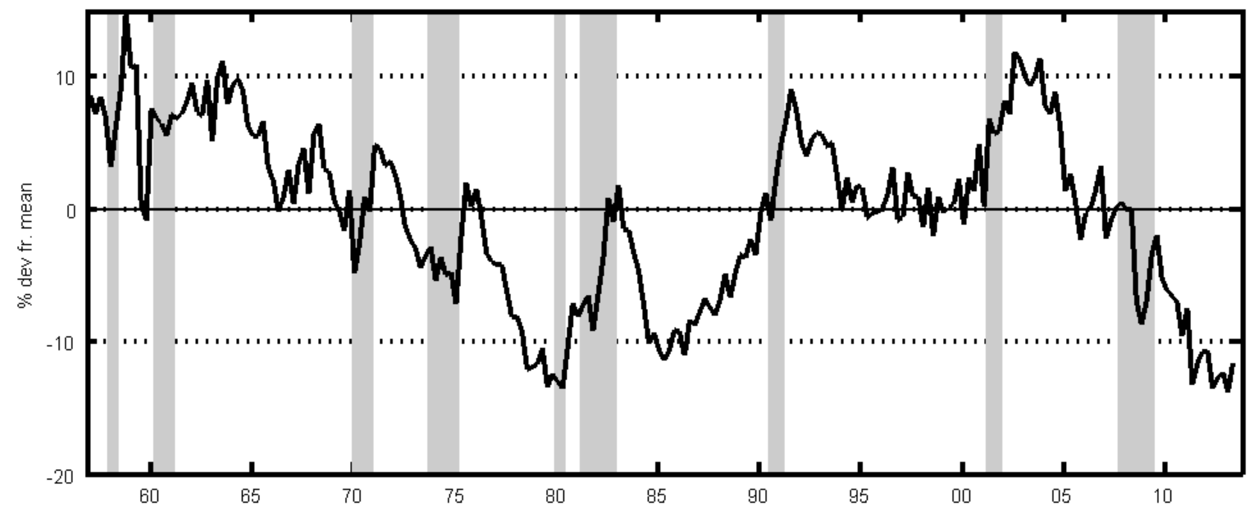


Fig A17: Unemployment Rates: Actual vs Natural (σ= 0.55; estim period: 57-08Q3)

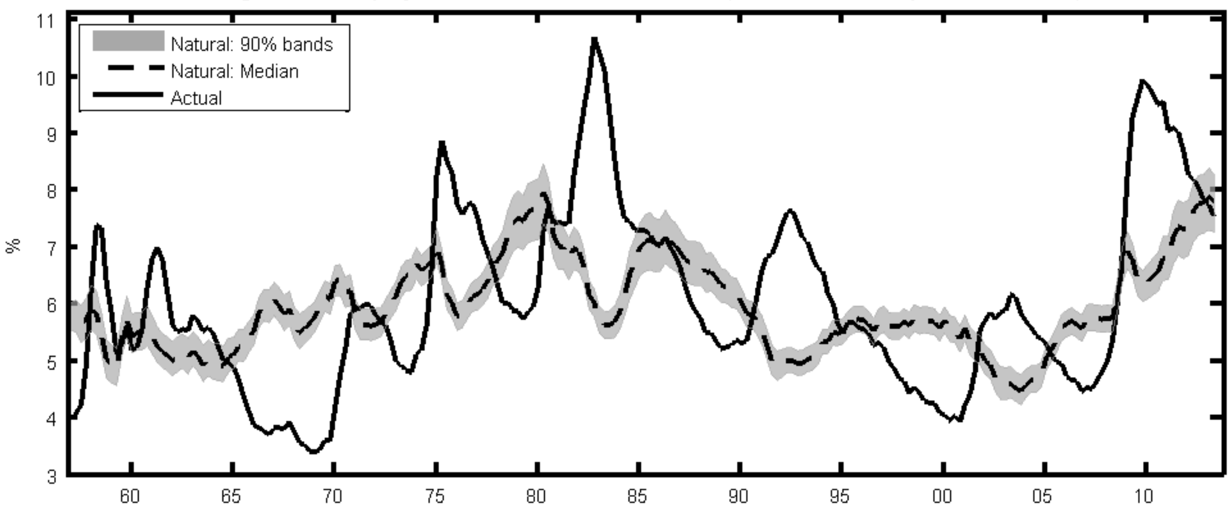
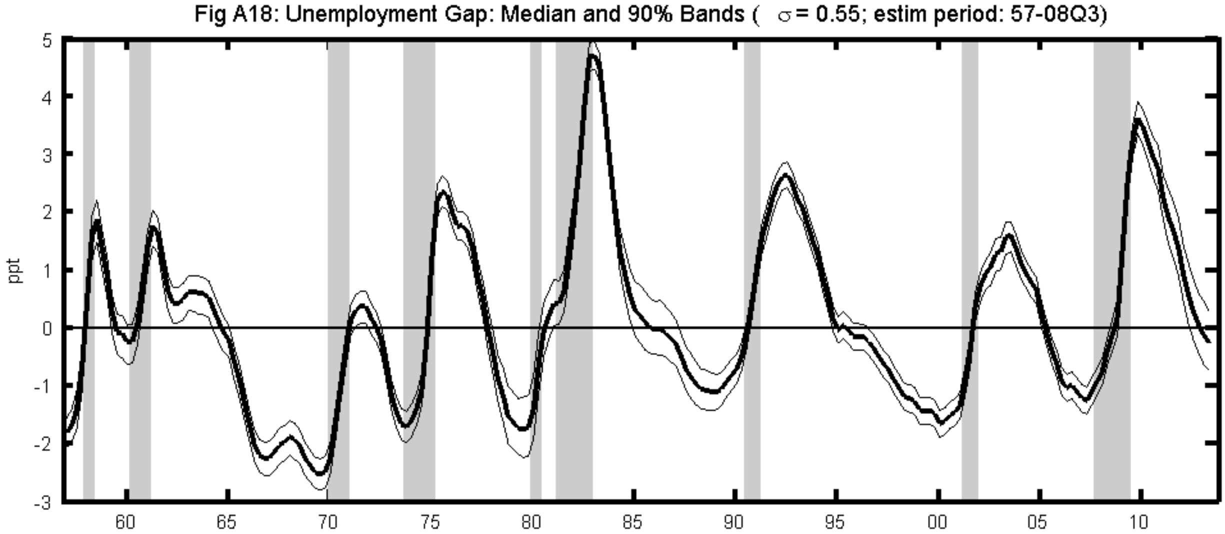


Fig A18: Unemployment Gap: Median and 90% Bands (σ= 0.55; estim period: 57-08Q3)



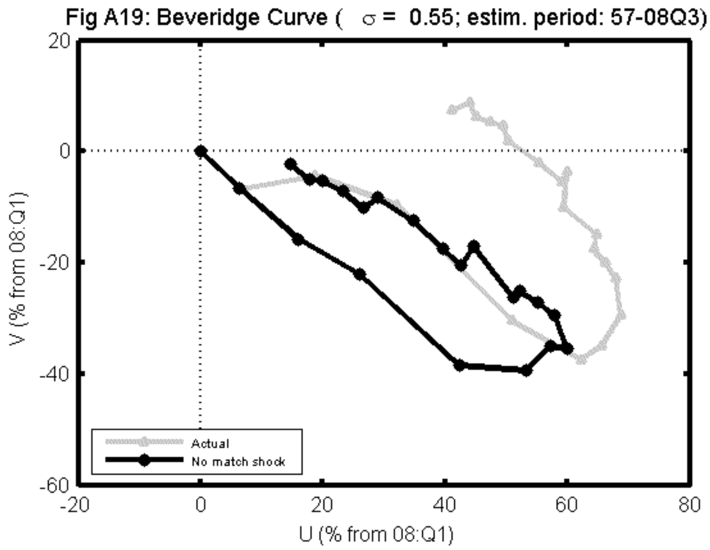


Fig A19: Beveridge Curve (σ = 0.55; estim. period: 57-08Q3)
V (% from 08:Q1)
U (% from 08:Q1)
Actual
No match shock

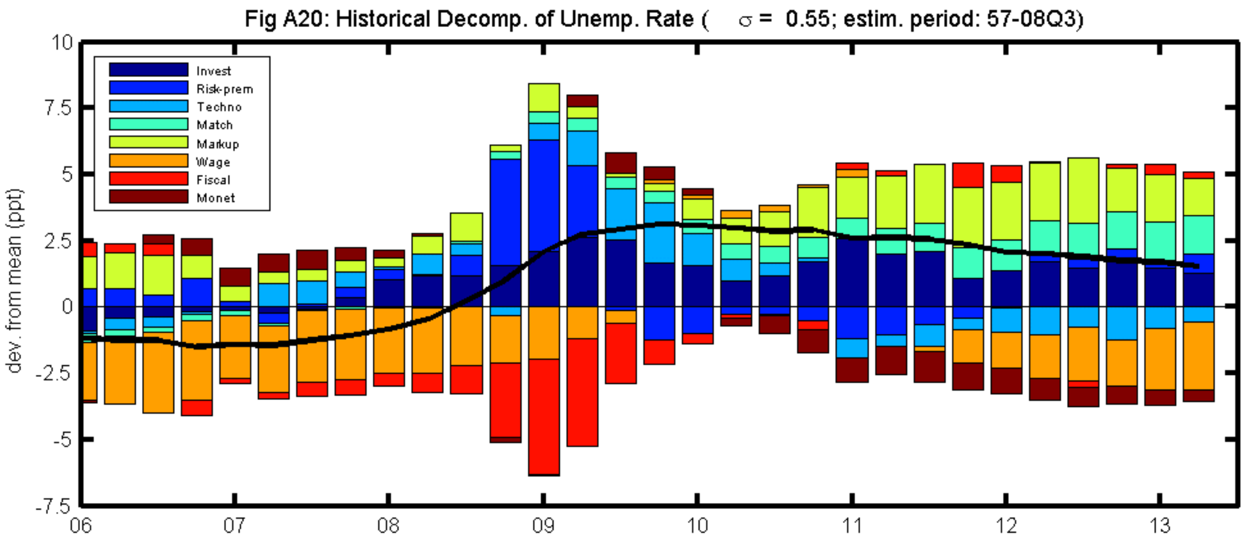


Fig A20: Historical Decomp. of Unemp. Rate (σ = 0.55; estim. period: 57-08Q3)
dev. from mean (ppt)
Invest
Risk-prem
Techno
Match
Markup
Wage
Fiscal
Monet

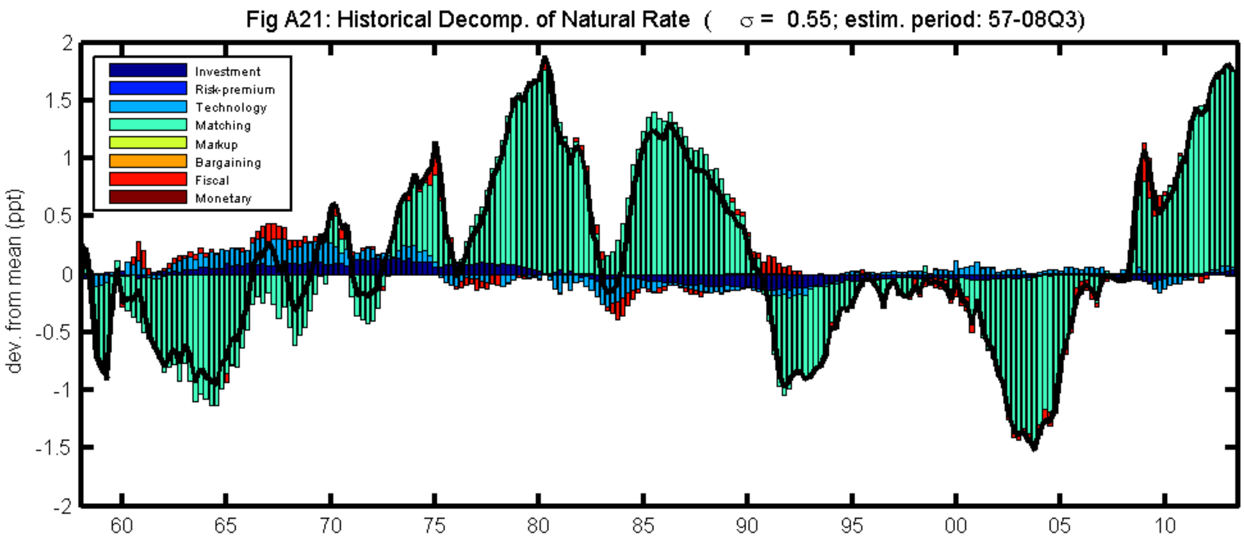


Fig A21: Historical Decomp. of Natural Rate (σ = 0.55; estim. period: 57-08Q3)
dev. from mean (ppt)
Investment
Risk-premium
Technology
Matching
Markup
Bargaining
Fiscal
Monetary

Robustness Check #4 - $\sigma = 0.75$ (Est. Per.: 57:Q1 - 08:Q3)

Calibrated Parameters: Check #4 - High Sigma

Capital depreciation rate	δ	0.0250
Capital share	α	0.33
Elasticity of substitution btw goods	θ	6
Backward-looking price setting	ς	0.01
Replacement rate	τ	0.40
Job destruction rate	ρ	0.085
Elasticity of matches to unemp.	σ	$\mathbf{0.75}$
Probability to fill a vacancy within a quarter	q	0.70
Exogenous spending/output ratio	g/y	0.20
Unemployment rate	U	0.0578
Quarterly gross growth rate	z	1.0039
Quarterly gross inflation rate	π	1.0088
Quarterly gross nominal interest rate	R	1.0139

Priors and Posteriors: Check #4 - High Sigma

		Priors	Post. Mode
Weight of pre-match cost in total hiring cost	ϕ_V	Beta (0.5,0.2)	0.25
Hiring cost/output ratio	$1000\frac{\kappa}{2}\aleph^2$	IGamma (5,1)	2.39
Habit in consump.	h	Beta (0.7,0.1)	0.64
Invest. adj. cost	ϕ_I	IGamma (5,1)	3.03
Capital ut. cost	ϕ_{u2}	IGamma (0.5,0.1)	0.60
Price adjust. cost	ϕ_P	IGamma (60,10)	62.78
Wage adjust. cost	ϕ_W	IGamma (150,25)	188.35
Wage indexation	ϱ	Beta (0.5,0.2)	0.96
Interest smoothing	ρ_r	Beta (0.7,0.1)	0.33
Resp. to inflation	ρ_π	IGamma (1.5,0.1)	1.84
Resp. to growth	ρ_y	IGamma (0.5,0.1)	0.36

Fig A22: Matching Efficiency (σ = 0.75; estim period: 57-08Q3)

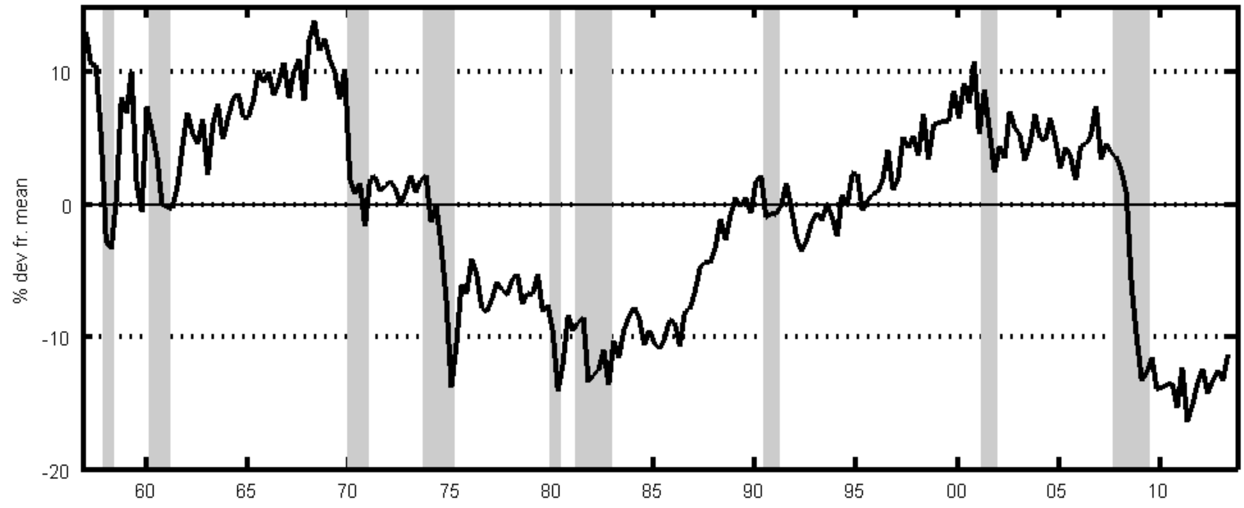


Fig A23: Unemployment Rates: Actual vs Natural (σ = 0.75; estim period: 57-08Q3)

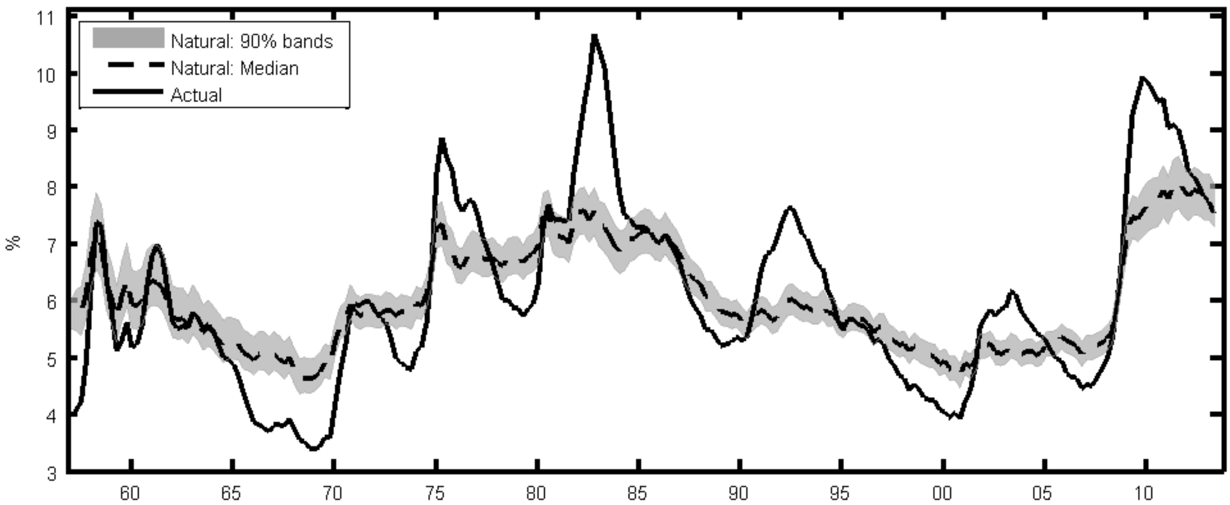
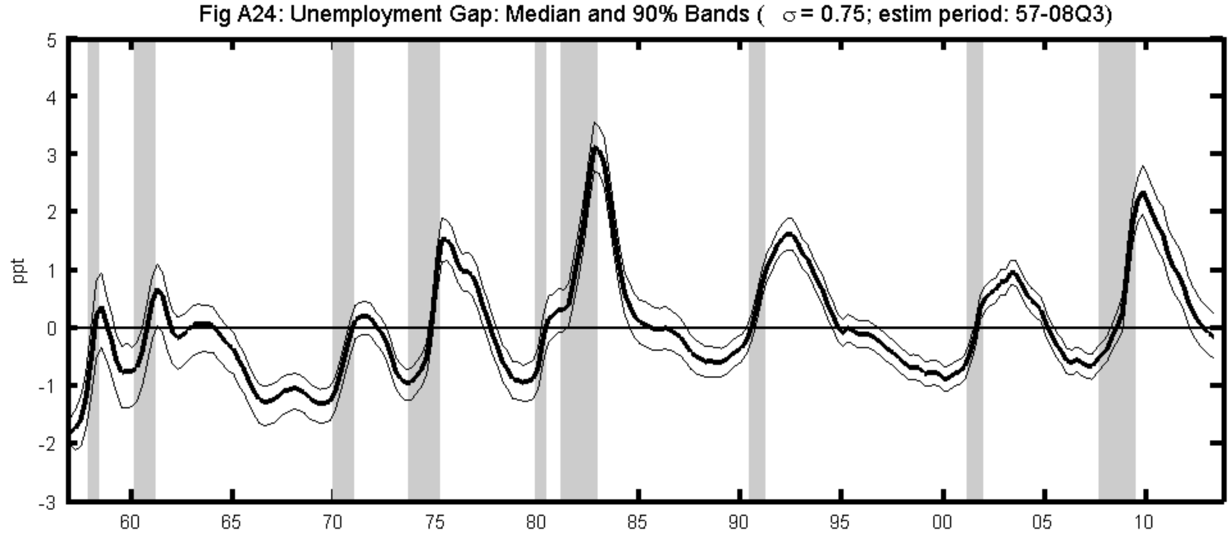


Fig A24: Unemployment Gap: Median and 90% Bands (σ = 0.75; estim period: 57-08Q3)



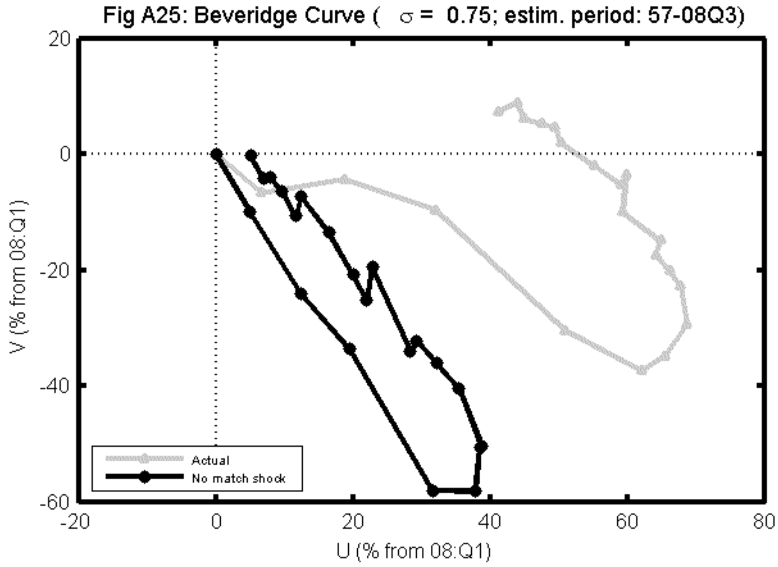


Fig A25: Beveridge Curve (σ = 0.75; estim. period: 57-08Q3)
V (% from 08:Q1)
U (% from 08:Q1)
Actual
No match shock

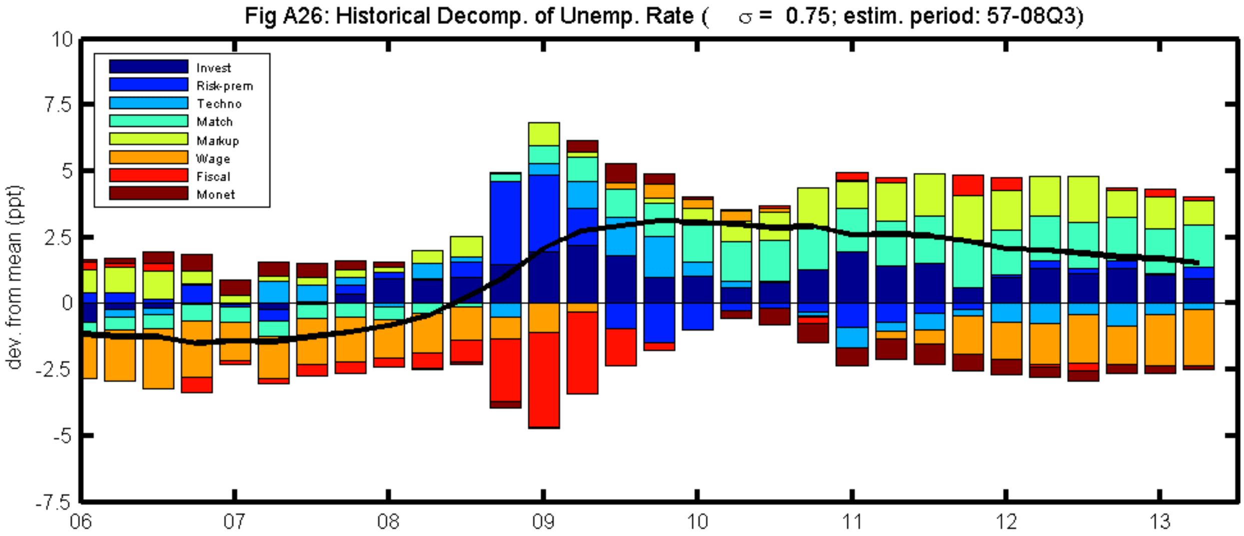


Fig A26: Historical Decomp. of Unemp. Rate (σ = 0.75; estim. period: 57-08Q3)
dev. from mean (ppt)
Invest
Risk-prem
Techno
Match
Markup
Wage
Fiscal
Monet

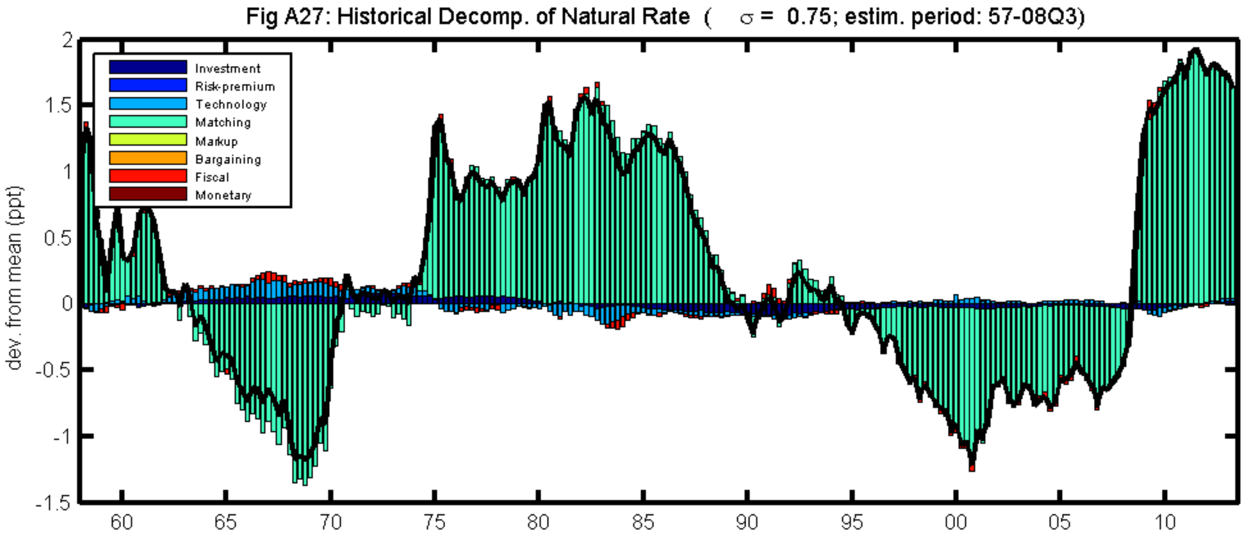


Fig A27: Historical Decomp. of Natural Rate (σ = 0.75; estim. period: 57-08Q3)
dev. from mean (ppt)
Investment
Risk-premium
Technology
Matching
Markup
Bargaining
Fiscal
Monetary

Robustness Check #5 - $\tau = 0.70$ (Est. Per.: 57:Q1 - 08:Q3)

Calibrated Parameters

Capital depreciation rate	δ	0.0250
Capital share	α	0.33
Elasticity of substitution btw goods	θ	6
Backward-looking price setting	ς	0.01
Replacement rate	τ	$\mathbf{0.70}$
Job destruction rate	ρ	0.085
Elasticity of matches to unemp.	σ	0.65
Probability to fill a vacancy within a quarter	q	0.70
Exogenous spending/output ratio	g/y	0.20
Unemployment rate	U	0.0578
Quarterly gross growth rate	z	1.0039
Quarterly gross inflation rate	π	1.0088
Quarterly gross nominal interest rate	R	1.0139

Priors and Posteriors

		Priors	Post. Mode
Weight of pre-match cost in total hiring cost	ϕ_V	Beta (0.5,0.2)	0.32
Hiring cost/output ratio	$1000\frac{\kappa}{2}\aleph^2$	IGamma (5,1)	2.46
Habit in consump.	h	Beta (0.7,0.1)	0.65
Invest. adj. cost	ϕ_I	IGamma (5,1)	3.06
Capital ut. cost	ϕ_{u2}	IGamma (0.5,0.1)	0.59
Price adjust. cost	ϕ_P	IGamma (60,10)	62.99
Wage adjust. cost	ϕ_W	IGamma (150,25)	192.91
Wage indexation	ϱ	Beta (0.5,0.2)	0.96
Interest smoothing	ρ_r	Beta (0.7,0.1)	0.34
Resp. to inflation	ρ_π	IGamma (1.5,0.1)	1.83
Resp. to growth	ρ_y	IGamma (0.5,0.1)	0.37

Fig A28: Matching Efficiency (τ = 0.70; estim period: 57-08Q3)

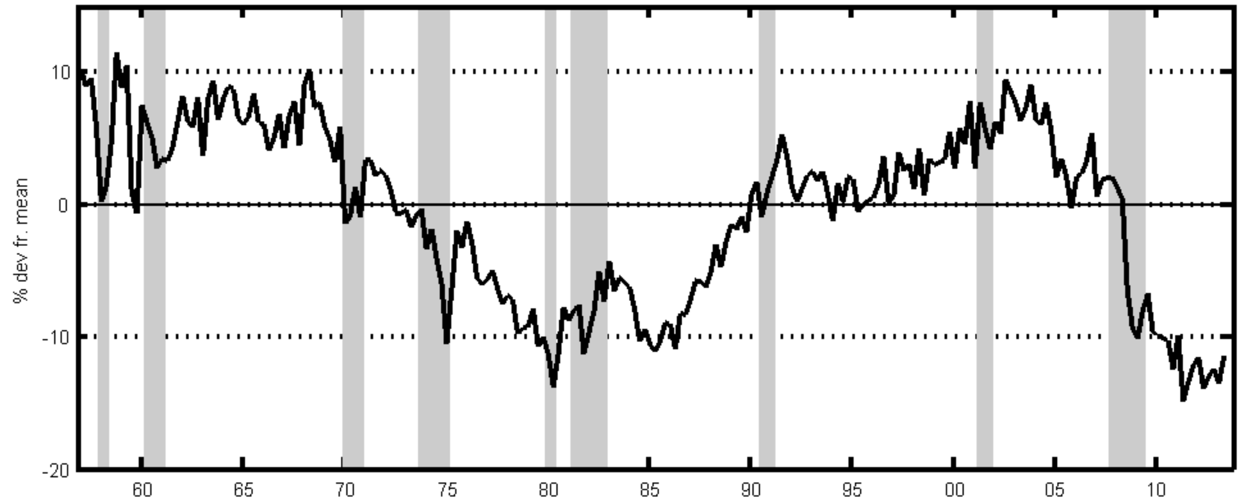


Fig A29: Unemployment Rates: Actual vs Natural (τ = 0.70; estim period: 57-08Q3)

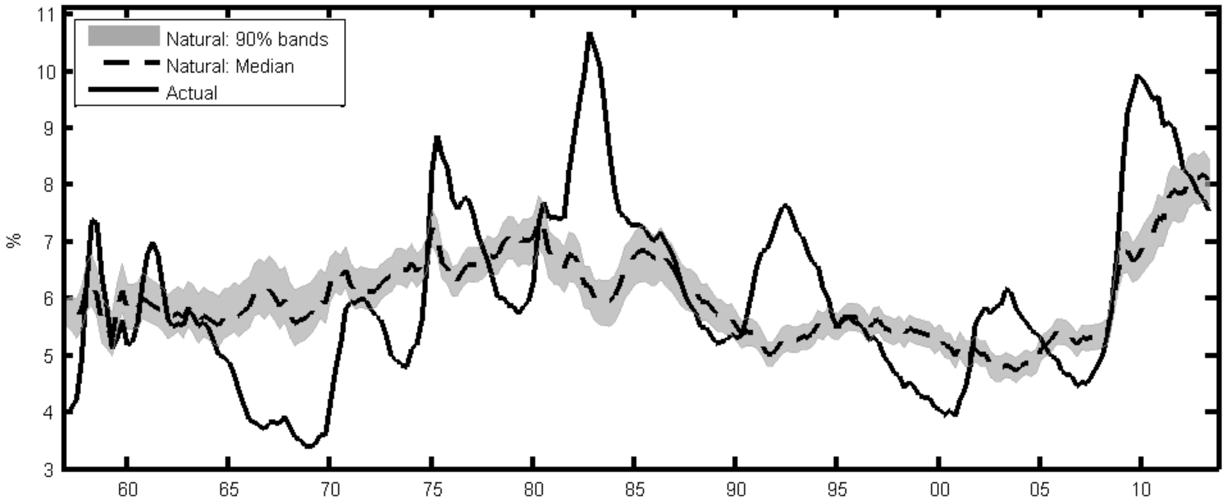


Fig A30: Unemployment Gap: Median and 90% Bands (τ = 0.70; estim period: 57-08Q3)

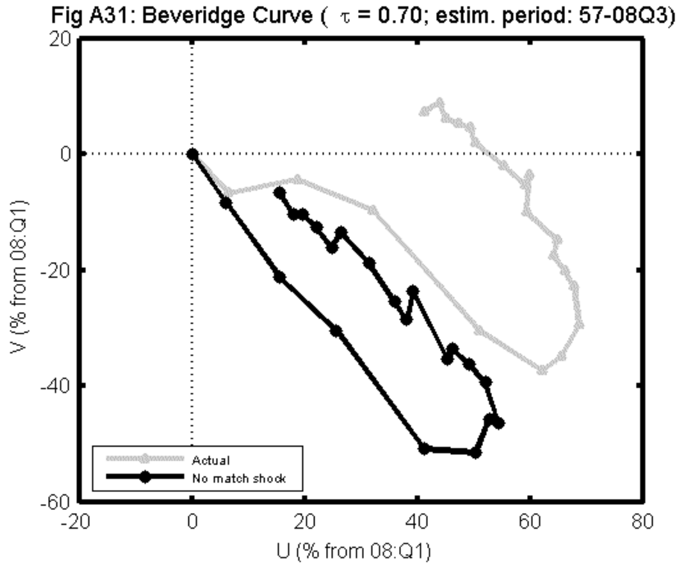


Fig A31: Beveridge Curve (τ = 0.70; estim. period: 57-08Q3)
V (% from 08:Q1)
U (% from 08:Q1)
Actual
No match shock
20
0
-20
-40
-60
-20
0
20
40
60
80

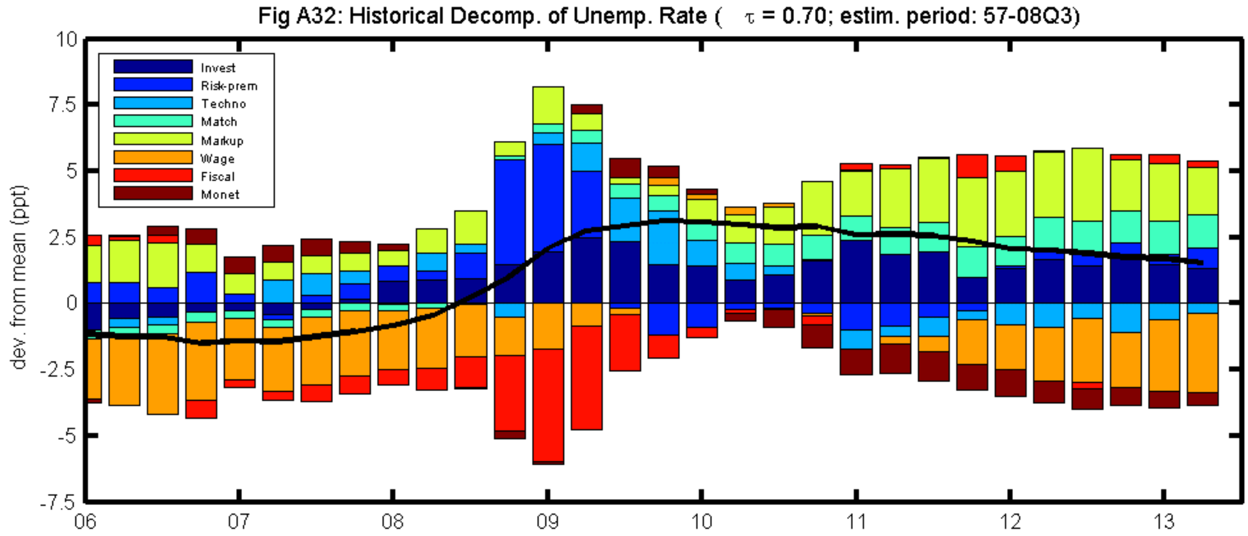


Fig A32: Historical Decomp. of Unemp. Rate (τ = 0.70; estim. period: 57-08Q3)
dev. from mean (ppt)
Invest
Risk-prem
Techno
Match
Markup
Wage
Fiscal
Monet
10
7.5
5
2.5
0
-2.5
-5
-7.5
06
07
08
09
10
11
12
13

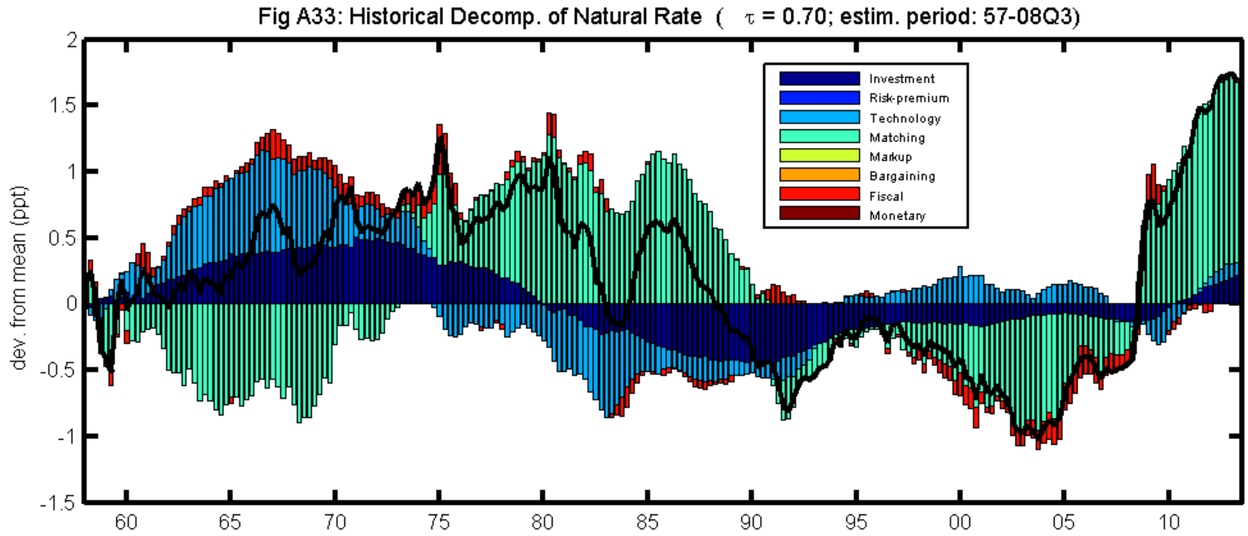


Fig A33: Historical Decomp. of Natural Rate (τ = 0.70; estim. period: 57-08Q3)
dev. from mean (ppt)
Investment
Risk-premium
Technology
Matching
Markup
Bargaining
Fiscal
Monetary
2
1.5
1
0.5
0
-0.5
-1
-1.5
60
65
70
75
80
85
90
95
00
05
10

#6 - Model with 9 shocks incl. match. and separ. shock

- 9 Observables including CPS 'E-U' separation rate over 1968:Q1-2008:Q3.
- Sep. rate follows: $\ln\rho_t = (1-\rho_\rho)\ln\rho + \rho_\rho \ln\rho_{t-1} - \delta_z\varepsilon_{zt} - \delta_\mu\varepsilon_{\mu t} + \varepsilon_{\rho t}$, $\delta_z \geq 0$, $\delta_\mu \geq 0$.

Capital depreciation rate	δ	0.0250
Capital share	α	0.33
Elasticity of substitution btw goods	θ	6
Backward-looking price setting	ς	0.01
Replacement rate	τ	0.40
Job destruction rate	ρ	0.085
Elasticity of matches to unemp.	σ	0.65
Probability to fill a vacancy within a quarter	q	0.70
Exogenous spending/output ratio	g/y	0.20
Unemployment rate	U	0.0599
Quarterly gross growth rate	z	1.0033
Quarterly gross inflation rate	π	1.0099
Quarterly gross nominal interest rate	R	1.0158

		Priors	Post. Mode
Weight of pre-match cost in total hiring cost	ϕ_V	Beta (0.5,0.2)	0.20
Hiring cost/output ratio	$1000\frac{\kappa}{2}\aleph^2$	IGamma (5,1)	2.92
Habit in consump.	h	Beta (0.7,0.1)	0.64
Invest. adj. cost	ϕ_I	IGamma (5,1)	3.60
Capital ut. cost	ϕ_{u2}	IGamma (0.5,0.1)	0.66
Price adjust. cost	ϕ_P	IGamma (60,10)	62.96
Wage adjust. cost	ϕ_W	IGamma (150,25)	113.08
Wage indexation	ϱ	Beta (0.5,0.2)	0.92
Interest smoothing	ρ_r	Beta (0.7,0.1)	0.28
Resp. to inflation	ρ_π	IGamma (1.5,0.1)	2.14
Resp. to growth	ρ_y	IGamma (0.5,0.1)	0.47
Elasticity of separation wrt tech. shock	δ_z	Uniform (0,3)	1.14
Elasticity of separation wrt IST. shock	δ_μ	Uniform (0,1)	0.20

Fig. A34: Matching Efficiency and Separation Rate

Fig. A35: Unemployment Rates: Actual vs Natural

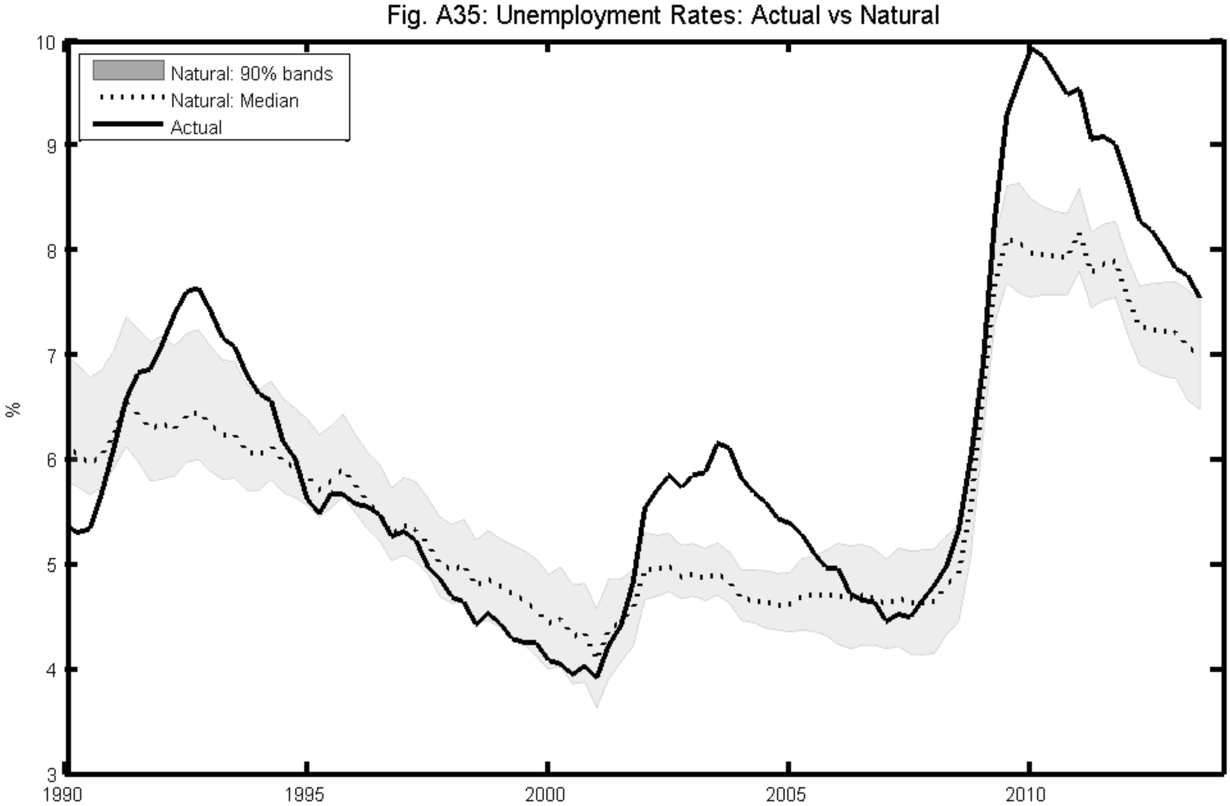


Fig A36: Historical Decomp. of Separation Rate (estim. period: 68Q1-08Q2; 9 observ. incl. CPS E-U separ. rate)

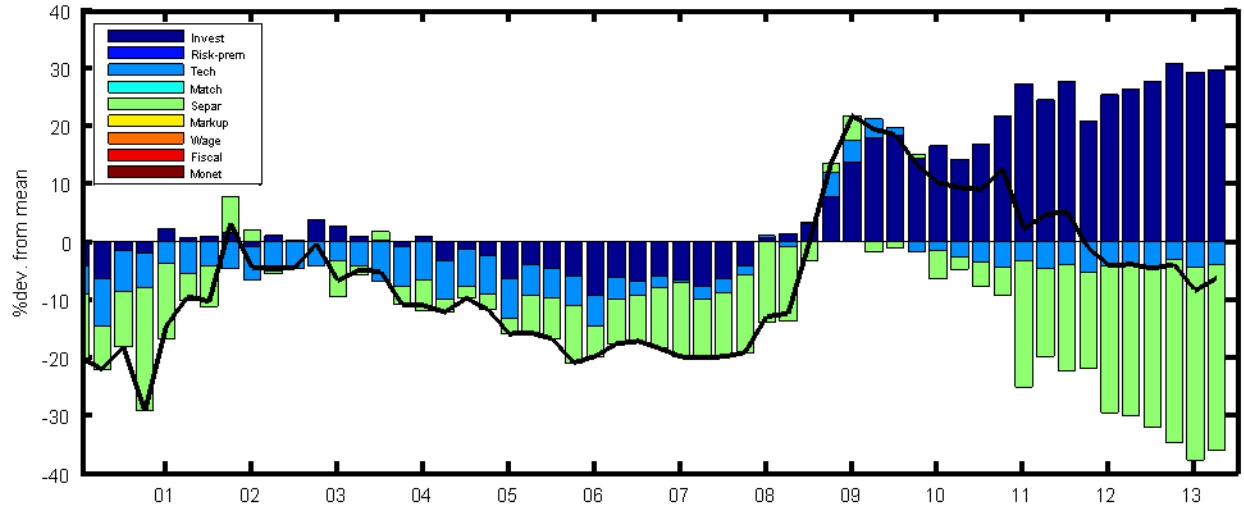


Fig A37: Historical Decomp. of Unemp. Rate (estim. period: 68Q1-08Q2; 9 observ. incl. CPS E-U separ. rate)

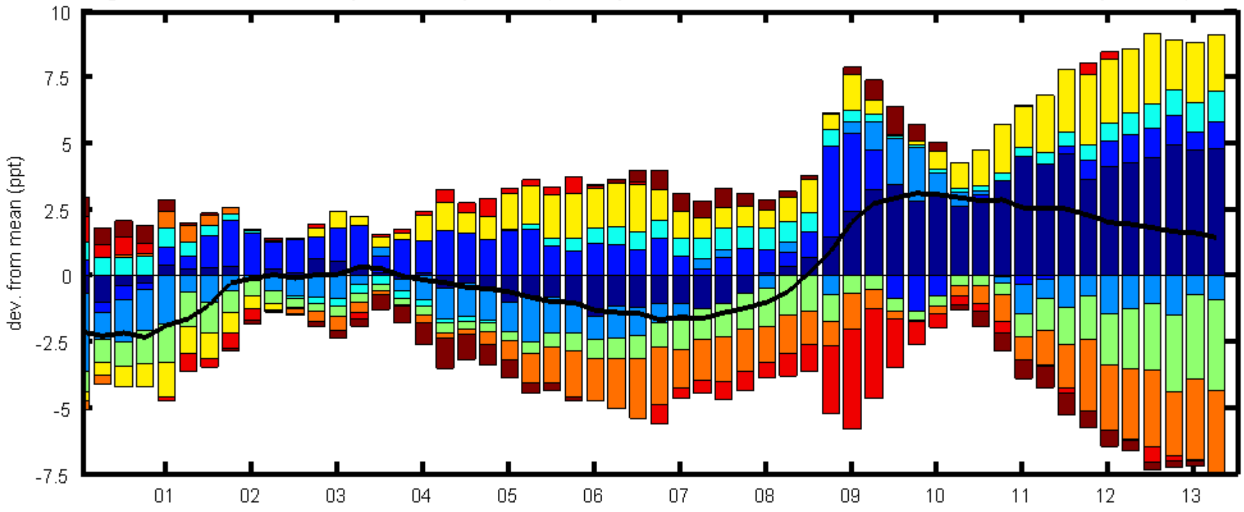
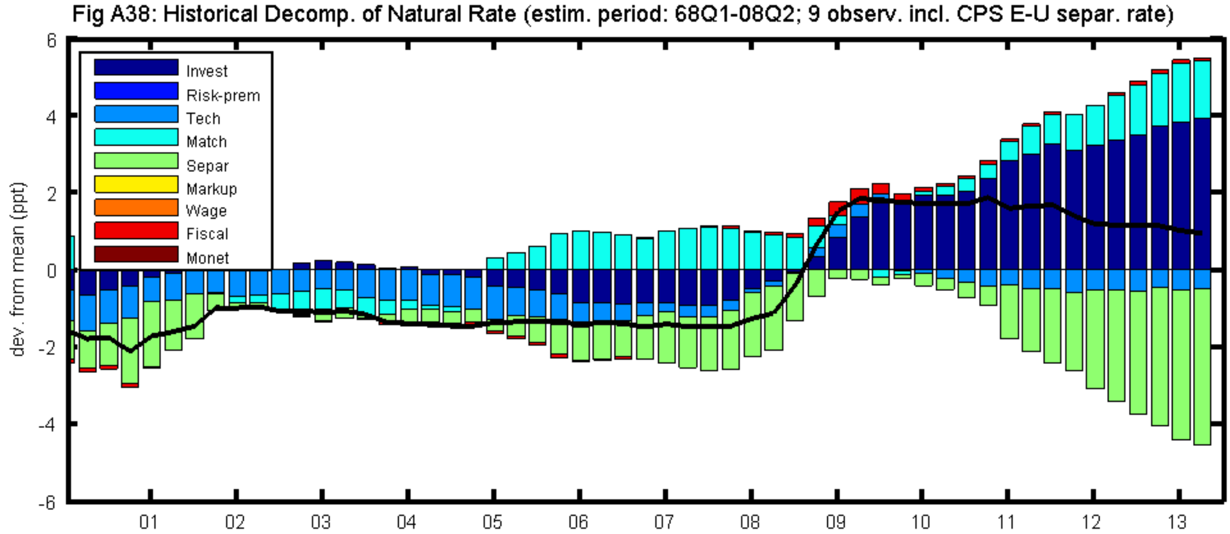


Fig A38: Historical Decomp. of Natural Rate (estim. period: 68Q1-08Q2; 9 observ. incl. CPS E-U separ. rate)



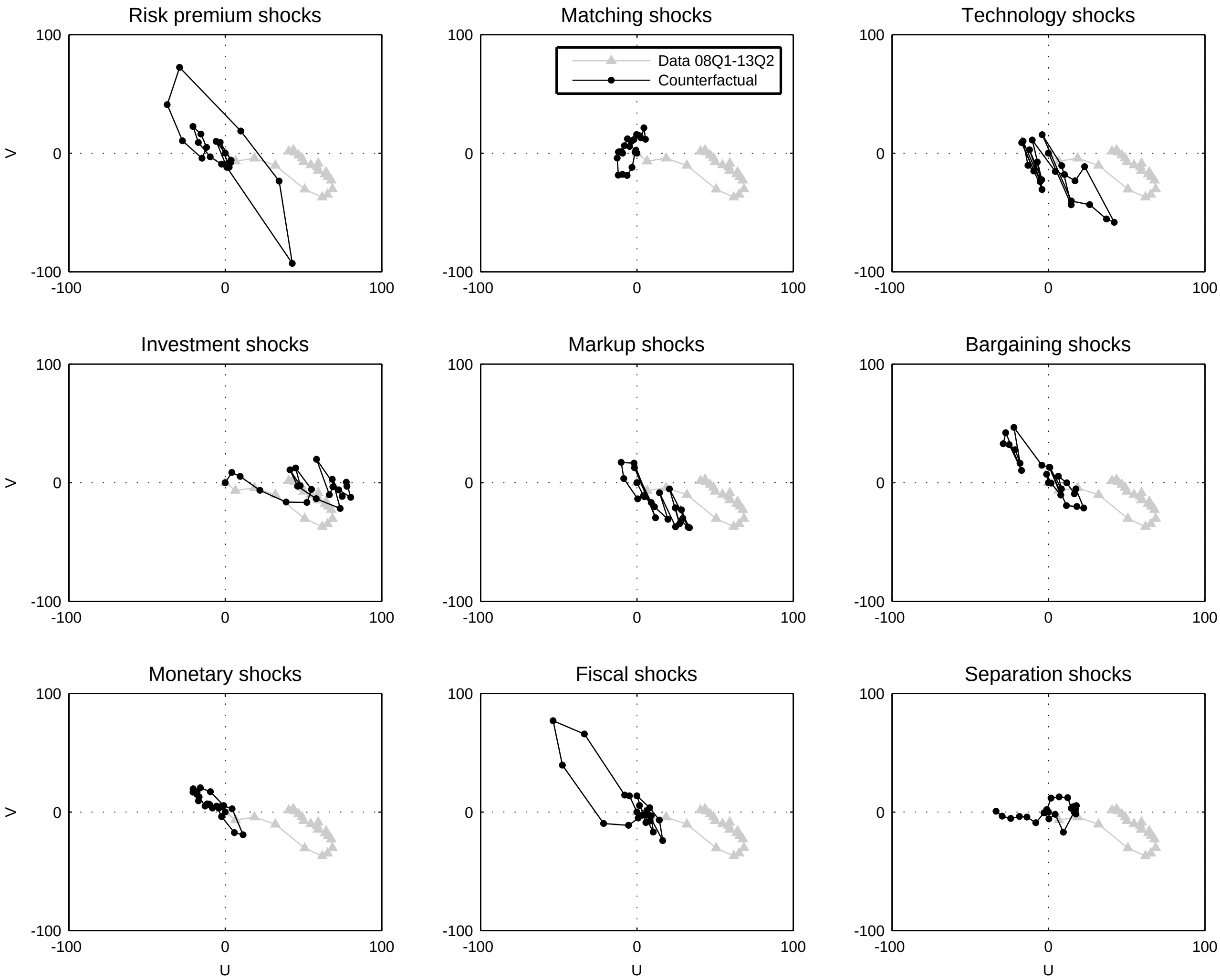


Figure A39: Conditional Beveridge curves. Model with both matching and separation shocks estimated with 9 observables over 1968:Q1 - 2008:Q3.

Check #7 - Model with 9 shocks incl. match. and separ. shock

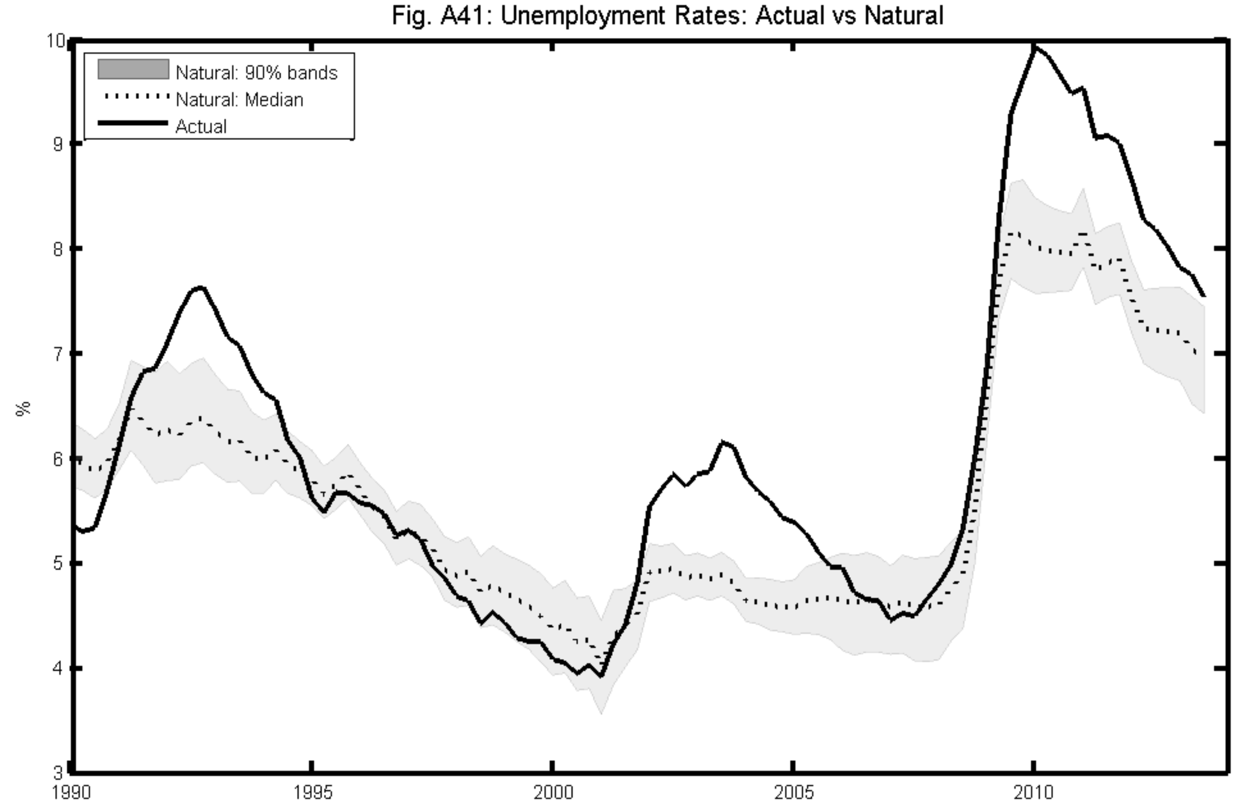
- 9 Observables including CPS 'E-U' separation rate over 1968:Q1-2008:Q3.
- Sep. rate follows: $\ln \rho_t = (1-\rho_\rho)\ln\rho + \rho_\rho \ln\rho_{t-1} + \varepsilon_{\rho t}$.

Capital depreciation rate	δ	0.0250
Capital share	α	0.33
Elasticity of substitution btw goods	θ	6
Backward-looking price setting	ς	0.01
Replacement rate	τ	0.40
Job destruction rate	ρ	0.085
Elasticity of matches to unemp.	σ	0.65
Probability to fill a vacancy within a quarter	q	0.70
Exogenous spending/output ratio	g/y	0.20
Unemployment rate	U	0.0599
Quarterly gross growth rate	z	1.0033
Quarterly gross inflation rate	π	1.0099
Quarterly gross nominal interest rate	R	1.0158

		Priors	Post. Mode
Weight of pre-match cost in total hiring cost	ϕ_V	Beta (0.5,0.2)	0.19
Hiring cost/output ratio	$1000\frac{\kappa}{2}\aleph^2$	IGamma (5,1)	2.87
Habit in consump.	h	Beta (0.7,0.1)	0.65
Invest. adj. cost	ϕ_I	IGamma (5,1)	3.30
Capital ut. cost	ϕ_{u2}	IGamma (0.5,0.1)	0.59
Price adjust. cost	ϕ_P	IGamma (60,10)	60.53
Wage adjust. cost	ϕ_W	IGamma (150,25)	107.06
Wage indexation	ϱ	Beta (0.5,0.2)	0.95
Interest smoothing	ρ_r	Beta (0.7,0.1)	0.26
Resp. to inflation	ρ_π	IGamma (1.5,0.1)	2.12
Resp. to growth	ρ_y	IGamma (0.5,0.1)	0.44

Fig. A40: Matching Efficiency and Separation Rate

Fig. A41: Unemployment Rates: Actual vs Natural



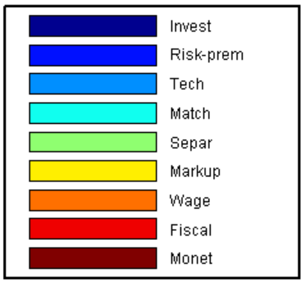


Fig A42: Beveridge Curve (estim. period: 68Q1-08Q3)

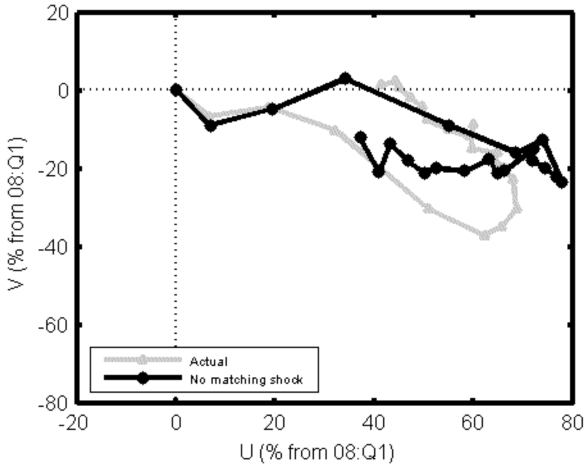


Fig A43: Historical Decomp. of Unemp. Rate (estim. period: 68Q1-08Q2; 9 observ. incl. CPS E-U separ. rate)

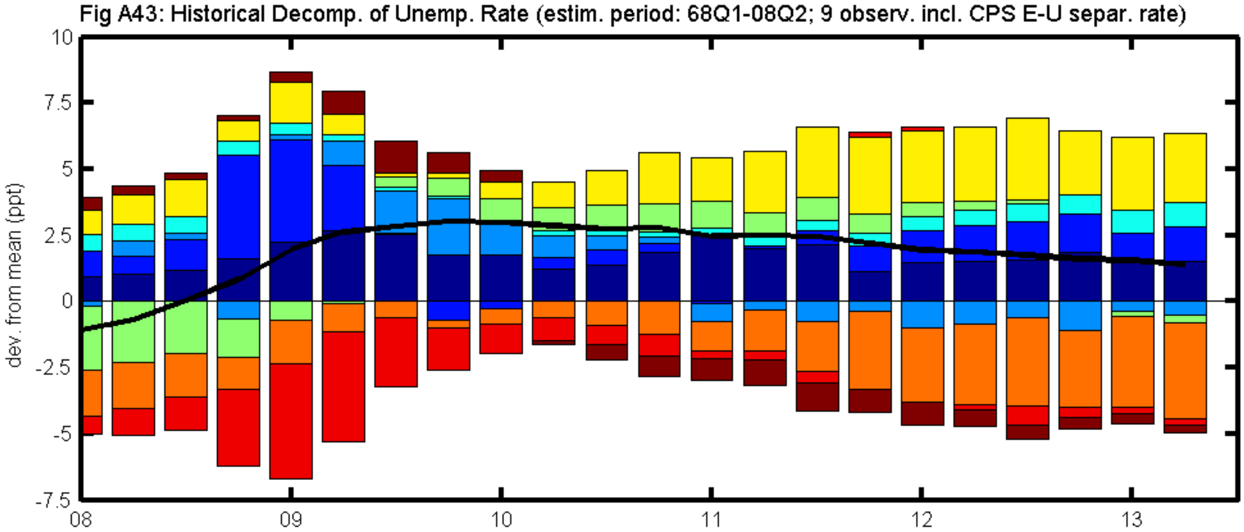
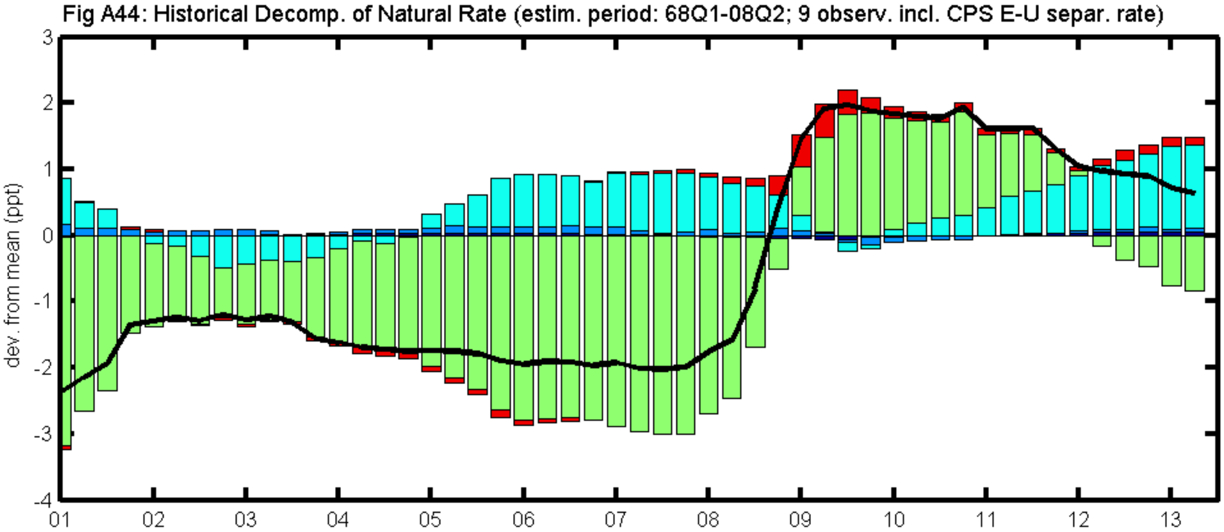


Fig A44: Historical Decomp. of Natural Rate (estim. period: 68Q1-08Q2; 9 observ. incl. CPS E-U separ. rate)



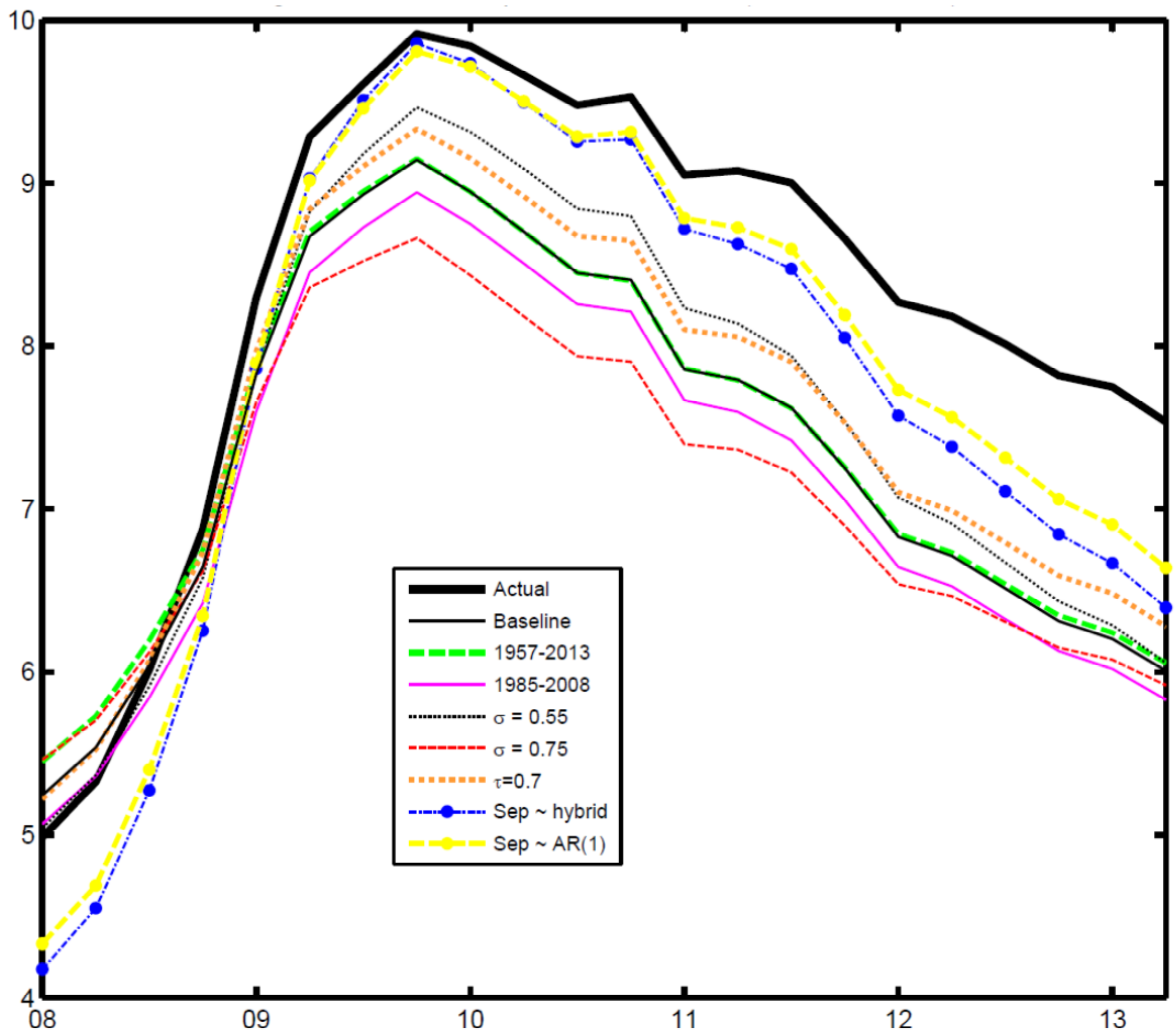


Figure A45: Actual unemployment rate (thick black line) versus counterfactual unemployment rates (when matching efficiency shocks are switched off between 2008:Q1 to 2013:Q2) for all robustness checks. Actual and counterfactual unemployment rates are expressed in percent of the labor force.