Do High-Frequency Data Improve High-Dimensional Portfolio Allocations?

Web Appendix

Nikolaus Hautsch Lada M. Kyj Peter Malec

July 2013

Contents

List of Figures

List of Tables

1 Analytical Solution for the Performance Fee

Consider the GMV framework (1*) and the preference structure (4*).[1] In addition, let

$$\overline{\mu_i^p} := \frac{1}{T-h} \sum_{t=1}^{T-h} \widehat{w}_{t,t+h}^{i\prime}\, \mu_{t,t+h},\ \overline{\mu_i^{2,p}} := \frac{1}{T-h} \sum_{t=1}^{T-h} \left(\widehat{w}_{t,t+h}^{i\prime}\, \mu_{t,t+h} \right)^2,\ i = \mathrm{I}, \mathrm{II}, \tag{1}$$

and $\vartheta := 2\,(1+\gamma)\,/\gamma$. Then, exploiting the fact that

$$\mathrm{E}\left[\left(r_{t,t+h}^{p,i} \right)^2 \middle| \mathcal{F}_t \right] = \widehat{w}_{t,t+h}^{i\prime}\, \Sigma_{t,t+h}\, \widehat{w}_{t,t+h}^{i} + \left(\widehat{w}_{t,t+h}^{i\prime}\, \mu_{t,t+h} \right)^2,\ i = \mathrm{I}, \mathrm{II}, \tag{2}$$

and using basic algebra, condition (5*) can be rearranged to

$$\Delta_\gamma^2 + \Delta_\gamma \left[\vartheta - 2\left(1 + \overline{\mu_{\mathrm{II}}^p}\right) \right] = (\vartheta - 2)\left(\overline{\mu_{\mathrm{II}}^p} - \overline{\mu_{\mathrm{I}}^p} \right) + \overline{\mu_{\mathrm{I}}^{2,p}} - \overline{\mu_{\mathrm{II}}^{2,p}} + \overline{\sigma_{\mathrm{I}}^{2,p}} - \overline{\sigma_{\mathrm{II}}^{2,p}}, \tag{3}$$

where $\overline{\sigma_i^{2,p}}$, $i = \mathrm{I}, \mathrm{II}$, is defined as in (6*). If we assume that $\mu_{t,t+h} = (h/252)\, \mu^{id}\, \iota$, $t = 1, \ldots, T-h$, (3) becomes

$$\Delta_\gamma^2 + \Delta_\gamma \left[\vartheta - 2\left(1 + \frac{h\, \mu^{id}}{252} \right) \right] = \overline{\sigma_{\mathrm{I}}^{2,p}} - \overline{\sigma_{\mathrm{II}}^{2,p}}, \tag{4}$$

yielding the solution

$$\Delta_\gamma = \frac{h\, \mu^{id}}{252} - \frac{1}{\gamma} + \sqrt{\left(\frac{h\, \mu^{id}}{252} - \frac{1}{\gamma} \right)^2 + \overline{\sigma_{\mathrm{I}}^{2,p}} - \overline{\sigma_{\mathrm{II}}^{2,p}}}, \tag{5}$$

which, under the assumption that $(h/252)\, \mu^{id} \leq 1/\gamma$, is strictly positive only if $\overline{\sigma_{\mathrm{I}}^{2,p}} > \overline{\sigma_{\mathrm{II}}^{2,p}}$.

[1]Numbers marked by an asterisk refer to equations in the paper.

2 Eigenvalue Cleaning

Eigenvalue cleaning is a regularization technique proposed by Laloux et al. (1999) and further developed by Tola et al. (2008) that draws upon random matrix theory to determine the distribution of the eigenvalues of a correlation matrix estimate R depending on the ratio of n observations and m dimensions, $q := n/m$. The idea is to compare empirical correlation eigenvalues with those implied by the null hypothesis of independent Gaussian asset returns, which allows for an identification of those eigenvalues that deviate from the "noisy" ones and hence constitute "signals".

Denote by $\Lambda := \text{diag}(\lambda_1, \ldots, \lambda_m)$ the diagonal matrix of eigenvalues of R ordered from largest to smallest and by Q the matrix of corresponding eigenvectors, yielding the spectral decomposition $R = Q\,\Lambda\,Q'$. For $n \to \infty$, under the null hypothesis R is given by the identity matrix implying that all eigenvalues are equal to one. However, if $m, n \to \infty$ with $q \geq 1$ fixed, the eigenvalues of R follow a Marchenko–Pastur distribution with maximum eigenvalue $\lambda_{\text{max}} := \left(1 + 1/q + 2\sqrt{1/q}\right)$. Hautsch et al. (2012) argue that, for practical purposes, the above threshold should be tightened to $\lambda^*_{\text{max}} := (1 - \lambda_1/m)\left(1 + 1/q + 2\sqrt{1/q}\right)$. This adjustment allows for a better identification of smaller signals, as it accounts for the fact that the largest empirical eigenvalue λ_1 often is associated with a dominating "market factor". Then, eigenvalue cleaning requires that all eigenvalues below λ^*_{max} are transformed according to

$$\tilde{\lambda}_i := \begin{cases} \lambda_i & \text{if} \quad \lambda_i \geq \lambda^*_{\text{max}}, \\ \delta & \text{otherwise}, \end{cases} \tag{6}$$

where δ is the average of the positive parts of all "noisy" eigenvalues, i.e.

$$\delta := \frac{\sum_{(\lambda_i < \lambda^*_{\text{max}})} \lambda_i^+}{\left(\#\text{ of } \lambda_i < \lambda^*_{\text{max}}\right)}. \tag{7}$$

Finally, the cleaned correlation matrix estimate is obtained as $\tilde{R} = Q\,\tilde{\Lambda}\,Q'$, where $\tilde{\Lambda} := \text{diag}\big(\tilde{\lambda}_i\big)$, $i = 1, \ldots, m$. We apply the procedure to (smoothed) correlation matrix estimates based on the blocked realized kernel, $R^{BRK}_{t,S}$, by setting the number of observations n equal to the minimum number of refresh times in any block averaged over the smoothing window. For the regularization of the rolling window sample covariance of daily returns, C_t, we apply eigenvalue cleaning to the corresponding sample correlation matrix R^c_t with n equal to the window length L.

3 Selection of the Number of Factors

To select the number of factors for the regularization approach discussed in Section 3.1 of the paper, we employ the criteria by Bai and Ng (2002) developed for linear factor models with m assets and n observations. In the context of smoothed BRK estimates, we consider a factor model defined in refresh time. Let $r^{(i)}_{t,S,l}$, $i = 1, \ldots, m$, denote the l-th refresh time return from days $t - S + 1$ to t. The resulting factor structure reads

$$r^{(i)}_{t,S,l} = \psi'_{t,S,i} \, F_{t,S,l} + \varepsilon^{(i)}_{t,S,l}, \ i = 1, \ldots, m, \ l = 1, \ldots, n_{t,S}, \tag{8}$$

where $F_{t,S,l}$ is the $(k_{t,S} \times 1)$ vector of common factors, $\psi_{t,S,i}$ denotes the corresponding vector of factor loadings and $\varepsilon^{(i)}_{t,S,l}$ is the idiosyncratic component of $r^{(i)}_{t,S,l}$, $i = 1, \ldots, m$. Following Bai and Ng (2002), we determine $k_{t,S}$ by employing the minima of the criteria

$$\begin{aligned} C^{m,1}_{t,S}(k_{t,S}) &= \hat{\sigma}^2_{t,S}(k_{t,S}) + k_{t,S} \, \hat{\sigma}^2_{t,S}(k_{\max}) \left(\frac{m + n_{t,S}}{m \, n_{t,S}} \right) \ln\left(\frac{m \, n_{t,S}}{m + n_{t,S}} \right), \\ C^{m,2}_{t,S}(k_{t,S}) &= \hat{\sigma}^2_{t,S}(k_{t,S}) + k_{t,S} \, \hat{\sigma}^2_{t,S}(k_{\max}) \left(\frac{m + n_{t,S}}{m \, n_{t,S}} \right) \ln\left[\min\left(\sqrt{m}, \sqrt{n_{t,S}}\right)^2 \right], \end{aligned} \tag{9}$$

where $\hat{\sigma}^2_{t,S}(k_{t,S}) := \frac{1}{m} \sum_{i=1}^{m} \hat{\sigma}^{2,(i)}_{t,S}(k_{t,S})$ with $\hat{\sigma}^{2,(i)}_{t,S}(k_{t,S})$ being an estimate of the residual variance $\mathrm{V}\big[\varepsilon^{(i)}_{t,S,l}\big]$, while $k_{\max}$ is the exogenously fixed maximum number of factors.

In practice, we let $n_{t,S}$ be the minimum number of refresh times in any block of the blocked realized kernel averaged over days $t - S + 1$ to t. Further, we set $\hat{\sigma}^{2,(i)}_{t,S}(k_{t,S})$ equal to the i-th diagonal element of $V^{RK}_{t,S} \left(I_m - \mathcal{Q}_{t,S,(k_{t,S})} \right) V^{RK}_{t,S}$, $i = 1, \ldots, m$, where $V^{RK}_{t,S}$ and $\mathcal{Q}_{t,S,(k_{t,S})}$ are defined as in (12*) and (14*), respectively. For the factor structure based on the rolling window sample covariance of daily returns in (20*), the number of observations is equal to the window length L. The factor residual variance is estimated by $\hat{\sigma}^2_t(k_t) := \frac{1}{m} \sum_{i=1}^{m} \hat{\sigma}^{2,(i)}_t(k_t)$, where $\hat{\sigma}^{2,(i)}_t(k_t)$ is the i-th diagonal element of $\left(V^c_t - \mathcal{Q}^c_{t,(k_t)} \right)$, $i = 1, \ldots, m$.

4 Cleaning Procedure for S&P 500 Quote Data

The raw dataset described in Section 4.1 of the paper is cleaned by performing the following steps:

1. Delete entries with negative bid-ask spreads.
2. Delete entries with non-positive bid or ask prices.
3. Delete entries with non-positive bid or ask sizes.
4. Delete entries with bid-ask spread greater than 1% of the current mid-quote.
5. Delete entries for which the mid-quote price is more than 5 times the median mid-quote on the given day.
6. Delete entries for which the mid-quote price deviated by more than 5 mean absolute deviations from a rolling median (excluding the observation under consideration) of 50 observations (25 observations before and 25 after).

A more detailed discussion of data filtering procedures can be found in Barndorff-Nielsen et al. (2008).

5 Details on Sensitivity Analysis and Robustness Checks

5.1 Number of Liquidity Groups

Table 5.1 reports (for daily horizons) the forecasting performance of non-smoothed BRK estimates regularized by eigenvalue cleaning (ERnB(1)) for different numbers of liquidity groups. Prior to the crisis, using four liquidity groups ($G = 4$) yields the lowest volatility. In this case, the choice of four liquidity groups seems to (empirically) balance the tradeoff between efficiency gains and the need of a tighter regularization. During the volatile crisis period, however, the effect of additional efficiency gains by increasing G seems to become more crucial. In this case, we observe the median portfolio volatility monotonically declining for rising G. Nonetheless, as soon as G exceeds four, the magnitude of additional reductions in portfolio volatility exhibits a noticeable decay and becomes smaller than one standard deviation. These results are in line with Hautsch et al. (2012) reporting that blocking-based efficiency gains are mainly due to a separation between liquid and illiquid assets which is ensured by a moderate number of liquidity groups.

Table 5.1: Number of Liquidity Groups G and GMV Portfolio Volatility of ERnB(1) Forecasts
Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights for the horizon $h = 1$ (in percentage points). Each random sample contains 350 assets out of the entire 400 asset universe. Evaluation is performed for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	Pre-Crisis		**Crisis**	
G	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$
1	8.38	0.28	14.43	0.11
2	8.25	0.29	14.25	0.11
4	7.49	0.07	14.02	0.11
5	8.15	0.30	13.98	0.11
8	8.13	0.30	13.94	0.11
10	8.12	0.30	13.93	0.11

5.2 Length of the Estimation Window

We investigate the impact of the (local) estimation window used for the sample covariance of daily returns on the median performance fees for switching to FRnB(5) and ERnB(252) forecasts, as well as on the corresponding median break-even transaction costs. Table 5.2 reports the results based on the crisis period. The corresponding analysis for the pre-crisis sample along with the complete results of the benchmark selection procedure outlined in Section 4.4 of the paper can be found in Section 6. For the low-volatility benchmarks, reducing the window length from 252 to, ultimately, 20 days implies a severe precision loss, as the median performance fees for switching to both FRnB(5) and ERnB(252) forecasts increase sharply. In these cases, the portfolio turnover of the LF benchmarks rises considerably making HF forecasts even more superior and thus leading to an increase in the median break-even transaction costs. A further lengthening of estimation windows to 378 days, however, causes only small additional reductions of median performance fees, thus indicating rather mild precision gains due to even longer local windows.

Reducing the local window length in case of the LF benchmark implying the smallest portfolio turnover (one-factor structure) yields lower median performance fees for switching to HF-based forecasts. This finding suggests that the loss of efficiency induced by a smaller observation window is outweighed by a higher responsiveness of forecasts induced by the use of more recent information. This is particularly true in case of a relatively tight regularization (as, e.g., induced by a one-factor structure), where the imposed structure itself limits the efficiency loss caused by shrinking local windows.

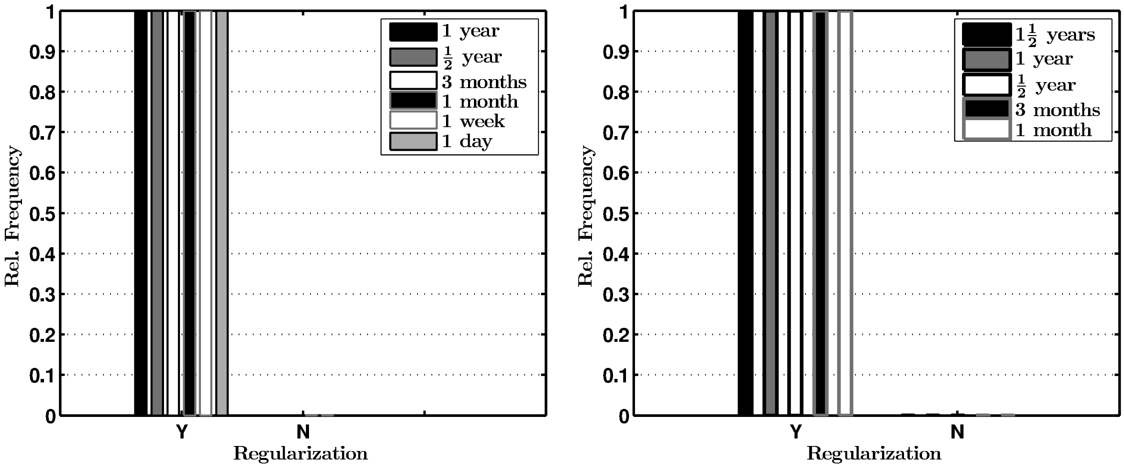
Notably, shortening the estimation window does not necessarily imply an *excessive* rise in portfolio turnover, as we observe decreasing median break-even transaction costs vis-a-vis FRnB(5) forecasts. However, when compared to the more severely smoothed ERnB(252) forecasts, break-even transaction costs increase or even become negative as long as the median performance fee is positive. In this situation, negative (median) break-even transaction costs reflect the higher (average) turnover induced by LF-based covariance forecasts in comparison to their long-term smoothed HF counterparts.

Table 5.2: Impact of Estimation Window on Basis Point Fees for Switching from Low-Frequency to High-Frequency Forecasts (Crisis)

Medians ($m(\cdot)$) across 1,000 random samples of the annualized basis point fee (Δ_{γ}^{a}) a risk-averse investor with quadratic utility and relative risk aversion γ would pay to switch from covariance forecasts using regularized sample covariances of daily data computed over different windows to high-frequency-based forecasts. We assume the constant conditional mean return being identical across all stocks and set it to $\mu^{id} = 0.05$ (annualized). Moreover, we report the break-even transaction costs (c_{γ}^{*}) in percentage points, defined as the ratio of Δ_{γ}^{a} and the difference of average portfolio turnovers. Each random sample contains 350 assets out of the entire 400 asset universe. Evaluation is conducted for the period including the crisis, 07/2008 to 12/2009. For each window length of the sample covariance, we consider the regularization yielding the lowest median realized portfolio volatility ("low volatility") or median portfolio turnover ("low turnover"). Low-volatility benchmarks: eigenvalue cleaning (378, 252 and 126 days), shrinkage towards single-factor model (63) and shrinkage towards equicorrelation model (20). Low-turnover benchmarks: imposing one-factor model (all windows). The entries corresponding to the 252-day window and the window yielding the smallest performance fee are marked in bold.

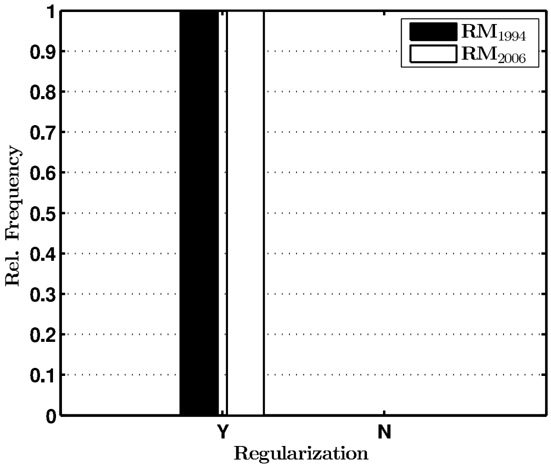
	$\mathbf{h=1}$				$\mathbf{h=5}$				$\mathbf{h=20}$			
Wind.	$m(\Delta_1^a)$	$m(c_1^*)$	$m(\Delta_{10}^a)$	$m(c_{10}^*)$	$m(\Delta_1^a)$	$m(c_1^*)$	$m(\Delta_{10}^a)$	$m(c_{10}^*)$	$m(\Delta_1^a)$	$m(c_1^*)$	$m(\Delta_{10}^a)$	$m(c_{10}^*)$
FRnB(5) **vs. low volatility (top) & low turnover benchmarks (bottom)**												
378	19.91	0.21	199.40	2.15	15.65	0.08	156.71	0.75	9.71	0.05	97.27	0.47
252	**19.91**	**0.23**	**199.39**	**2.33**	**15.85**	**0.08**	**158.70**	**0.83**	**9.84**	**0.05**	**98.54**	**0.55**
126	22.95	0.32	229.77	3.22	19.04	0.12	190.63	1.17	14.04	0.11	140.63	1.11
63	26.52	0.39	265.54	3.88	23.99	0.16	240.24	1.65	22.12	0.28	221.55	2.82
20	31.98	0.59	320.17	5.89	29.55	0.26	295.82	2.63	28.01	0.76	280.42	7.59
378	71.72	0.67	717.44	6.75	67.81	0.28	678.44	2.82	62.30	0.23	623.37	2.30
252	**69.13**	**0.65**	**691.55**	**6.53**	**65.65**	**0.27**	**656.84**	**2.75**	**62.01**	**0.23**	**620.43**	**2.33**
126	61.72	0.59	617.59	5.93	60.18	0.26	602.12	2.58	62.34	0.25	623.78	2.47
63	48.12	0.48	481.64	4.77	47.19	0.21	472.29	2.10	52.59	0.22	526.27	2.24
20	**40.39**	**0.47**	**404.35**	**4.73**	**37.58**	**0.20**	**376.17**	**2.00**	**38.84**	**0.23**	**388.82**	**2.32**
ERnB(252) **vs. low volatility (top) & low turnover benchmarks (bottom)**												
378	**-30.66**	**2.98**	**-307.32**	**29.90**	**-29.85**	**1.43**	**-299.22**	**14.30**	**-27.45**	**1.02**	**-275.13**	**10.20**
252	**-30.62**	**1.78**	**-306.96**	**17.80**	**-29.55**	**0.79**	**-296.15**	**7.89**	**-27.29**	**0.52**	**-273.52**	**5.16**
126	-27.70	0.88	-277.67	8.86	-26.54	0.40	-266.00	4.04	-23.28	0.22	-233.30	2.21
63	-24.01	0.70	-240.65	7.04	-21.55	0.26	-215.98	2.61	-15.15	0.10	-151.77	0.98
20	-18.59	0.38	-186.27	3.85	-15.85	0.14	-158.87	1.38	-9.19	0.05	-92.04	0.47
378	21.32	5.80	213.46	58.08	22.37	1.80	224.02	18.07	25.11	0.67	251.38	6.67
252	**18.65**	**5.96**	**186.78**	**59.64**	**20.25**	**1.89**	**202.74**	**18.91**	**24.86**	**0.74**	**248.91**	**7.37**
126	11.19	8.09	112.10	81.07	14.73	2.72	147.56	27.24	25.17	1.25	252.07	12.50
63	-2.37	1.35	-23.78	13.49	1.71	-0.57	17.12	-5.67	15.48	6.45	155.01	64.59
20	**-10.13**	**0.58**	**-101.53**	**5.81**	**-7.87**	**0.19**	**-78.87**	**1.91**	**1.74**	**-0.03**	**17.42**	**-0.27**

6 Additional Results for 400 Assets



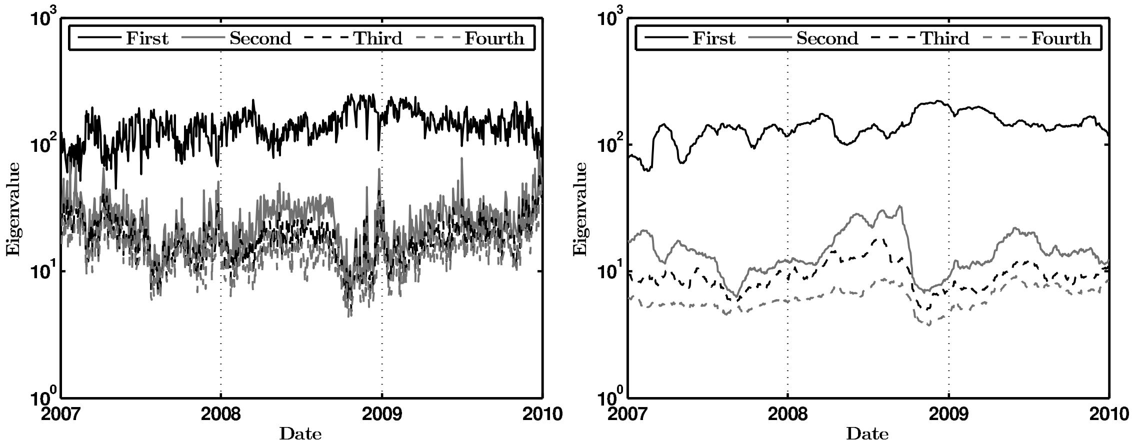
(a) BRK Estimates ($G = 4$)

(b) Rolling-Window Sample Covariance



(c) RiskMetrics

Figure 6.1: Regularization Frequencies
BRK estimates are regularized if any correlation eigenvalue is negative or the condition number of the correlation matrix is greater than 10×400. The rolling-window sample covariance of daily returns and RiskMetrics forecasts are regularized if the condition number of the corresponding correlation matrix is greater than the above threshold.



(a) Unsmoothed (b) Monthly Smoothing Window

Figure 6.2: Eigenvalues of BRK Correlation Matrix Estimates (Logarithmic Scale)

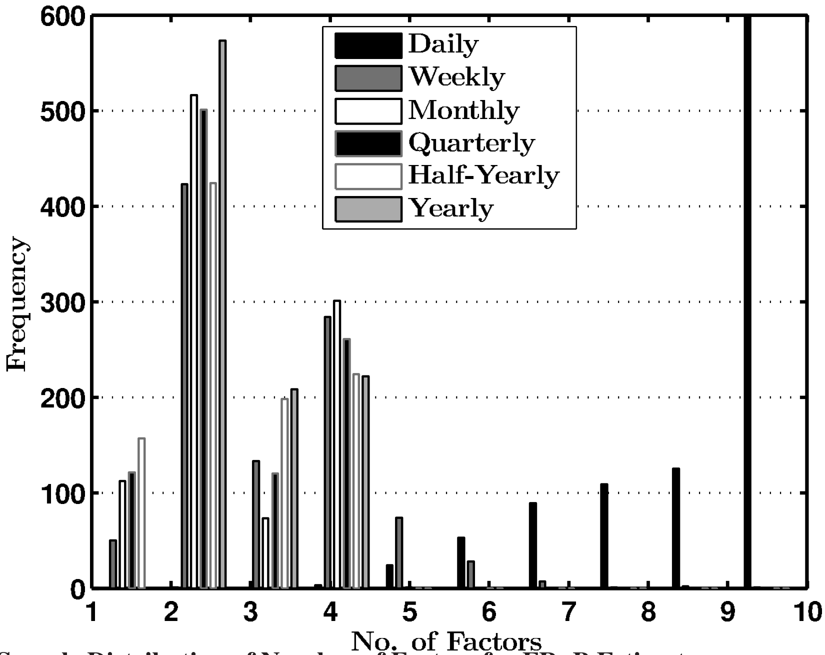
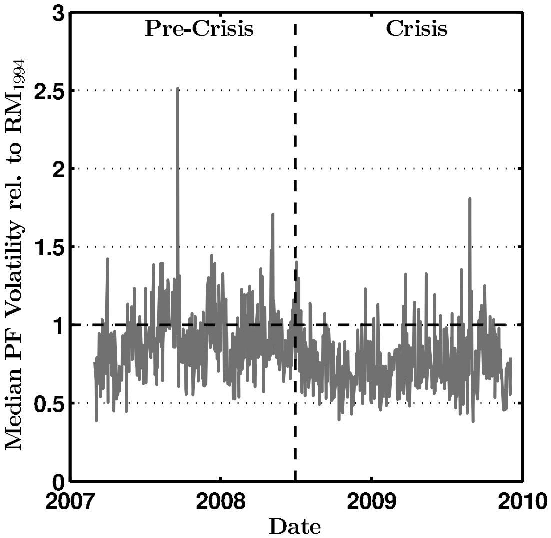
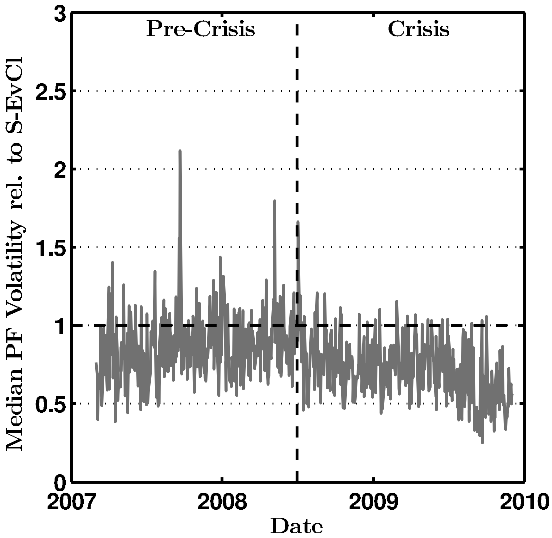


Figure 6.3: Sample Distribution of Number of Factors for FRnB Estimates
Number of factors is determined by applying the Bai and Ng (2002) criteria from Section 3 to BRK estimates smoothed over different windows.

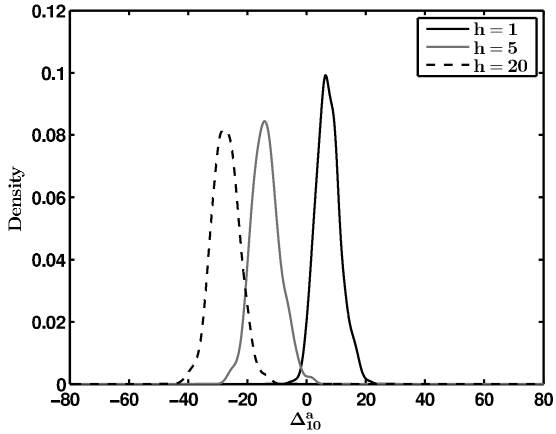


(a) Rel. to RM_{1994}

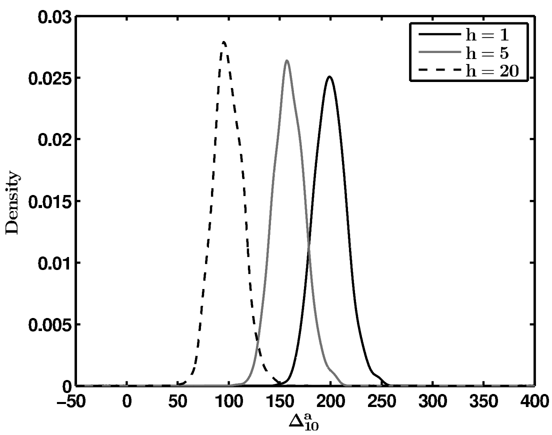


(b) Rel. to S-EvCl

Figure 6.4: Median Portfolio Volatility of CCHAR Forecasts Relative to Benchmarks ($\mathbf{h = 1}$)
Time series of ratios $m(\sigma_{t,t+1}^{p,\text{CCHAR}})/m(\sigma_{t,t+1}^{p,\text{bench}})$, where $\sigma_{t,t+1}^{p}$ is the square root of the realized portfolio variance in (24*) computed for $h = 1$. $m(\cdot)$ denotes the median across 1,000 random samples with each random sample containing 350 assets out of the entire 400 asset universe.



(a) Pre-Crisis: $\text{FRnB}(5)$ vs. RM_{1994}



(b) Crisis: $\text{FRnB}(5)$ vs. S-EvCl

Figure 6.5: Kernel Estimates of Performance Fee Density for Switching to FRnB(5) Forecasts
Kernel density estimates across 1,000 random samples of the annualized basis point fee (Δ_{γ}^{a}) a risk-averse investor with quadratic utility and relative risk aversion $\gamma = 10$ would pay to switch from covariance forecasts using daily data to high-frequency-based forecasts. Each random sample contains 350 assets out of the entire 400 asset universe. The assumed constant conditional mean return is identical across all stocks and set to $\mu^{id} = 0.05$ (annualized). Density estimates are based on the Gaussian kernel and the rule-of-thumb bandwidth with normal reference.

Table 6.1: GMV Portfolio Performance of FRnB Forecasts

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 350 assets out of the entire 400 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
FRnB(1)	156.59	5422.90	2558.26	-6.37	2.47	172.30	4857.46	541.36	-6.37	2.47	180.64	467.77	138.90	-6.37	2.47
FRnB(5)	7.62	0.07	84.16	-0.89	0.23	7.98	0.08	39.68	-0.89	0.23	8.38	0.01	12.20	-0.89	0.23
FRnB(20)	8.72	0.10	30.05	-0.95	0.24	8.81	0.10	16.77	-0.95	0.24	9.13	0.01	9.54	-0.95	0.24
FRnB(63)	9.43	0.11	11.77	-1.08	0.25	9.53	0.11	7.89	-1.08	0.25	9.88	0.01	5.49	-1.08	0.25
FRnB(126)	10.24	0.12	7.44	-1.10	0.25	10.32	0.12	4.90	-1.10	0.25	10.64	0.01	3.59	-1.10	0.25
FRnB(252)	11.29	0.14	6.43	-1.05	0.23	11.38	0.14	3.55	-1.05	0.23	11.75	0.01	2.63	-1.05	0.23
Crisis															
FRnB(1)	486.51	21433.67	5261.36	-12.74	4.47	511.83	21325.70	1034.49	-12.74	4.47	519.58	1867.49	264.31	-12.74	4.47
FRnB(5)	13.54	0.11	111.82	-1.08	0.27	13.99	0.11	50.74	-1.08	0.27	14.68	0.01	15.06	-1.08	0.27
FRnB(20)	14.46	0.13	36.90	-1.19	0.28	14.63	0.13	20.11	-1.19	0.28	15.13	0.01	11.94	-1.19	0.28
FRnB(63)	15.50	0.15	18.53	-1.31	0.29	15.59	0.15	11.02	-1.31	0.29	15.90	0.01	7.76	-1.31	0.29
FRnB(126)	16.20	0.18	13.90	-1.50	0.32	16.27	0.18	8.12	-1.50	0.32	16.50	0.02	5.27	-1.50	0.32
FRnB(252)	17.13	0.19	10.13	-1.59	0.34	17.21	0.19	5.36	-1.59	0.34	17.30	0.02	3.41	-1.59	0.34

Table 6.2: GMV Portfolio Performance of 3FRnB and 1FRnB Forecasts

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 350 assets out of the entire 400 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
3FRnB(1)	39.16	1507.31	484.87	-1.33	0.62	39.74	1190.52	96.54	-1.33	0.62	44.27	146.28	26.61	-1.33	0.62
3FRnB(5)	7.78	0.08	76.81	-0.90	0.23	8.08	0.09	38.07	-0.90	0.23	8.46	0.01	11.62	-0.90	0.23
3FRnB(20)	8.41	0.10	25.63	-1.02	0.25	8.54	0.10	15.06	-1.02	0.25	8.85	0.01	9.23	-1.02	0.25
3FRnB(63)	9.15	0.11	10.53	-1.13	0.26	9.22	0.11	6.52	-1.13	0.26	9.49	0.01	4.63	-1.13	0.26
3FRnB(126)	9.74	0.12	6.93	-1.18	0.26	9.81	0.12	4.16	-1.18	0.26	10.09	0.01	3.02	-1.18	0.26
3FRnB(252)	10.19	0.13	5.57	-1.17	0.26	10.26	0.13	3.13	-1.17	0.26	10.60	0.01	2.23	-1.17	0.26
1FRnB(1)	14.37	673.38	184.73	-0.61	0.23	12.75	540.84	40.68	-0.61	0.23	13.48	56.65	10.60	-0.61	0.23
1FRnB(5)	8.98	0.10	47.93	-0.70	0.20	9.18	0.10	25.76	-0.70	0.20	9.57	0.01	8.71	-0.70	0.20
1FRnB(20)	9.57	0.11	16.89	-0.78	0.21	9.66	0.11	10.91	-0.78	0.21	10.05	0.01	7.17	-0.78	0.21
1FRnB(63)	10.12	0.12	7.62	-0.89	0.22	10.18	0.12	4.96	-0.89	0.22	10.51	0.01	3.71	-0.89	0.22
1FRnB(126)	10.57	0.12	5.47	-0.94	0.22	10.62	0.12	3.47	-0.94	0.22	10.97	0.01	2.72	-0.94	0.22
1FRnB(252)	11.16	0.13	4.44	-0.95	0.22	11.25	0.13	2.60	-0.95	0.22	11.72	0.01	1.96	-0.95	0.22
Crisis															
3FRnB(1)	62.40	3535.05	521.49	-1.42	0.67	67.93	3682.46	111.85	-1.42	0.67	68.28	359.74	28.88	-1.42	0.67
3FRnB(5)	13.86	0.11	101.90	-1.06	0.26	14.28	0.11	48.39	-1.06	0.26	14.92	0.01	14.47	-1.06	0.26
3FRnB(20)	14.82	0.14	36.57	-1.26	0.29	14.98	0.14	20.11	-1.26	0.29	15.38	0.01	11.89	-1.26	0.29
3FRnB(63)	15.82	0.17	16.21	-1.39	0.31	15.91	0.16	9.34	-1.39	0.31	16.16	0.02	6.33	-1.39	0.31
3FRnB(126)	16.56	0.19	11.85	-1.48	0.32	16.62	0.19	6.77	-1.48	0.32	16.75	0.02	4.58	-1.48	0.32
3FRnB(252)	17.36	0.20	8.73	-1.56	0.33	17.41	0.20	4.55	-1.56	0.33	17.46	0.02	2.87	-1.56	0.33
1FRnB(1)	31.46	3359.64	190.62	-0.68	0.33	38.58	4499.48	35.19	-0.68	0.33	37.96	432.67	9.97	-0.68	0.33
1FRnB(5)	16.33	0.14	32.08	-0.56	0.18	16.53	0.14	17.72	-0.56	0.18	16.90	0.01	6.38	-0.56	0.18
1FRnB(20)	17.12	0.15	11.64	-0.60	0.17	17.22	0.15	7.43	-0.60	0.17	17.65	0.02	4.98	-0.60	0.17
1FRnB(63)	18.19	0.17	6.79	-0.65	0.18	18.34	0.17	4.09	-0.65	0.18	18.95	0.02	2.99	-0.65	0.18
1FRnB(126)	19.20	0.19	6.42	-0.74	0.20	19.33	0.19	3.63	-0.74	0.20	19.79	0.02	2.57	-0.74	0.20
1FRnB(252)	20.40	0.21	6.44	-0.87	0.22	20.46	0.21	3.24	-0.87	0.22	20.57	0.02	2.11	-0.87	0.22

Table 6.3: Basis Point Fees for Switching from Low-Frequency to High-Frequency Covariance Matrix Forecasts ($\mu^{\mathbf{id}} = -0.05$)

Medians ($m(\cdot)$) across 1,000 random samples of annualized basis point fees (Δ^a_γ) a risk-averse investor with quadratic utility and relative risk aversion γ would pay to switch from covariance forecasts using daily data to high-frequency-based forecasts. We assume that the constant conditional mean return is identical across all stocks and set it to $\mu^{id} = -0.05$ (annualized). Also reported are break-even transaction costs (c^*_γ) in percentage points, defined as the ratio of Δ^a_γ and the difference of average portfolio turnovers. Each random sample contains 350 assets out of the entire 400 asset universe. Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009. For each length of the smoothing window applied to BRK estimates, the regularization method that minimizes median realized portfolio volatility is chosen. CCHAR is based on unsmoothed ERnB estimates. The low-frequency benchmarks are the RiskMetrics1994 estimator (RM_{1994}) as well as the sample covariance computed over 252 days regularized by eigenvalue cleaning (S-EvCl) and by imposing a one factor model (1F).

	$\mathbf{h = 1}$				$\mathbf{h = 5}$				$\mathbf{h = 20}$			
	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$
Pre-Crisis: vs. $\mathbf{RM_{1994}}$ (top) & 1F (bottom)												
ERnB(1)	1.68	0.01	16.78	0.10	-0.64	-0.00	-6.40	-0.04	-4.85	-0.09	-48.38	-0.91
FRnB(5)	0.70	0.01	7.03	0.10	-1.40	-0.01	-13.94	-0.15	-2.73	-0.05	-27.25	-0.53
3FRnB(20)	-5.66	0.68	-56.52	6.84	-6.04	0.45	-60.34	4.51	-6.84	1.69	-68.25	16.88
3FRnB(63)	-12.16	0.54	-121.45	5.38	-12.08	0.23	-120.58	2.26	-12.71	0.14	-126.91	1.43
3FRnB(126)	-17.66	0.67	-176.39	6.72	-17.59	0.27	-175.70	2.69	-18.49	0.15	-184.60	1.54
3FRnB(252)	-22.17	0.80	-221.40	7.97	-22.16	0.31	-221.30	3.12	-23.79	0.18	-237.55	1.75
CCHAR	4.01	0.18	40.06	1.78	2.19	0.72	21.86	7.20	-0.81	0.01	-8.10	0.11
ERnB(1)	16.76	0.08	167.25	0.83	14.43	0.07	143.99	0.69	11.71	0.06	116.86	0.58
FRnB(5)	15.78	0.20	157.43	2.00	13.68	0.07	136.56	0.74	13.80	0.07	137.67	0.68
3FRnB(20)	9.42	0.45	94.04	4.51	9.05	0.14	90.35	1.41	9.69	0.07	96.70	0.65
3FRnB(63)	2.94	0.46	29.34	4.60	3.01	0.12	30.00	1.24	3.81	0.06	37.98	0.61
3FRnB(126)	-2.59	-5.71	-25.86	-57.03	-2.54	-1.01	-25.34	-10.12	-1.99	-0.15	-19.84	-1.48
3FRnB(252)	-7.09	12.01	-70.80	119.90	-7.09	5.96	-70.76	59.52	-7.26	-2.52	-72.53	-25.16
CCHAR	19.08	0.38	190.36	3.75	17.25	0.22	172.13	2.24	15.71	0.26	156.74	2.64
Crisis: vs. S-EvCl (top) & 1F (bottom)												
ERnB(1)	13.20	0.07	131.72	0.69	6.94	0.04	69.24	0.40	-4.12	-0.03	-41.14	-0.31
FRnB(5)	19.90	0.23	198.60	2.32	15.84	0.08	158.08	0.83	9.83	0.05	98.15	0.55
FRnB(20)	6.88	0.66	68.63	6.56	6.68	0.18	66.68	1.77	3.04	0.03	30.35	0.26
FRnB(63)	-8.70	1.11	-86.86	11.11	-7.93	1.00	-79.19	10.00	-8.89	-0.26	-88.79	-2.60
FRnB(126)	-19.76	1.58	-197.32	15.83	-18.72	0.83	-186.93	8.32	-18.56	1.17	-185.34	11.70
ERnB(252)	-30.61	1.77	-305.75	17.73	-29.53	0.79	-294.98	7.86	-27.28	0.51	-272.44	5.14
CCHAR	32.72	0.86	326.43	8.56	29.54	0.47	294.68	4.73	23.73	0.53	236.75	5.27
ERnB(1)	62.44	0.30	622.52	2.97	56.70	0.26	565.35	2.59	48.09	0.22	479.54	2.17
FRnB(5)	69.10	0.65	688.82	6.50	65.63	0.27	654.24	2.74	61.98	0.23	617.97	2.32
FRnB(20)	56.05	1.81	558.92	18.07	56.34	0.66	561.81	6.55	55.18	0.27	550.22	2.69
FRnB(63)	40.47	3.23	403.65	32.21	41.89	1.04	417.77	10.33	43.21	0.36	431.01	3.57
FRnB(126)	29.43	3.72	293.58	37.14	31.03	1.20	309.61	11.93	33.50	0.47	334.14	4.72
ERnB(252)	18.64	5.95	186.05	59.40	20.24	1.89	201.93	18.84	24.85	0.74	247.93	7.34
CCHAR	81.87	1.40	815.92	13.93	79.35	0.72	790.85	7.15	75.90	0.58	756.54	5.75

Table 6.4: Basis Point Fees for Switching from Low-Frequency to High-Frequency Covariance Matrix Forecasts ($\mu^{\mathbf{id}} = \mathbf{0}$)
Medians ($m(\cdot)$) across 1,000 random samples of annualized basis point fees (Δ^a_γ) a risk-averse investor with quadratic utility and relative risk aversion γ would pay to switch from covariance forecasts using daily data to high-frequency-based forecasts. We assume that the constant conditional mean return is identical across all stocks and set it to $\mu^{id} = 0$ (annualized). Also reported are break-even transaction costs (c^*_γ) in percentage points, defined as the ratio of Δ^a_γ and the difference of average portfolio turnovers. Each random sample contains 350 assets out of the entire 400 asset universe. Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009. For each length of the smoothing window applied to BRK estimates, the regularization method that minimizes median realized portfolio volatility is chosen. CCHAR is based on unsmoothed ERnB estimates. The low-frequency benchmarks are the RiskMetrics1994 estimator (RM_{1994}) as well as the sample covariance computed over 252 days regularized by eigenvalue cleaning (S-EvCl) and by imposing a one factor model (1F).

	$\mathbf{h = 1}$				$\mathbf{h = 5}$				$\mathbf{h = 20}$			
	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$
Pre-Crisis: vs. $\mathbf{RM_{1994}}$ (top) & 1F (bottom)												
ERnB(1)	1.68	0.01	16.82	0.10	-0.64	-0.00	-6.41	-0.04	-4.85	-0.09	-48.47	-0.91
FRnB(5)	0.70	0.01	7.05	0.10	-1.40	-0.01	-13.97	-0.15	-2.73	-0.05	-27.31	-0.53
3FRnB(20)	-5.66	0.69	-56.63	6.85	-6.05	0.45	-60.46	4.52	-6.84	1.69	-68.39	16.91
3FRnB(63)	-12.17	0.54	-121.70	5.39	-12.08	0.23	-120.81	2.27	-12.71	0.14	-127.16	1.43
3FRnB(126)	-17.67	0.67	-176.74	6.73	-17.60	0.27	-176.05	2.69	-18.49	0.15	-184.96	1.54
3FRnB(252)	-22.17	0.80	-221.84	7.99	-22.16	0.31	-221.74	3.13	-23.79	0.18	-238.02	1.76
CCHAR	4.01	0.18	40.14	1.79	2.19	0.72	21.91	7.22	-0.81	0.01	-8.12	0.11
ERnB(1)	16.76	0.08	167.58	0.83	14.43	0.07	144.28	0.69	11.71	0.06	117.10	0.59
FRnB(5)	15.78	0.20	157.74	2.01	13.69	0.07	136.83	0.74	13.80	0.07	137.94	0.68
3FRnB(20)	9.42	0.45	94.23	4.52	9.05	0.14	90.53	1.41	9.69	0.07	96.89	0.65
3FRnB(63)	2.94	0.46	29.40	4.61	3.01	0.12	30.06	1.24	3.81	0.06	38.06	0.61
3FRnB(126)	-2.59	-5.71	-25.92	-57.14	-2.54	-1.01	-25.39	-10.14	-1.99	-0.15	-19.88	-1.48
3FRnB(252)	-7.09	12.01	-70.94	120.14	-7.09	5.96	-70.90	59.64	-7.27	-2.52	-72.67	-25.21
CCHAR	19.08	0.38	190.74	3.76	17.25	0.22	172.47	2.24	15.71	0.26	157.05	2.64
Crisis: vs. S-EvCl (top) & 1F (bottom)												
ERnB(1)	13.20	0.07	131.98	0.70	6.94	0.04	69.37	0.41	-4.12	-0.03	-41.22	-0.31
FRnB(5)	19.91	0.23	198.99	2.32	15.84	0.08	158.39	0.83	9.84	0.05	98.34	0.55
FRnB(20)	6.88	0.66	68.76	6.57	6.68	0.18	66.82	1.77	3.04	0.03	30.41	0.26
FRnB(63)	-8.70	1.11	-87.03	11.13	-7.93	1.00	-79.35	10.02	-8.89	-0.26	-88.96	-2.61
FRnB(126)	-19.76	1.59	-197.71	15.86	-18.72	0.83	-187.30	8.34	-18.56	1.17	-185.70	11.73
ERnB(252)	-30.62	1.78	-306.35	17.76	-29.54	0.79	-295.56	7.87	-27.28	0.51	-272.98	5.15
CCHAR	32.73	0.86	327.08	8.58	29.54	0.47	295.26	4.74	23.73	0.53	237.22	5.28
ERnB(1)	62.45	0.30	623.75	2.97	56.71	0.26	566.47	2.59	48.09	0.22	480.49	2.18
FRnB(5)	69.11	0.65	690.18	6.51	65.64	0.27	655.54	2.74	62.00	0.23	619.20	2.32
FRnB(20)	56.06	1.81	560.03	18.11	56.35	0.66	562.92	6.56	55.19	0.27	551.31	2.70
FRnB(63)	40.48	3.23	404.45	32.27	41.89	1.04	418.60	10.35	43.22	0.36	431.86	3.58
FRnB(126)	29.43	3.72	294.16	37.21	31.04	1.20	310.22	11.96	33.50	0.47	334.80	4.73
ERnB(252)	18.65	5.95	186.41	59.52	20.24	1.89	202.33	18.88	24.85	0.74	248.42	7.35
CCHAR	81.88	1.40	817.53	13.95	79.36	0.72	792.41	7.16	75.92	0.58	758.03	5.76

Table 6.5: Basis Point Fees for Switching from Low-Frequency to High-Frequency Covariance Matrix Forecasts ($\mu^{\mathbf{id}} = \mathbf{0.1}$)
Medians ($m(\cdot)$) across 1,000 random samples of annualized basis point fees (Δ^a_γ) a risk-averse investor with quadratic utility and relative risk aversion γ would pay to switch from covariance forecasts using daily data to high-frequency-based forecasts. We assume that the constant conditional mean return is identical across all stocks and set it to $\mu^{id} = 0.1$ (annualized). Also reported are break-even transaction costs (c^*_γ) in percentage points, defined as the ratio of Δ^a_γ and the difference of average portfolio turnovers. Each random sample contains 350 assets out of the entire 400 asset universe. Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009. For each length of the smoothing window applied to BRK estimates, the regularization method that minimizes median realized portfolio volatility is chosen. CCHAR is based on unsmoothed ERnB estimates. The low-frequency benchmarks are the RiskMetrics1994 estimator (RM_{1994}) as well as the sample covariance computed over 252 days regularized by eigenvalue cleaning (S-EvCl) and by imposing a one factor model (1F).

	$\mathbf{h=1}$				$\mathbf{h=5}$				$\mathbf{h=20}$			
	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$
Pre-Crisis: vs. RM_{1994} (top) & 1F (bottom)												
ERnB(1)	1.68	0.01	16.88	0.10	-0.64	-0.00	-6.44	-0.04	-4.85	-0.09	-48.67	-0.92
FRnB(5)	0.70	0.01	7.07	0.10	-1.40	-0.01	-14.03	-0.15	-2.73	-0.05	-27.42	-0.53
3FRnB(20)	-5.66	0.69	-56.85	6.88	-6.05	0.45	-60.71	4.53	-6.84	1.69	-68.66	16.98
3FRnB(63)	-12.17	0.54	-122.18	5.42	-12.08	0.23	-121.30	2.27	-12.72	0.14	-127.67	1.43
3FRnB(126)	-17.68	0.67	-177.45	6.76	-17.61	0.27	-176.75	2.70	-18.50	0.15	-185.70	1.55
3FRnB(252)	-22.18	0.80	-222.72	8.02	-22.17	0.31	-222.62	3.14	-23.80	0.18	-238.97	1.76
CCHAR	4.02	0.18	40.30	1.79	2.19	0.72	21.99	7.24	-0.81	0.01	-8.15	0.11
ERnB(1)	16.77	0.08	168.25	0.83	14.44	0.07	144.85	0.69	11.72	0.06	117.56	0.59
FRnB(5)	15.79	0.20	158.37	2.01	13.69	0.07	137.37	0.74	13.80	0.07	138.49	0.69
3FRnB(20)	9.43	0.45	94.60	4.53	9.06	0.14	90.89	1.42	9.69	0.07	97.28	0.65
3FRnB(63)	2.94	0.46	29.52	4.62	3.01	0.12	30.18	1.25	3.81	0.06	38.21	0.62
3FRnB(126)	-2.59	-5.72	-26.02	-57.37	-2.54	-1.01	-25.49	-10.18	-1.99	-0.15	-19.96	-1.49
3FRnB(252)	-7.10	12.02	-71.22	120.62	-7.09	5.96	-71.18	59.87	-7.27	-2.52	-72.96	-25.31
CCHAR	19.09	0.38	191.50	3.77	17.26	0.22	173.15	2.25	15.72	0.26	157.68	2.66
Crisis: vs. S-EvCl (top) & 1F (bottom)												
ERnB(1)	13.21	0.07	132.51	0.70	6.94	0.04	69.65	0.41	-4.12	-0.03	-41.39	-0.31
FRnB(5)	19.92	0.23	199.79	2.33	15.85	0.08	159.02	0.83	9.84	0.05	98.74	0.55
FRnB(20)	6.88	0.66	69.04	6.60	6.69	0.18	67.08	1.78	3.04	0.03	30.53	0.26
FRnB(63)	-8.71	1.11	-87.38	11.18	-7.94	1.00	-79.67	10.06	-8.90	-0.26	-89.32	-2.62
FRnB(126)	-19.77	1.59	-198.50	15.92	-18.73	0.83	-188.05	8.37	-18.57	1.17	-186.44	11.77
ERnB(252)	-30.63	1.78	-307.57	17.83	-29.55	0.79	-296.74	7.90	-27.29	0.52	-274.07	5.17
CCHAR	32.74	0.86	328.38	8.62	29.55	0.47	296.43	4.76	23.74	0.53	238.17	5.30
ERnB(1)	62.48	0.30	626.23	2.99	56.73	0.26	568.72	2.60	48.11	0.22	482.40	2.19
FRnB(5)	69.14	0.65	692.92	6.54	65.66	0.27	658.14	2.75	62.02	0.23	621.66	2.33
FRnB(20)	56.09	1.81	562.25	18.18	56.38	0.66	565.16	6.59	55.21	0.27	553.50	2.71
FRnB(63)	40.49	3.23	406.06	32.40	41.91	1.04	420.26	10.39	43.24	0.36	433.58	3.59
FRnB(126)	29.44	3.72	295.33	37.36	31.05	1.20	311.45	12.00	33.52	0.47	336.13	4.75
ERnB(252)	18.66	5.96	187.16	59.76	20.25	1.89	203.14	18.95	24.86	0.74	249.41	7.38
CCHAR	81.92	1.40	820.78	14.01	79.40	0.72	795.56	7.19	75.95	0.58	761.04	5.78

Table 6.6: GMV Portfolio Performance of Regularized Rolling Window Sample Covariance Matrix of Daily Returns over 378 Days

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 350 assets out of the entire 400 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on the rolling window sample covariance matrix of daily returns over 378 days regularized by a one- or three-factor structure (1F or 3F), a factor structure based on the Bai and Ng (2002) criteria (BN-F), eigenvalue cleaning (S-EvCl) and shrinkage towards an equicorrelation or one-factor model (SHRK$_{EC}$ or SHRK$_{SF}$). Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
1F	10.63	0.11	4.93	-0.66	0.18	10.70	0.11	2.68	-0.66	0.18	11.12	0.01	1.84	-0.66	0.18
3F	10.47	0.13	7.80	-0.79	0.20	10.53	0.13	4.34	-0.79	0.20	10.81	0.01	2.67	-0.79	0.20
BN-F	9.27	0.11	10.75	-1.11	0.25	9.30	0.11	5.35	-1.11	0.25	9.49	0.01	3.00	-1.11	0.25
S-EvCl	8.59	0.09	20.12	-1.23	0.27	8.65	0.09	9.20	-1.23	0.27	8.90	0.01	4.71	-1.23	0.27
SHRK$_{EC}$	9.44	0.10	32.10	-1.92	0.36	9.50	0.10	16.10	-1.92	0.36	9.75	0.01	8.48	-1.92	0.36
SHRK$_{SF}$	8.80	0.09	24.21	-1.57	0.31	8.86	0.09	12.23	-1.57	0.31	9.12	0.01	6.32	-1.57	0.31
Crisis															
1F	18.07	0.17	5.44	-0.61	0.17	18.19	0.18	2.59	-0.61	0.17	18.43	0.02	1.53	-0.61	0.17
3F	16.93	0.15	11.39	-0.99	0.23	17.11	0.15	5.82	-0.99	0.23	17.33	0.02	3.48	-0.99	0.23
BN-F	15.67	0.14	14.58	-1.41	0.29	15.81	0.14	7.23	-1.41	0.29	16.03	0.01	3.61	-1.41	0.29
S-EvCl	14.93	0.14	19.39	-1.55	0.33	15.06	0.13	9.24	-1.55	0.33	15.33	0.01	4.77	-1.55	0.33
SHRK$_{EC}$	15.77	0.15	25.81	-1.98	0.38	15.92	0.15	12.91	-1.98	0.38	16.17	0.02	6.86	-1.98	0.38
SHRK$_{SF}$	15.92	0.15	29.46	-2.11	0.39	16.07	0.15	14.62	-2.11	0.39	16.37	0.02	7.67	-2.11	0.39

Table 6.7: GMV Portfolio Performance of Regularized Rolling Window Sample Covariance Matrix of Daily Returns over 252 Days

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 350 assets out of the entire 400 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on the rolling window sample covariance matrix of daily returns over 252 days regularized by a one- or three-factor structure (1F or 3F), a factor structure based on the Bai and Ng (2002) criteria (BN-F), eigenvalue cleaning (S-EvCl) and shrinkage towards an equicorrelation or one-factor model (SHRK$_{EC}$ or SHRK$_{SF}$). Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
								Pre-Crisis							
1F	10.13	0.10	6.73	-0.72	0.19	10.22	0.10	3.71	-0.72	0.19	10.61	0.01	2.32	-0.72	0.19
3F	9.16	0.09	9.70	-0.88	0.22	9.21	0.09	5.21	-0.88	0.22	9.43	0.01	3.01	-0.88	0.22
BN-F	9.20	0.13	13.59	-1.08	0.25	9.24	0.12	6.67	-1.08	0.25	9.43	0.01	3.49	-1.08	0.25
S-EvCl	8.44	0.09	21.28	-1.18	0.27	8.51	0.09	10.00	-1.18	0.27	8.73	0.01	4.94	-1.18	0.27
SHRK$_{EC}$	8.95	0.09	24.65	-1.60	0.32	9.01	0.09	12.63	-1.60	0.32	9.24	0.01	6.63	-1.60	0.32
SHRK$_{SF}$	8.56	0.09	24.26	-1.44	0.29	8.63	0.09	12.34	-1.44	0.29	8.86	0.01	6.47	-1.44	0.29
								Crisis							
1F	17.92	0.18	5.98	-0.58	0.17	18.07	0.18	2.92	-0.58	0.17	18.41	0.02	1.73	-0.58	0.17
3F	16.89	0.16	13.59	-0.92	0.22	17.13	0.16	6.79	-0.92	0.22	17.44	0.02	3.93	-0.92	0.22
BN-F	15.43	0.13	20.38	-1.33	0.28	15.58	0.13	9.87	-1.33	0.28	15.85	0.01	4.92	-1.33	0.28
S-EvCl	14.93	0.14	26.39	-1.46	0.32	15.08	0.14	12.59	-1.46	0.32	15.34	0.01	6.06	-1.46	0.32
SHRK$_{EC}$	15.53	0.15	32.40	-1.89	0.37	15.70	0.15	16.19	-1.89	0.37	15.98	0.02	8.28	-1.89	0.37
SHRK$_{SF}$	15.59	0.15	35.98	-1.99	0.37	15.77	0.15	17.79	-1.99	0.37	16.07	0.02	9.07	-1.99	0.37

Table 6.8: GMV Portfolio Performance of Regularized Rolling Window Sample Covariance Matrix of Daily Returns over 126 Days

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 350 assets out of the entire 400 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on the rolling window sample covariance matrix of daily returns over 126 days regularized by a one- or three-factor structure (1F or 3F), a factor structure based on the Bai and Ng (2002) criteria (BN-F), eigenvalue cleaning (S-EvCl) and shrinkage towards an equicorrelation or one-factor model (SHRK$_{EC}$ or SHRK$_{SF}$). Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
1F	9.60	0.09	8.72	-0.66	0.18	9.66	0.09	4.84	-0.66	0.18	9.95	0.01	2.97	-0.66	0.18
3F	9.37	0.09	13.10	-0.73	0.19	9.41	0.09	6.83	-0.73	0.19	9.63	0.01	4.02	-0.73	0.19
BN-F	8.95	0.14	17.63	-0.91	0.22	8.99	0.14	8.63	-0.91	0.22	9.19	0.01	4.73	-0.91	0.22
S-EvCl	8.38	0.09	22.32	-1.00	0.25	8.44	0.09	11.04	-1.00	0.25	8.71	0.01	5.94	-1.00	0.25
SHRK$_{EC}$	8.86	0.09	28.42	-1.34	0.30	8.92	0.09	14.60	-1.34	0.30	9.23	0.01	7.87	-1.34	0.30
SHRK$_{SF}$	8.34	0.09	27.63	-1.15	0.26	8.41	0.09	14.00	-1.15	0.26	8.67	0.01	7.28	-1.15	0.26
Crisis															
1F	17.50	0.17	7.72	-0.47	0.15	17.77	0.17	3.98	-0.47	0.15	18.44	0.02	2.40	-0.47	0.15
3F	16.78	0.16	18.50	-0.76	0.19	17.02	0.16	9.31	-0.76	0.19	17.34	0.02	5.07	-0.76	0.19
BN-F	15.63	0.14	30.47	-1.09	0.25	15.82	0.14	15.17	-1.09	0.25	16.12	0.01	7.61	-1.09	0.25
S-EvCl	15.13	0.13	40.38	-1.20	0.28	15.28	0.13	18.22	-1.20	0.28	15.61	0.01	8.70	-1.20	0.28
SHRK$_{EC}$	15.55	0.14	43.66	-1.63	0.34	15.71	0.14	22.04	-1.63	0.34	16.03	0.01	11.27	-1.63	0.34
SHRK$_{SF}$	15.35	0.14	42.10	-1.54	0.31	15.54	0.14	21.21	-1.54	0.31	15.93	0.01	10.85	-1.54	0.31

Table 6.9: GMV Portfolio Performance of Regularized Rolling Window Sample Covariance Matrix of Daily Returns over 63 Days

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 350 assets out of the entire 400 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on the rolling window sample covariance matrix of daily returns over 63 days regularized by a one- or three-factor structure (1F or 3F), a factor structure based on the Bai and Ng (2002) criteria (BN-F), eigenvalue cleaning (S-EvCl) and shrinkage towards an equicorrelation or one-factor model (SHRK$_{EC}$ or SHRK$_{SF}$). Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
1F	9.39	0.10	14.49	-0.57	0.18	9.47	0.10	7.89	-0.57	0.18	9.84	0.01	4.43	-0.57	0.18
3F	9.36	0.10	21.01	-0.64	0.18	9.42	0.10	10.83	-0.64	0.18	9.67	0.01	5.70	-0.64	0.18
BN-F	9.04	0.11	28.16	-0.74	0.20	9.12	0.11	13.44	-0.74	0.20	9.40	0.01	6.80	-0.74	0.20
S-EvCl	8.49	0.09	31.34	-0.79	0.23	8.60	0.08	15.16	-0.79	0.23	8.87	0.01	7.70	-0.79	0.23
SHRK$_{EC}$	8.95	0.10	34.06	-1.03	0.28	9.06	0.09	17.35	-1.03	0.28	9.40	0.01	9.07	-1.03	0.28
SHRK$_{SF}$	8.58	0.09	34.58	-0.90	0.23	8.68	0.09	17.18	-0.90	0.23	8.94	0.01	8.74	-0.90	0.23
Crisis															
1F	16.71	0.14	10.87	-0.41	0.14	17.02	0.14	5.68	-0.41	0.14	17.90	0.02	3.29	-0.41	0.14
3F	16.58	0.14	26.82	-0.64	0.18	16.87	0.15	13.47	-0.64	0.18	17.34	0.01	6.63	-0.64	0.18
BN-F	15.86	0.14	35.74	-0.81	0.21	16.07	0.14	17.51	-0.81	0.21	16.52	0.01	8.88	-0.81	0.21
S-EvCl	15.41	0.13	39.18	-0.88	0.25	15.61	0.13	18.97	-0.88	0.25	15.96	0.01	9.76	-0.88	0.25
SHRK$_{EC}$	15.47	0.13	47.15	-1.20	0.29	15.70	0.13	23.58	-1.20	0.29	16.13	0.01	12.17	-1.20	0.29
SHRK$_{SF}$	15.38	0.12	43.36	-1.04	0.25	15.61	0.12	21.61	-1.04	0.25	16.12	0.01	11.12	-1.04	0.25

Table 6.10: GMV Portfolio Performance of Regularized Rolling Window Sample Covariance Matrix of Daily Returns over 20 Days

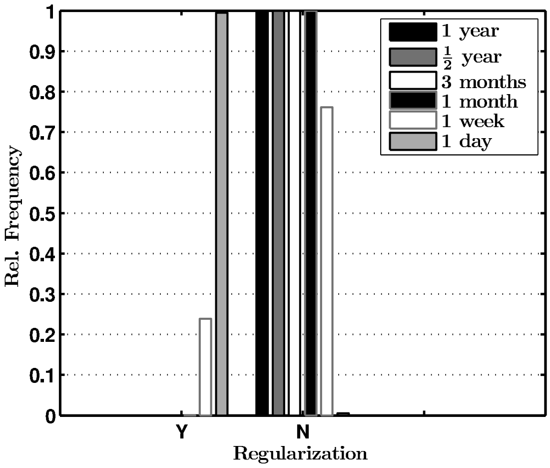
Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}^a_p$) using predicted GMV weights (in percentage points). Each random sample contains 350 assets out of the entire 400 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on the rolling window sample covariance matrix of daily returns over 20 days regularized by a one- or three-factor structure (1F or 3F), a factor structure based on the Bai and Ng (2002) criteria (BN-F), eigenvalue cleaning (S-EvCl) and shrinkage towards an equicorrelation or one-factor model (SHRK$_{EC}$ or SHRK$_{SF}$). Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}^a_p)$	$s(\bar{\sigma}^a_p)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}^a_p)$	$s(\bar{\sigma}^a_p)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}^a_p)$	$s(\bar{\sigma}^a_p)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
1F	9.67	0.09	34.21	-0.44	0.16	9.76	0.09	17.60	-0.44	0.16	9.98	0.01	8.57	-0.44	0.16
3F	9.63	0.09	46.54	-0.51	0.17	9.71	0.09	21.47	-0.51	0.17	9.88	0.01	9.88	-0.51	0.17
BN-F	9.63	0.10	64.13	-0.59	0.19	9.73	0.10	27.79	-0.59	0.19	9.92	0.01	12.08	-0.59	0.19
S-EvCl	9.35	0.11	48.95	-0.50	0.19	9.49	0.11	22.86	-0.50	0.19	9.61	0.01	10.70	-0.50	0.19
SHRK$_{EC}$	9.47	0.13	51.82	-0.73	0.24	9.55	0.13	25.77	-0.73	0.24	9.67	0.01	12.35	-0.73	0.24
SHRK$_{SF}$	9.47	0.09	50.24	-0.59	0.19	9.58	0.09	24.75	-0.59	0.19	9.74	0.01	11.87	-0.59	0.19
Crisis															
1F	16.25	0.12	26.44	-0.35	0.14	16.45	0.12	13.25	-0.35	0.14	17.12	0.01	6.67	-0.35	0.14
3F	16.50	0.13	45.79	-0.47	0.17	16.77	0.13	21.05	-0.47	0.17	17.51	0.01	9.63	-0.47	0.17
BN-F	16.65	0.13	66.98	-0.57	0.19	16.96	0.13	28.78	-0.57	0.19	17.72	0.01	12.42	-0.57	0.19
S-EvCl	16.12	0.13	50.02	-0.50	0.20	16.42	0.13	23.62	-0.50	0.20	17.13	0.01	11.29	-0.50	0.20
SHRK$_{EC}$	15.72	0.13	57.55	-0.71	0.24	15.96	0.13	28.20	-0.71	0.24	16.48	0.01	13.21	-0.71	0.24
SHRK$_{SF}$	16.22	0.13	53.89	-0.60	0.19	16.48	0.13	26.33	-0.60	0.19	17.11	0.01	12.57	-0.60	0.19

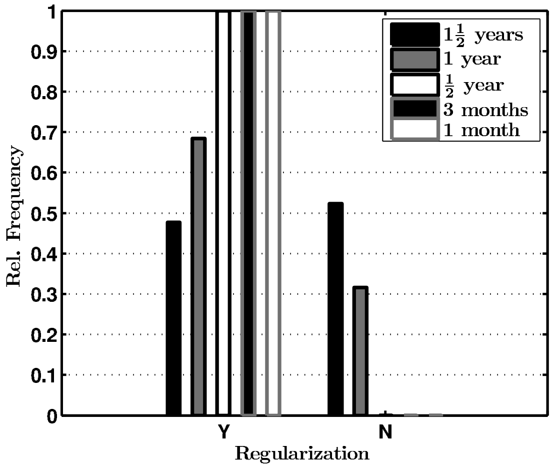
Table 6.11: Impact of Estimation Window on Basis Point Fees for Switching from Low-Frequency to High-Frequency Forecasts (Pre-Crisis, $\mu^{\mathbf{id}} = \mathbf{0.05}$) Medians ($m(\cdot)$) across 1,000 random samples of the annualized basis point fee (Δ^a_γ) a risk-averse investor with quadratic utility and relative risk aversion γ would pay to switch from covariance forecasts using regularized sample covariances of daily data computed over different windows to high-frequency-based forecasts. We assume that the constant conditional mean return is identical across all stocks and set it to $\mu^{id} = 0.05$ (annualized). Also reported are break-even transaction costs (c^*_γ) in percentage points, defined as the ratio of Δ^a_γ and the difference of average portfolio turnovers. Each random sample contains 350 assets out of the entire 400 asset universe. Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008. For each window length of the sample covariance, we consider the regularization yielding the lowest median realized portfolio volatility ("low volatility") or median portfolio turnover ("low turnover"). Low-volatility benchmarks: eigenvalue cleaning (378, 252, 63 and 20 days) and shrinkage towards single-factor model (126). Low-turnover benchmarks: imposing one-factor model (all windows).

	$\mathbf{h=1}$				$\mathbf{h=5}$				$\mathbf{h=20}$			
Wind.	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$
ERnB(1) **vs. low volatility (top) & low turnover benchmarks (bottom)**												
378	3.60	0.02	36.69	0.18	0.80	0.01	8.15	0.05	-2.77	-0.02	-28.16	-0.20
252	2.30	0.01	23.38	0.11	-0.43	-0.00	-4.43	-0.03	-4.39	-0.03	-44.67	-0.29
126	1.49	0.01	15.20	0.08	-1.30	-0.01	-13.25	-0.07	-4.80	-0.05	-48.87	-0.48
63	2.79	0.02	28.42	0.17	0.33	0.01	3.38	0.06	-3.13	-0.03	-31.86	-0.31
20	10.53	0.06	107.22	0.57	8.50	0.06	86.54	0.62	3.78	0.09	38.47	0.94
378	23.25	0.11	236.60	1.09	20.66	0.09	210.24	0.92	19.48	0.08	198.24	0.77
252	18.04	0.08	183.61	0.84	15.59	0.07	158.65	0.69	13.89	0.06	141.35	0.58
126	12.78	0.06	130.08	0.62	9.95	0.05	101.33	0.48	7.07	0.03	72.02	0.34
63	10.78	0.05	109.74	0.53	8.21	0.04	83.53	0.41	6.02	0.03	61.28	0.31
20	13.52	0.07	137.61	0.73	11.04	0.07	112.40	0.74	7.38	0.08	75.12	0.85
3FRnB(252) **vs. low volatility (top) & low turnover benchmarks (bottom)**												
378	-26.79	1.91	-272.96	19.42	-27.21	0.91	-277.22	9.27	-29.25	0.63	-297.98	6.39
252	-28.15	1.86	-286.79	18.93	-28.45	0.85	-289.87	8.61	-30.85	0.59	-314.35	6.05
126	-28.95	1.35	-294.98	13.78	-29.29	0.55	-298.41	5.57	-31.27	0.32	-318.56	3.27
63	-27.69	1.10	-282.14	11.18	-27.68	0.46	-282.04	4.71	-29.66	0.28	-302.20	2.83
20	-19.89	0.51	-202.63	5.18	-19.48	0.22	-198.43	2.23	-22.71	0.14	-231.35	1.46
378	-7.19	-14.52	-73.22	-147.89	-7.33	-4.57	-74.66	-46.58	-7.00	-1.23	-71.33	-12.58
252	-12.42	12.17	-126.52	124.00	-12.42	4.56	-126.51	46.45	-12.62	13.58	-128.49	138.27
126	-17.67	5.93	-180.00	60.40	-18.07	2.17	-184.08	22.07	-19.42	1.52	-197.82	15.49
63	-19.71	2.35	-200.77	23.94	-19.83	0.88	-201.98	8.92	-20.54	0.52	-209.24	5.33
20	-16.96	0.65	-172.75	6.58	-17.00	0.25	-173.14	2.58	-19.15	0.17	-195.11	1.69

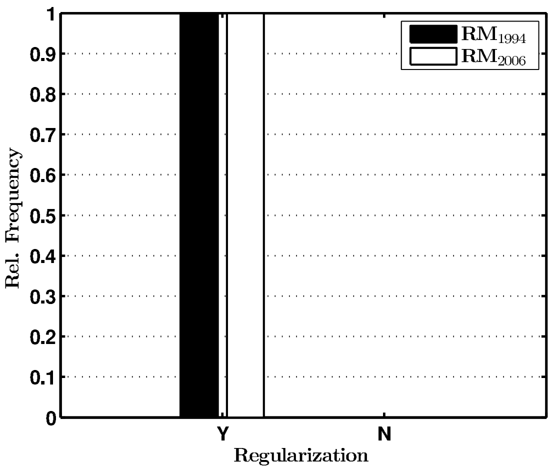
7 Additional Results for 100 Assets



(a) BRK Estimates ($G = 2$)



(b) Rolling-Window Sample Covariance



(c) RiskMetrics

Figure 7.1: Regularization Frequencies
BRK estimates are regularized if any correlation eigenvalue is negative or the condition number of the correlation matrix is greater than 10×100. The rolling-window sample covariance of daily returns and RiskMetrics forecasts are regularized if the condition number of the corresponding correlation matrix is greater than the above threshold.

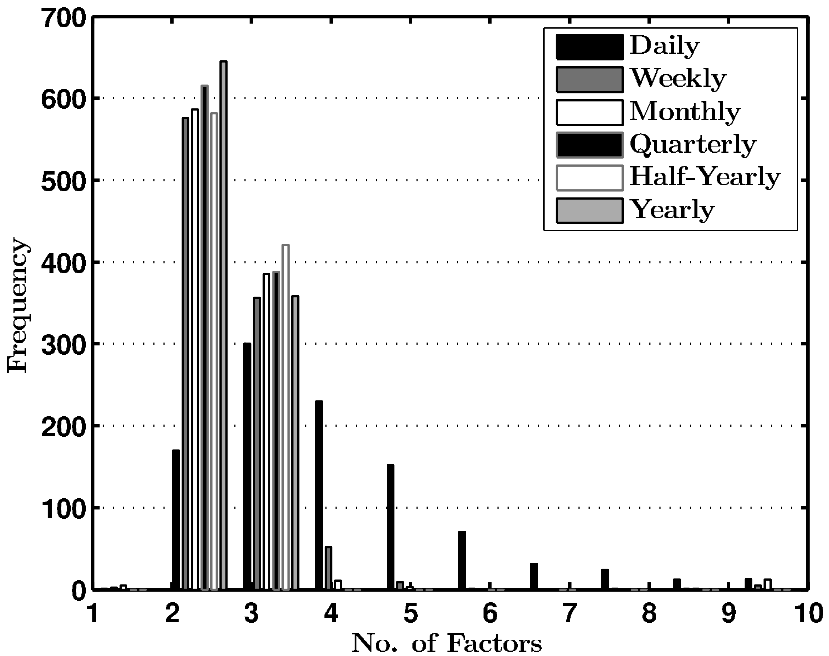


Figure 7.2: Sample Distribution of Number of Factors for FRnB Estimates
Number of factors is determined by applying Bai and Ng (2002) criteria from Section 3 to BRK estimates smoothed over different windows.

Table 7.1: Number of Liquidity Groups G and GMV Portfolio Volatility of ERnB(1) Forecasts
Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights for horizon $h = 1$ in percentage points. Each random sample contains 85 assets out of the entire 100 asset universe. Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	Pre-Crisis		**Crisis**	
G	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$
1	11.45	0.29	19.11	0.31
2	10.81	0.29	18.65	0.31
4	10.79	0.29	18.62	0.31
5	10.77	0.30	18.62	0.31
10	10.76	0.30	18.61	0.31

Table 7.2: GMV Portfolio Performance of ERnB, FRnB, 3FnB and 1FRnB Forecasts Based on Conditional Regularization

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 85 assets out of the entire 100 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. (Smoothed) BRK estimates are regularized if any correlation eigenvalue is negative or the condition number of the correlation matrix is greater than 10×100. BRK(S) represents random-walk-type forecasts based on unregularized BRK estimates smoothed over S days. Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
ERnB(1)	10.81	0.29	201.37	-0.70	0.43	11.20	0.32	44.17	-0.70	0.43	11.89	0.04	12.12	-0.70	0.43
ERnB(5)	10.51	0.29	87.13	-0.86	0.44	10.66	0.29	40.54	-0.86	0.44	11.25	0.03	12.14	-0.86	0.44
BRK(20)	10.27	0.29	24.12	-0.78	0.43	10.43	0.30	13.65	-0.78	0.43	10.88	0.03	8.30	-0.78	0.43
BRK(63)	10.47	0.30	9.14	-0.77	0.43	10.59	0.31	5.54	-0.77	0.43	10.90	0.03	3.89	-0.77	0.43
BRK(126)	10.58	0.32	5.36	-0.75	0.42	10.68	0.32	3.17	-0.75	0.42	10.94	0.03	2.28	-0.75	0.42
BRK(252)	10.89	0.34	3.90	-0.68	0.40	10.99	0.34	2.21	-0.68	0.40	11.24	0.04	1.58	-0.68	0.40
FRnB(1)	10.78	0.29	183.85	-0.72	0.41	11.20	0.32	41.37	-0.72	0.41	11.91	0.04	11.60	-0.72	0.41
FRnB(5)	10.53	0.29	87.05	-0.86	0.44	10.69	0.29	40.49	-0.86	0.44	11.28	0.03	12.16	-0.86	0.44
3FRnB(1)	10.78	0.29	180.68	-0.71	0.41	11.17	0.32	40.66	-0.71	0.41	11.88	0.04	11.43	-0.71	0.41
3FRnB(5)	10.50	0.29	87.33	-0.86	0.44	10.66	0.29	40.53	-0.86	0.44	11.25	0.03	12.15	-0.86	0.44
1FRnB(1)	12.55	0.38	142.41	-0.60	0.37	12.92	0.41	32.68	-0.60	0.37	13.91	0.05	9.71	-0.60	0.37
1FRnB(5)	10.61	0.29	87.20	-0.86	0.44	10.76	0.29	40.51	-0.86	0.44	11.35	0.03	12.17	-0.86	0.44
Crisis															
ERnB(1)	18.65	0.31	197.18	-0.71	0.46	19.36	0.32	43.45	-0.71	0.46	20.47	0.04	12.36	-0.71	0.46
ERnB(5)	17.96	0.29	81.19	-0.76	0.46	18.47	0.31	35.91	-0.76	0.46	19.40	0.03	11.25	-0.76	0.46
BRK(20)	17.94	0.29	24.26	-0.71	0.42	18.25	0.31	13.38	-0.71	0.42	19.09	0.03	7.91	-0.71	0.42
BRK(63)	18.55	0.31	10.39	-0.72	0.42	18.78	0.32	6.03	-0.72	0.42	19.39	0.03	4.05	-0.72	0.42
BRK(126)	18.93	0.32	7.92	-0.76	0.44	19.13	0.33	4.51	-0.76	0.44	19.66	0.03	3.03	-0.76	0.44
BRK(252)	19.11	0.32	6.05	-0.84	0.46	19.26	0.32	3.11	-0.84	0.46	19.66	0.03	1.91	-0.84	0.46
FRnB(1)	18.91	3.42	184.78	-0.74	0.44	19.46	4.37	41.47	-0.74	0.44	20.50	0.44	11.79	-0.74	0.44
FRnB(5)	18.29	0.29	80.03	-0.78	0.45	18.75	0.31	35.03	-0.78	0.45	19.63	0.03	11.02	-0.78	0.45
3FRnB(1)	18.84	0.29	162.34	-0.70	0.42	19.34	0.31	36.60	-0.70	0.42	20.38	0.03	10.53	-0.70	0.42
3FRnB(5)	18.24	0.29	78.42	-0.78	0.45	18.70	0.31	34.59	-0.78	0.45	19.62	0.03	10.93	-0.78	0.45
1FRnB(1)	20.38	0.38	97.15	-0.53	0.36	20.88	0.40	23.65	-0.53	0.36	21.88	0.04	7.59	-0.53	0.36
1FRnB(5)	18.97	0.33	76.20	-0.71	0.42	19.39	0.34	33.04	-0.71	0.42	20.07	0.04	10.60	-0.71	0.42

Table 7.3: GMV Portfolio Performance of ERnB and FRnB Forecasts Based on Unconditional Regularization

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 85 assets out of the entire 100 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. (Smoothed) BRK estimates are always regularized, which is indicated by an asterisk (*). Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
								Pre-Crisis							
ERnB(1)*	10.79	0.29	199.73	-0.70	0.42	11.19	0.32	43.91	-0.70	0.42	11.88	0.04	12.04	-0.70	0.42
ERnB(5)*	10.37	0.29	58.11	-0.71	0.44	10.60	0.31	28.92	-0.71	0.44	11.17	0.03	9.55	-0.71	0.44
ERnB(20)*	10.52	0.30	19.80	-0.74	0.45	10.66	0.32	11.94	-0.74	0.45	11.09	0.03	7.48	-0.74	0.45
ERnB(63)*	10.78	0.31	8.45	-0.77	0.45	10.87	0.31	5.56	-0.77	0.45	11.16	0.03	3.93	-0.77	0.45
ERnB(126)*	10.99	0.31	5.89	-0.77	0.45	11.07	0.32	3.57	-0.77	0.45	11.33	0.03	2.67	-0.77	0.45
ERnB(252)*	11.33	0.32	4.17	-0.72	0.43	11.41	0.33	2.51	-0.72	0.43	11.68	0.03	1.83	-0.72	0.43
FRnB(1)*	10.78	0.29	182.01	-0.72	0.41	11.19	0.32	41.05	-0.72	0.41	11.89	0.04	11.50	-0.72	0.41
FRnB(5)*	10.70	0.28	50.30	-0.75	0.42	10.92	0.30	26.77	-0.75	0.42	11.46	0.03	9.35	-0.75	0.42
FRnB(20)*	10.96	0.29	17.20	-0.78	0.43	11.11	0.30	11.33	-0.78	0.43	11.53	0.03	7.50	-0.78	0.43
FRnB(63)*	11.38	0.30	7.75	-0.82	0.44	11.49	0.31	5.31	-0.82	0.44	11.89	0.03	4.05	-0.82	0.44
FRnB(126)*	11.60	0.31	5.29	-0.80	0.44	11.70	0.32	3.51	-0.80	0.44	12.01	0.03	2.81	-0.80	0.44
FRnB(252)*	12.17	0.35	3.88	-0.75	0.42	12.29	0.36	2.29	-0.75	0.42	12.67	0.04	1.77	-0.75	0.42
								Crisis							
ERnB(1)*	18.65	0.31	197.18	-0.71	0.46	19.36	0.32	43.45	-0.71	0.46	20.47	0.04	12.36	-0.71	0.46
ERnB(5)*	18.17	0.29	61.91	-0.70	0.46	18.63	0.30	29.66	-0.70	0.46	19.46	0.03	9.79	-0.70	0.46
ERnB(20)*	18.66	0.30	24.92	-0.72	0.44	18.92	0.32	13.29	-0.72	0.44	19.64	0.03	7.72	-0.72	0.44
ERnB(63)*	19.45	0.33	11.08	-0.75	0.45	19.65	0.34	6.85	-0.75	0.45	20.22	0.04	4.63	-0.75	0.45
ERnB(126)*	19.85	0.33	9.92	-0.78	0.47	20.02	0.34	5.67	-0.78	0.47	20.53	0.04	3.66	-0.78	0.47
ERnB(252)*	20.15	0.33	7.83	-0.87	0.49	20.32	0.34	4.53	-0.87	0.49	20.66	0.04	3.02	-0.87	0.49
FRnB(1)*	18.91	3.42	184.78	-0.74	0.44	19.46	4.37	41.47	-0.74	0.44	20.50	0.44	11.79	-0.74	0.44
FRnB(5)*	18.80	0.29	52.46	-0.75	0.44	19.17	0.30	26.15	-0.75	0.44	19.89	0.03	8.72	-0.75	0.44
FRnB(20)*	19.27	0.30	17.14	-0.79	0.43	19.53	0.31	10.12	-0.79	0.43	20.20	0.03	6.29	-0.79	0.43
FRnB(63)*	20.05	0.32	8.98	-0.83	0.44	20.30	0.33	5.50	-0.83	0.44	20.85	0.03	3.91	-0.83	0.44
FRnB(126)*	20.89	0.34	7.82	-0.89	0.46	21.02	0.35	4.66	-0.89	0.46	21.46	0.04	3.25	-0.89	0.46
FRnB(252)*	21.02	0.33	7.91	-0.95	0.49	21.14	0.33	4.09	-0.95	0.49	21.43	0.03	2.41	-0.95	0.49

Table 7.4: GMV Portfolio Performance of 3FRnB and 1FRnB Forecasts Based on Unconditional Regularization

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 85 assets out of the entire 100 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. (Smoothed) BRK estimates are always regularized, which is indicated by an asterisk (*). Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
3FRnB(1)*	10.77	0.29	178.82	-0.71	0.41	11.15	0.32	40.34	-0.71	0.41	11.86	0.04	11.34	-0.71	0.41
3FRnB(5)*	10.56	0.29	51.88	-0.76	0.42	10.78	0.30	27.45	-0.76	0.42	11.37	0.03	9.48	-0.76	0.42
3FRnB(20)*	10.74	0.30	17.57	-0.79	0.43	10.90	0.32	11.42	-0.79	0.43	11.36	0.03	7.47	-0.79	0.43
3FRnB(63)*	11.02	0.31	7.51	-0.82	0.45	11.13	0.32	4.99	-0.82	0.45	11.46	0.03	3.81	-0.82	0.45
3FRnB(126)*	11.01	0.31	5.03	-0.81	0.44	11.11	0.32	3.13	-0.81	0.44	11.40	0.03	2.39	-0.81	0.44
3FRnB(252)*	11.19	0.32	3.96	-0.76	0.43	11.29	0.33	2.27	-0.76	0.43	11.55	0.03	1.68	-0.76	0.43
1FRnB(1)*	12.57	0.38	139.68	-0.59	0.37	12.93	0.41	32.23	-0.59	0.37	13.91	0.05	9.57	-0.59	0.37
1FRnB(5)*	12.98	0.40	39.30	-0.66	0.39	13.20	0.42	21.83	-0.66	0.39	14.15	0.05	8.17	-0.66	0.39
1FRnB(20)*	13.35	0.40	13.91	-0.69	0.40	13.55	0.42	9.59	-0.69	0.40	14.37	0.05	6.50	-0.69	0.40
1FRnB(63)*	13.41	0.35	5.94	-0.74	0.42	13.51	0.36	3.96	-0.74	0.42	13.83	0.04	2.99	-0.74	0.42
1FRnB(126)*	14.08	0.39	4.21	-0.74	0.41	14.18	0.40	2.60	-0.74	0.41	14.56	0.04	2.01	-0.74	0.41
1FRnB(252)*	15.40	0.49	3.46	-0.71	0.39	15.53	0.50	1.98	-0.71	0.39	16.03	0.05	1.49	-0.71	0.39
Crisis															
3FRnB(1)*	18.84	0.29	162.34	-0.70	0.42	19.34	0.31	36.60	-0.70	0.42	20.38	0.03	10.53	-0.70	0.42
3FRnB(5)*	18.73	0.29	46.11	-0.75	0.44	19.07	0.31	24.30	-0.75	0.44	19.91	0.03	8.36	-0.75	0.44
3FRnB(20)*	19.35	0.32	15.92	-0.79	0.43	19.59	0.33	9.75	-0.79	0.43	20.29	0.04	6.23	-0.79	0.43
3FRnB(63)*	20.19	0.34	8.27	-0.83	0.44	20.35	0.34	5.01	-0.83	0.44	20.85	0.04	3.61	-0.83	0.44
3FRnB(126)*	20.72	0.34	7.11	-0.89	0.46	20.86	0.35	4.07	-0.89	0.46	21.31	0.04	2.82	-0.89	0.46
3FRnB(252)*	20.97	0.33	6.34	-0.95	0.49	21.08	0.34	3.25	-0.95	0.49	21.38	0.04	1.99	-0.95	0.49
1FRnB(1)*	20.38	0.38	97.15	-0.53	0.36	20.88	0.40	23.65	-0.53	0.36	21.88	0.04	7.59	-0.53	0.36
1FRnB(5)*	21.10	0.41	27.49	-0.57	0.36	21.46	0.42	16.11	-0.57	0.36	22.24	0.04	6.35	-0.57	0.36
1FRnB(20)*	22.11	0.48	11.40	-0.61	0.35	22.38	0.49	7.49	-0.61	0.35	23.11	0.05	5.03	-0.61	0.35
1FRnB(63)*	22.97	0.56	6.88	-0.66	0.37	23.03	0.56	4.34	-0.66	0.37	23.70	0.06	3.27	-0.66	0.37
1FRnB(126)*	22.74	0.53	6.10	-0.73	0.39	22.79	0.53	3.54	-0.73	0.39	23.24	0.06	2.51	-0.73	0.39
1FRnB(252)*	23.04	0.48	5.70	-0.81	0.43	23.07	0.48	2.83	-0.81	0.43	23.14	0.05	1.79	-0.81	0.43

Table 7.5: GMV Portfolio Performance of Covariance Matrix Forecasts Employing Daily Returns

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 85 assets out of the entire 100 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on S-VEC and DCC models, regularized RiskMetrics1994 and RiskMetrics2006 estimators (RM_{1994} and RM_{2006}), as well as the rolling window sample covariance matrix of daily returns over 252 days regularized by a one- or three-factor structure (1F or 3F), a factor structure based on the Bai and Ng (2002) criteria (BN-F), eigenvalue cleaning (S-EvCl) and shrinkage towards an equicorrelation or one-factor model (SHRK_{EC} or SHRK_{SF}). In addition, results for the equally-weighted portfolio (EQW) are reported. Rolling-window sample covariance and RiskMetrics forecasts are regularized if the condition number of the corresponding correlation matrix is greater than 10×100. Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
EQW	18.43	0.19	1.04	0.00	0.11	18.51	0.19	0.45	0.00	0.11	18.75	0.02	0.23	0.00	0.11
1F	12.24	0.44	20.61	-1.12	0.54	12.31	0.44	10.76	-1.12	0.54	12.63	0.05	5.78	-1.12	0.54
3F	11.69	0.40	20.95	-1.16	0.54	11.74	0.40	10.83	-1.16	0.54	11.94	0.04	5.78	-1.16	0.54
BN-F	11.79	0.41	23.20	-1.19	0.55	11.85	0.41	11.73	-1.19	0.55	12.03	0.04	6.10	-1.19	0.55
S-EvCl	11.85	0.41	23.12	-1.21	0.59	11.91	0.41	11.60	-1.21	0.59	12.11	0.04	6.04	-1.21	0.59
SHRK_{EC}	11.67	0.40	22.28	-1.22	0.58	11.73	0.40	11.34	-1.22	0.58	11.92	0.04	5.87	-1.22	0.58
SHRK_{SF}	11.82	0.41	23.75	-1.27	0.59	11.89	0.41	11.97	-1.27	0.59	12.08	0.04	6.09	-1.27	0.59
RM_{1994}	12.39	0.42	49.21	-1.25	0.58	12.50	0.43	25.50	-1.25	0.58	12.83	0.04	12.62	-1.25	0.58
RM_{2006}	11.64	0.37	41.91	-1.17	0.57	11.74	0.38	21.14	-1.17	0.57	12.03	0.04	9.64	-1.19	0.57
S-VECH	12.53	0.45	44.49	-1.45	0.63	12.52	0.45	22.57	-1.43	0.62	12.50	0.04	10.09	-1.37	0.61
DCC	11.61	0.41	78.75	-1.00	0.52	11.57	0.43	24.83	-1.00	0.52	11.68	0.04	8.90	-0.97	0.51
Crisis															
EQW	37.75	0.46	1.68	0.00	0.11	37.72	0.45	0.72	0.00	0.11	37.57	0.05	0.38	0.00	0.11
1F	21.04	0.44	7.00	-0.70	0.40	21.13	0.44	3.23	-0.70	0.40	21.30	0.04	1.81	-0.70	0.40
3F	21.84	0.52	10.15	-0.82	0.41	21.93	0.52	4.47	-0.82	0.41	22.07	0.05	2.42	-0.82	0.41
BN-F	22.33	0.51	17.84	-1.08	0.49	22.37	0.51	8.51	-1.08	0.49	22.58	0.05	4.37	-1.08	0.49
S-EvCl	21.73	0.36	17.61	-1.08	0.55	21.79	0.36	8.54	-1.08	0.55	21.98	0.04	4.35	-1.08	0.55
SHRK_{EC}	20.86	0.38	17.46	-1.14	0.56	20.97	0.37	8.66	-1.14	0.56	21.19	0.04	4.43	-1.14	0.56
SHRK_{SF}	21.99	0.42	23.46	-1.43	0.61	22.07	0.41	11.60	-1.43	0.61	22.27	0.04	5.81	-1.43	0.61
RM_{1994}	22.87	0.41	56.33	-1.37	0.58	23.12	0.41	29.22	-1.37	0.58	23.52	0.04	14.57	-1.37	0.58
RM_{2006}	22.01	0.40	49.93	-1.35	0.58	22.21	0.39	25.05	-1.35	0.58	22.49	0.04	11.03	-1.36	0.58
S-VECH	29.84	0.67	88.16	-2.47	0.88	29.55	0.64	44.34	-2.43	0.87	28.60	0.06	20.74	-2.31	0.83
DCC	22.09	0.42	64.40	-1.06	0.54	22.35	0.42	29.69	-1.05	0.53	22.43	0.04	11.45	-1.00	0.53

Table 7.6: GMV Portfolio Performance of Rolling Window Sample Covariance Matrix of Daily Returns Regularized Unconditionally
Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 85 assets out of the entire 100 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on the rolling window sample covariance matrix of daily returns over 252 days regularized by a one- or three-factor structure (1F* or 3F*), a factor structure based on the Bai and Ng (2002) criteria (BN-F*), eigenvalue cleaning (S-EvCl*) and shrinkage towards an equicorrelation or one-factor model (SHRK^*_{EC} or SHRK^*_{SF}). Regularization is always carried out, which is indicated by an asterisk (*). Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
1F*	13.39	0.46	5.10	-0.57	0.36	13.48	0.46	2.72	-0.57	0.36	13.82	0.05	1.67	-0.57	0.36
3F*	11.40	0.37	7.61	-0.74	0.41	11.49	0.38	4.00	-0.74	0.41	11.73	0.04	2.37	-0.74	0.41
BN-F*	11.39	0.38	14.26	-0.82	0.45	11.45	0.39	6.54	-0.82	0.45	11.67	0.04	3.39	-0.82	0.45
S-EvCl*	11.37	0.38	13.45	-0.84	0.52	11.40	0.39	6.39	-0.84	0.52	11.58	0.04	3.33	-0.84	0.52
SHRK^*_{EC}	11.09	0.37	14.05	-0.94	0.52	11.14	0.38	7.08	-0.94	0.52	11.34	0.04	3.73	-0.94	0.52
SHRK^*_{SF}	11.29	0.39	16.86	-1.03	0.53	11.35	0.39	8.46	-1.03	0.53	11.54	0.04	4.36	-1.03	0.53
Crisis															
1F*	21.04	0.44	7.00	-0.70	0.40	21.13	0.44	3.23	-0.70	0.40	21.30	0.04	1.81	-0.70	0.40
3F*	21.84	0.52	10.15	-0.82	0.41	21.93	0.52	4.47	-0.82	0.41	22.07	0.05	2.42	-0.82	0.41
BN-F*	22.33	0.51	17.84	-1.08	0.49	22.37	0.51	8.51	-1.08	0.49	22.58	0.05	4.37	-1.08	0.49
S-EvCl*	21.73	0.36	17.61	-1.08	0.55	21.79	0.36	8.54	-1.08	0.55	21.98	0.04	4.35	-1.08	0.55
SHRK^*_{EC}	20.86	0.38	17.46	-1.14	0.56	20.97	0.37	8.66	-1.14	0.56	21.19	0.04	4.43	-1.14	0.56
SHRK^*_{SF}	21.99	0.42	23.46	-1.43	0.61	22.07	0.41	11.60	-1.43	0.61	22.27	0.04	5.81	-1.43	0.61

Table 7.7: Basis Point Fees for Switching from Low-Frequency to High-Frequency Forecasts (Pre-Crisis, $\mu^{\mathbf{id}} = \mathbf{0.05}$)
Medians ($m(\cdot)$) across 1,000 random samples of the annualized basis point fee (Δ^a_γ) a risk-averse investor with quadratic utility and relative risk aversion γ would pay to switch from covariance forecasts using daily data to high-frequency-based forecasts. We assume that the constant conditional mean return is identical across all stocks and set it to $\mu^{id} = 0.05$ (annualized). Also reported are break-even transaction costs (c^*_γ) in percentage points, defined as the ratio of Δ^a_γ and the difference of average portfolio turnovers. Each random sample contains 85 assets out of the 100 asset universe. Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008. For each length of the smoothing window applied to BRK estimates, the regularization method (conditional or unconditional(*)) that minimizes median realized portfolio volatility is chosen. BRK(S) represents random-walk-type forecasts based on unregularized BRK estimates smoothed over S days. The low-frequency benchmarks are the sample covariance computed over 252 days and unconditionally regularized by shrinkage towards an equicorrelation model (SHRK$^*_{EC}$) or by imposing a one factor model (1F*).

	$\mathbf{h=1}$				$\mathbf{h=5}$				$\mathbf{h=20}$			
	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$
vs. SHRK$^*_{EC}$												
3FRnB(1)*	3.80	0.03	38.06	0.26	0.21	0.00	2.06	0.03	-5.81	-0.04	-58.18	-0.39
ERnB(5)*	8.00	0.19	80.11	1.90	6.15	0.06	61.63	0.59	2.08	0.02	20.87	0.19
BRK(20)	9.00	0.97	90.16	9.68	7.91	0.25	79.27	2.55	5.27	0.06	52.84	0.61
BRK(63)	6.82	-1.48	68.36	-14.84	6.19	-0.87	62.02	-8.67	5.04	1.37	50.53	13.67
BRK(126)	5.77	-0.72	57.75	-7.23	5.32	-0.30	53.30	-3.05	4.59	-0.17	45.97	-1.75
BRK(252)	2.40	-0.27	24.03	-2.75	1.96	-0.10	19.63	-0.99	1.35	-0.04	13.52	-0.38
vs. 1F*												
3FRnB(1)*	31.83	0.19	318.70	1.86	28.70	0.15	287.30	1.53	25.14	0.13	251.75	1.30
ERnB(5)*	36.08	0.69	361.23	6.87	34.80	0.27	348.36	2.66	33.19	0.21	332.23	2.13
BRK(20)	37.09	1.98	371.30	19.85	36.50	0.67	365.37	6.74	36.35	0.28	363.85	2.79
BRK(63)	34.86	8.69	349.00	86.99	34.81	2.49	348.53	24.88	36.17	0.82	362.05	8.25
BRK(126)	33.77	111.22	338.04	1113.44	33.88	15.13	339.17	151.44	35.60	3.00	356.39	29.98
BRK(252)	30.35	-25.00	303.87	-250.29	30.44	-11.70	304.71	-117.09	32.19	-16.87	322.29	-168.91

Table 7.8: GMV Portfolio Performance of Rolling Window Sample Covariance Matrix of Daily Returns over 378 Days Regularized Conditionally or Unconditionally

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 85 assets out of the entire 100 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on the rolling window sample covariance matrix of daily returns over 378 days regularized by a one- or three-factor structure (1F or 3F), a factor structure based on the Bai and Ng (2002) criteria (BN-F), eigenvalue cleaning (S-EvCl) and shrinkage towards an equicorrelation or one-factor model (SHRK$_{EC}$ or SHRK$_{SF}$). Regularization is carried out if the condition number of the corresponding correlation matrix is greater than 10×100. In addition, unconditional regularization is considered and indicated by an asterisk (*). SCOV denotes the unregularized rolling window sample covariance. Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
SCOV	12.83	0.47	18.71	-1.28	0.60	12.93	0.47	9.36	-1.28	0.60	13.15	0.05	5.01	-1.28	0.60
1F*	15.99	0.66	4.40	-0.59	0.37	16.14	0.67	2.34	-0.59	0.37	16.77	0.07	1.47	-0.59	0.37
3F*	12.68	0.44	7.28	-0.79	0.43	12.78	0.45	3.79	-0.79	0.43	13.03	0.05	2.31	-0.79	0.43
BN-F*	12.20	0.43	10.55	-0.85	0.47	12.28	0.44	5.22	-0.85	0.47	12.55	0.05	3.00	-0.85	0.47
S-EvCl*	12.16	0.42	10.01	-0.85	0.55	12.24	0.43	4.78	-0.85	0.55	12.45	0.04	2.67	-0.85	0.55
SHRK$^*_{EC}$	11.99	0.43	11.79	-0.96	0.53	12.07	0.44	6.00	-0.96	0.53	12.30	0.05	3.28	-0.96	0.53
SHRK$^*_{SF}$	12.18	0.44	13.65	-1.03	0.54	12.27	0.45	6.91	-1.03	0.54	12.51	0.05	3.74	-1.03	0.54
Crisis															
1F	21.01	0.44	9.40	-0.84	0.44	21.15	0.44	4.62	-0.84	0.44	21.57	0.04	2.67	-0.84	0.44
3F	21.72	0.52	10.74	-0.94	0.46	21.80	0.52	5.15	-0.94	0.46	22.02	0.05	2.94	-0.94	0.46
BN-F	22.20	0.51	17.56	-1.17	0.52	22.25	0.51	7.92	-1.17	0.52	22.40	0.05	4.24	-1.17	0.52
S-EvCl	21.41	0.36	13.54	-1.21	0.56	21.51	0.36	6.68	-1.21	0.56	21.74	0.04	3.66	-1.21	0.56
SHRK$_{EC}$	20.96	0.39	13.99	-1.21	0.57	21.07	0.39	6.91	-1.21	0.57	21.28	0.04	3.65	-1.21	0.57
SHRK$_{SF}$	21.75	0.41	16.85	-1.41	0.60	21.84	0.41	8.25	-1.41	0.60	22.01	0.04	4.26	-1.41	0.60
1F*	21.32	0.46	6.14	-0.72	0.41	21.39	0.46	2.75	-0.72	0.41	21.47	0.05	1.56	-0.72	0.41
3F*	21.60	0.52	7.73	-0.84	0.42	21.69	0.52	3.37	-0.84	0.42	21.81	0.05	1.91	-0.84	0.42
BN-F*	22.08	0.51	15.50	-1.08	0.50	22.09	0.51	6.71	-1.08	0.50	22.23	0.05	3.51	-1.08	0.50
S-EvCl*	21.25	0.34	11.45	-1.13	0.56	21.34	0.35	5.49	-1.13	0.56	21.51	0.04	2.96	-1.13	0.56
SHRK$^*_{EC}$	20.72	0.38	12.41	-1.15	0.56	20.81	0.37	6.07	-1.15	0.56	20.96	0.04	3.19	-1.15	0.56
SHRK$^*_{SF}$	21.59	0.41	15.94	-1.37	0.60	21.67	0.40	7.79	-1.37	0.60	21.80	0.04	4.01	-1.37	0.60

Table 7.9: GMV Portfolio Performance of Rolling Window Sample Covariance Matrix of Daily Returns over 252 Days Regularized Conditionally or Unconditionally

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 85 assets out of the entire 100 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on the rolling window sample covariance matrix of daily returns over 252 days regularized by a one- or three-factor structure (1F or 3F), a factor structure based on the Bai and Ng (2002) criteria (BN-F), eigenvalue cleaning (S-EvCl) and shrinkage towards an equicorrelation or one-factor model (SHRK$_{EC}$ or SHRK$_{SF}$). Regularization is carried out if the condition number of the corresponding correlation matrix is greater than 10×100. In addition, unconditional regularization is considered and indicated by an asterisk (*). Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h = 1}$					$\mathbf{h = 5}$					$\mathbf{h = 20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
1F	13.29	0.51	17.93	-1.01	0.51	13.39	0.52	9.77	-1.01	0.51	13.74	0.05	5.56	-1.01	0.51
3F	12.61	0.46	18.39	-1.07	0.51	12.69	0.46	9.85	-1.07	0.51	12.88	0.05	5.56	-1.07	0.51
BN-F	12.73	0.48	21.45	-1.12	0.54	12.81	0.48	11.10	-1.12	0.54	12.98	0.05	6.00	-1.12	0.54
S-EvCl	12.81	0.48	21.38	-1.14	0.59	12.90	0.48	10.92	-1.14	0.59	13.09	0.05	5.92	-1.14	0.59
SHRK$_{EC}$	12.58	0.46	20.23	-1.15	0.58	12.67	0.46	10.56	-1.15	0.58	12.85	0.05	5.68	-1.15	0.58
SHRK$_{SF}$	12.78	0.47	22.22	-1.22	0.58	12.88	0.47	11.41	-1.22	0.58	13.05	0.05	5.98	-1.22	0.58
1F*	13.39	0.46	5.10	-0.57	0.36	13.48	0.46	2.72	-0.57	0.36	13.82	0.05	1.67	-0.57	0.36
3F*	11.40	0.37	7.61	-0.74	0.41	11.49	0.38	4.00	-0.74	0.41	11.73	0.04	2.37	-0.74	0.41
BN-F*	11.39	0.38	14.26	-0.82	0.45	11.45	0.39	6.54	-0.82	0.45	11.67	0.04	3.39	-0.82	0.45
S-EvCl*	11.37	0.38	13.45	-0.84	0.52	11.40	0.39	6.39	-0.84	0.52	11.58	0.04	3.33	-0.84	0.52
SHRK$^*_{EC}$	11.09	0.37	14.05	-0.94	0.52	11.14	0.38	7.08	-0.94	0.52	11.34	0.04	3.73	-0.94	0.52
SHRK$^*_{SF}$	11.29	0.39	16.86	-1.03	0.53	11.35	0.39	8.46	-1.03	0.53	11.54	0.04	4.36	-1.03	0.53
Crisis															
1F	21.04	0.44	7.00	-0.70	0.40	21.13	0.44	3.23	-0.70	0.40	21.30	0.04	1.81	-0.70	0.40
3F	21.84	0.52	10.15	-0.82	0.41	21.93	0.52	4.47	-0.82	0.41	22.07	0.05	2.42	-0.82	0.41
BN-F	22.33	0.51	17.84	-1.08	0.49	22.37	0.51	8.51	-1.08	0.49	22.58	0.05	4.37	-1.08	0.49
S-EvCl	21.73	0.36	17.61	-1.08	0.55	21.79	0.36	8.54	-1.08	0.55	21.98	0.04	4.35	-1.08	0.55
SHRK$_{EC}$	20.86	0.38	17.46	-1.14	0.56	20.97	0.37	8.66	-1.14	0.56	21.19	0.04	4.43	-1.14	0.56
SHRK$_{SF}$	21.99	0.42	23.46	-1.43	0.61	22.07	0.41	11.60	-1.43	0.61	22.27	0.04	5.81	-1.43	0.61
1F*	21.04	0.44	7.00	-0.70	0.40	21.13	0.44	3.23	-0.70	0.40	21.30	0.04	1.81	-0.70	0.40
3F*	21.84	0.52	10.15	-0.82	0.41	21.93	0.52	4.47	-0.82	0.41	22.07	0.05	2.42	-0.82	0.41
BN-F*	22.33	0.51	17.84	-1.08	0.49	22.37	0.51	8.51	-1.08	0.49	22.58	0.05	4.37	-1.08	0.49
S-EvCl*	21.73	0.36	17.61	-1.08	0.55	21.79	0.36	8.54	-1.08	0.55	21.98	0.04	4.35	-1.08	0.55
SHRK$^*_{EC}$	20.86	0.38	17.46	-1.14	0.56	20.97	0.37	8.66	-1.14	0.56	21.19	0.04	4.43	-1.14	0.56
SHRK$^*_{SF}$	21.99	0.42	23.46	-1.43	0.61	22.07	0.41	11.60	-1.43	0.61	22.27	0.04	5.81	-1.43	0.61

Table 7.10: GMV Portfolio Performance of Regularized Rolling Window Sample Covariance Matrix of Daily Returns over 126 Days

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 85 assets out of the entire 100 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on the rolling window sample covariance matrix of daily returns over 126 days regularized by a one- or three-factor structure (1F or 3F), a factor structure based on the Bai and Ng (2002) criteria (BN-F), eigenvalue cleaning (S-EvCl) and shrinkage towards an equicorrelation or one-factor model (SHRK$_{EC}$ or SHRK$_{SF}$). Regularization is carried out if the condition number of the corresponding correlation matrix is greater than 10×100. Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
1F	12.89	0.44	8.96	-0.66	0.42	13.00	0.45	4.74	-0.66	0.42	13.45	0.05	2.87	-0.66	0.42
3F	11.98	0.40	11.28	-0.76	0.43	12.06	0.41	5.94	-0.76	0.43	12.37	0.04	3.45	-0.76	0.43
BN-F	12.15	0.40	19.37	-0.84	0.45	12.24	0.41	9.41	-0.84	0.45	12.57	0.04	4.78	-0.84	0.45
S-EvCl	12.25	0.43	17.61	-0.81	0.51	12.34	0.43	8.80	-0.81	0.51	12.62	0.05	4.88	-0.81	0.51
SHRK$_{EC}$	11.98	0.40	15.24	-0.86	0.53	12.08	0.41	7.86	-0.86	0.53	12.39	0.04	4.46	-0.86	0.53
SHRK$_{SF}$	12.21	0.41	21.75	-0.97	0.52	12.32	0.42	10.98	-0.97	0.52	12.63	0.04	5.86	-0.97	0.52
Crisis															
1F	21.22	0.45	9.53	-0.58	0.35	21.45	0.46	4.70	-0.58	0.35	21.94	0.05	2.72	-0.58	0.35
3F	21.82	0.49	14.33	-0.72	0.39	21.95	0.48	6.63	-0.72	0.39	22.17	0.05	3.54	-0.72	0.39
BN-F	22.67	0.49	27.81	-0.95	0.45	22.76	0.49	12.84	-0.95	0.45	23.06	0.05	6.38	-0.95	0.45
S-EvCl	22.22	0.39	25.09	-0.93	0.50	22.36	0.40	12.17	-0.93	0.50	22.68	0.04	6.20	-0.93	0.50
SHRK$_{EC}$	21.20	0.40	27.77	-1.04	0.52	21.36	0.40	13.75	-1.04	0.52	21.67	0.04	7.11	-1.04	0.52
SHRK$_{SF}$	22.62	0.44	37.36	-1.30	0.56	22.73	0.44	18.28	-1.30	0.56	23.02	0.04	9.34	-1.30	0.56

Table 7.11: GMV Portfolio Performance of Regularized Rolling Window Sample Covariance Matrix of Daily Returns over 63 Days

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 85 assets out of the entire 100 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on the rolling window sample covariance matrix of daily returns over 63 days regularized by a one- or three-factor structure (1F or 3F), a factor structure based on the Bai and Ng (2002) criteria (BN-F), eigenvalue cleaning (S-EvCl) and shrinkage towards an equicorrelation or one-factor model (SHRK$_{EC}$ or SHRK$_{SF}$). Regularization is carried out if the condition number of the corresponding correlation matrix is greater than 10×100. Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
1F	12.84	0.44	16.09	-0.61	0.41	12.95	0.45	8.43	-0.61	0.41	13.57	0.05	4.72	-0.61	0.41
3F	12.09	0.40	18.53	-0.70	0.41	12.18	0.40	9.64	-0.70	0.41	12.55	0.04	5.38	-0.70	0.41
BN-F	12.36	0.39	27.29	-0.78	0.43	12.52	0.41	13.17	-0.78	0.43	12.86	0.04	6.84	-0.78	0.43
S-EvCl	12.20	0.44	26.25	-0.70	0.47	12.33	0.45	12.03	-0.70	0.47	12.61	0.05	6.09	-0.70	0.47
SHRK$_{EC}$	12.01	0.42	20.89	-0.76	0.50	12.12	0.44	10.72	-0.76	0.50	12.51	0.04	5.96	-0.76	0.50
SHRK$_{SF}$	12.28	0.42	32.30	-0.88	0.49	12.41	0.44	16.17	-0.88	0.49	12.79	0.04	8.20	-0.88	0.49
Crisis															
1F	21.46	0.50	13.56	-0.50	0.32	21.89	0.52	6.90	-0.50	0.32	22.80	0.05	3.84	-0.50	0.32
3F	21.27	0.45	20.89	-0.65	0.37	21.45	0.44	9.90	-0.65	0.37	21.78	0.04	4.97	-0.65	0.37
BN-F	22.02	0.45	36.31	-0.85	0.42	22.16	0.45	16.13	-0.85	0.42	22.51	0.04	7.99	-0.85	0.42
S-EvCl	21.53	0.39	32.64	-0.80	0.46	21.77	0.40	15.79	-0.80	0.46	22.20	0.04	7.89	-0.80	0.46
SHRK$_{EC}$	21.05	0.39	34.29	-0.93	0.49	21.31	0.39	17.08	-0.93	0.49	21.74	0.04	8.84	-0.93	0.49
SHRK$_{SF}$	22.06	0.42	44.06	-1.07	0.50	22.27	0.42	21.53	-1.07	0.50	22.68	0.04	10.80	-1.07	0.50

Table 7.12: GMV Portfolio Performance of Regularized Rolling Window Sample Covariance Matrix of Daily Returns over 20 Days
Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}^a_p$) using predicted GMV weights (in percentage points). Each random sample contains 85 assets out of the entire 100 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on the rolling window sample covariance matrix of daily returns over 20 days regularized by a one- or three-factor structure (1F or 3F), a factor structure based on the Bai and Ng (2002) criteria (BN-F), eigenvalue cleaning (S-EvCl) and shrinkage towards an equicorrelation or one-factor model (SHRK$_{EC}$ or SHRK$_{SF}$). Regularization is carried out if the condition number of the corresponding correlation matrix is greater than 10×100. Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}^a_p)$	$s(\bar{\sigma}^a_p)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}^a_p)$	$s(\bar{\sigma}^a_p)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}^a_p)$	$s(\bar{\sigma}^a_p)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
1F	13.86	0.45	39.24	-0.49	0.36	14.03	0.47	19.57	-0.49	0.36	14.80	0.05	9.41	-0.49	0.36
3F	12.78	0.39	47.84	-0.59	0.39	12.95	0.40	22.46	-0.59	0.39	13.23	0.04	10.15	-0.59	0.39
BN-F	13.41	0.44	76.18	-0.72	0.42	13.61	0.46	31.11	-0.72	0.42	13.83	0.05	12.89	-0.72	0.42
S-EvCl	12.54	0.44	43.77	-0.54	0.42	12.72	0.48	20.94	-0.54	0.42	12.96	0.05	9.82	-0.54	0.42
SHRK$_{EC}$	12.40	0.44	41.78	-0.66	0.48	12.52	0.47	20.62	-0.66	0.48	12.87	0.05	9.63	-0.66	0.48
SHRK$_{SF}$	12.83	0.41	51.95	-0.66	0.43	13.05	0.43	25.19	-0.66	0.43	13.30	0.05	11.66	-0.66	0.43
Crisis															
1F	21.04	0.43	32.70	-0.44	0.31	21.31	0.44	15.59	-0.44	0.31	22.13	0.05	7.50	-0.44	0.31
3F	21.46	0.41	46.18	-0.57	0.35	21.75	0.41	21.38	-0.57	0.35	22.51	0.04	9.73	-0.57	0.35
BN-F	22.84	0.41	87.41	-0.79	0.41	23.25	0.41	35.93	-0.79	0.41	23.92	0.04	14.62	-0.79	0.41
S-EvCl	21.29	0.41	48.56	-0.53	0.39	21.50	0.42	22.14	-0.53	0.39	22.33	0.04	10.28	-0.53	0.39
SHRK$_{EC}$	20.70	0.39	49.54	-0.68	0.44	20.89	0.39	23.95	-0.68	0.44	21.54	0.04	11.17	-0.68	0.44
SHRK$_{SF}$	21.64	0.39	59.39	-0.71	0.41	22.06	0.41	28.53	-0.71	0.41	22.84	0.04	13.36	-0.71	0.41

Table 7.13: Impact of Estimation Window on Basis Point Fees for Switching from Low-Frequency to High-Frequency Forecasts (Pre-Crisis, $\mu^{\mathbf{id}} = \mathbf{0.05}$) Medians ($m(\cdot)$) across 1,000 random samples of the annualized basis point fee (Δ_γ^a) a risk-averse investor with quadratic utility and relative risk aversion γ would pay to switch from covariance forecasts using regularized sample covariances of daily data computed over different windows to high-frequency-based forecasts. We assume that the constant conditional mean return is identical across all stocks and set it to $\mu^{id} = 0.05$ (annualized). Also reported are break-even transaction costs (c_γ^*) in percentage points, defined as the ratio of Δ_γ^a and the difference of average portfolio turnovers. Each random sample contains 85 assets out of the entire 100 asset universe. Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008. For each window length of the sample covariance, we consider the (conditional or unconditional) regularization yielding the lowest median realized portfolio volatility ("low volatility") or median portfolio turnover ("low turnover"). Low-volatility benchmarks: unconditional shrinkage towards equicorrelation model (378 and 252 days), shrinkage towards equicorrelation model (63 and 20 days) and imposing one-factor model (126). Low-turnover benchmarks: imposing one-factor model unconditionally (378 and 252 days) and imposing one-factor model (126, 63 and 20 days).

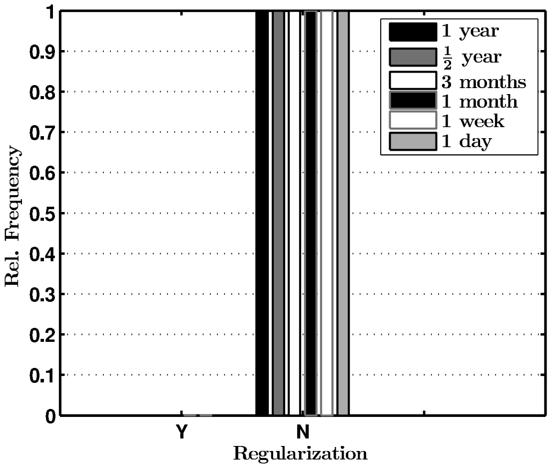
	$\mathbf{h = 1}$				$\mathbf{h = 5}$				$\mathbf{h = 20}$			
Wind.	$m(\Delta_1^a)$	$m(c_1^*)$	$m(\Delta_{10}^a)$	$m(c_{10}^*)$	$m(\Delta_1^a)$	$m(c_1^*)$	$m(\Delta_{10}^a)$	$m(c_{10}^*)$	$m(\Delta_1^a)$	$m(c_1^*)$	$m(\Delta_{10}^a)$	$m(c_{10}^*)$
					ERnB$(5)^*$ **vs. low volatility (top) & low turnover benchmarks (bottom)**							
378	9.20	0.21	93.61	2.12	7.24	0.07	73.68	0.68	2.87	0.02	29.26	0.19
252	9.23	0.20	93.97	2.04	7.32	0.06	74.54	0.64	2.66	0.02	27.04	0.18
126	9.02	0.19	91.84	1.96	7.01	0.06	71.41	0.59	3.72	0.02	37.89	0.18
63	9.31	0.25	94.73	2.55	7.76	0.08	79.02	0.83	5.49	0.05	55.92	0.51
20	14.10	0.90	143.55	9.15	12.55	0.31	127.74	3.20	9.95	-1.10	101.27	-11.22
378	65.02	1.26	661.18	12.77	64.51	0.50	655.98	5.08	67.67	0.41	688.13	4.19
252	40.28	0.76	409.76	7.71	39.17	0.29	398.52	2.97	38.05	0.23	387.14	2.30
126	20.22	0.39	205.83	3.96	18.64	0.14	189.70	1.42	17.28	0.10	175.87	1.03
63	19.61	0.44	199.55	4.50	18.17	0.16	184.96	1.63	19.16	0.16	195.06	1.65
20	33.19	1.89	337.72	19.22	32.60	0.76	331.73	7.70	36.61	-7.06	372.53	-71.83
					BRK(252) **vs. low volatility (top) & low turnover benchmarks (bottom)**							
378	3.19	-0.55	32.52	-5.60	2.63	-0.23	26.80	-2.38	1.97	-0.15	20.07	-1.49
252	3.25	-0.42	33.14	-4.30	2.76	-0.17	28.13	-1.76	1.78	-0.11	18.10	-1.16
126	2.92	-0.49	29.69	-5.01	2.33	-0.18	23.70	-1.87	2.83	-0.13	28.78	-1.30
63	3.35	-0.23	34.09	-2.39	3.23	-0.10	32.85	-0.98	4.62	-0.07	47.04	-0.68
20	8.13	-0.23	82.74	-2.33	7.98	-0.10	81.19	-0.99	8.97	-0.07	91.28	-0.71
378	58.95	-167.78	599.50	-1706.19	59.86	-123.17	608.79	-1252.50	66.64	17.61	677.64	179.01
252	34.14	-17.80	347.33	-181.09	34.44	-7.11	350.39	-72.35	37.07	-4.96	377.13	-50.48
126	14.04	-2.69	142.93	-27.42	13.91	-1.05	141.54	-10.73	16.34	-0.63	166.34	-6.41
63	13.52	-1.07	137.59	-10.90	13.46	-0.41	136.99	-4.20	18.18	-0.28	185.00	-2.84
20	27.14	-0.80	276.21	-8.15	27.96	-0.34	284.55	-3.48	35.71	-0.25	363.33	-2.55

Table 7.14: Impact of Estimation Window on Basis Point Fees for Switching from Low-Frequency to High-Frequency Forecasts (Crisis, $\mu^{\mathbf{id}} = \mathbf{0.05}$)

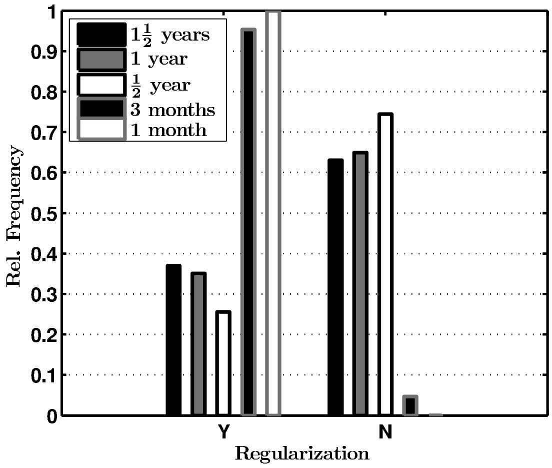
Medians ($m(\cdot)$) across 1,000 random samples of the annualized basis point fee (Δ^a_γ) a risk-averse investor with quadratic utility and relative risk aversion γ would pay to switch from covariance forecasts using regularized sample covariances of daily data computed over different windows to high-frequency-based forecasts. We assume that the constant conditional mean return is identical across all stocks and set it to $\mu^{id} = 0.05$ (annualized). Also reported are break-even transaction costs (c^*_γ) in percentage points, defined as the ratio of Δ^a_γ and the difference of average portfolio turnovers. Each random sample contains 85 assets out of the entire 100 asset universe. Evaluation is conducted for the period including the crisis, 07/2008 to 12/2009. For each window length of the sample covariance, we consider the (conditional or unconditional) regularization yielding the lowest median realized portfolio volatility ("low volatility") or median portfolio turnover ("low turnover"). Low-volatility benchmarks: unconditional shrinkage towards equicorrelation model (378 days) and shrinkage towards equicorrelation model (252, 126, 63 and 20 days). Low-turnover benchmarks: imposing one-factor model unconditionally (378 days) and imposing one-factor model (252, 126, 63 and 20 days).

	$\mathbf{h=1}$				$\mathbf{h=5}$				$\mathbf{h=20}$			
Wind.	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$
ERnB$(5)^*$ **vs. low volatility (top) & low turnover benchmarks (bottom)**												
378	50.10	1.01	509.60	10.28	43.58	0.37	443.33	3.76	30.79	0.23	313.27	2.38
252	52.71	1.18	536.12	12.03	46.64	0.44	474.41	4.50	35.15	0.33	357.68	3.34
126	59.85	1.75	608.68	17.80	54.93	0.69	558.64	7.01	45.67	0.85	464.61	8.66
63	57.00	2.06	579.76	20.97	54.15	0.86	550.78	8.75	47.03	2.39	478.38	24.31
20	49.17	3.95	500.11	40.21	44.95	1.57	457.27	15.94	42.65	-1.57	433.91	-15.99
378	62.09	1.11	631.42	11.29	55.41	0.41	563.55	4.18	41.20	0.25	419.14	2.54
252	56.42	1.02	573.80	10.42	49.94	0.38	507.94	3.84	37.49	0.24	381.49	2.39
126	60.10	1.15	611.18	11.68	56.81	0.45	577.75	4.62	51.34	0.36	522.17	3.69
63	65.07	1.34	661.66	13.65	66.00	0.58	671.16	5.90	70.33	0.59	715.09	6.01
20	56.09	1.92	570.50	19.48	53.52	0.76	544.36	7.72	55.44	1.22	563.86	12.37
BRK(252) **vs. low volatility (top) & low turnover benchmarks (bottom)**												
378	32.74	-5.16	333.15	-52.46	31.63	-2.14	321.86	-21.77	26.87	-1.05	273.47	-10.68
252	35.20	-3.09	358.12	-31.42	34.40	-1.24	350.05	-12.65	31.31	-0.62	318.61	-6.36
126	42.37	-1.96	431.02	-19.96	42.82	-0.81	435.57	-8.21	41.87	-0.40	426.01	-4.11
63	39.37	-1.40	400.52	-14.21	41.91	-0.60	426.36	-6.12	43.24	-0.31	439.91	-3.18
20	31.46	-0.72	320.14	-7.38	32.82	-0.32	333.97	-3.21	38.72	-0.21	393.98	-2.13
378	44.41	-209.85	451.76	-2134.58	43.20	24.85	439.47	252.85	37.28	5.33	379.33	54.26
252	38.87	-40.60	395.47	-413.06	37.82	-58.52	384.83	-595.43	33.80	14.19	343.94	144.41
126	42.72	-12.27	434.58	-124.87	44.66	-5.61	454.32	-57.04	47.54	-2.96	483.61	-30.11
63	47.67	-6.38	484.86	-64.89	53.98	-2.86	549.00	-29.08	66.68	-1.73	677.99	-17.63
20	38.39	-1.44	390.63	-14.66	41.18	-0.66	418.99	-6.74	51.62	-0.46	525.04	-4.70

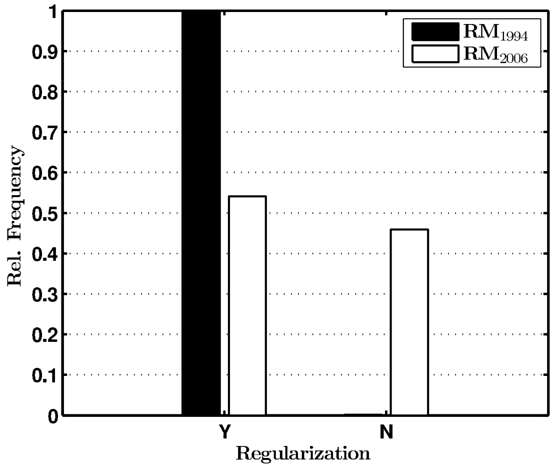
8 Additional Results for 30 Assets



(a) BRK Estimates ($G = 1$)



(b) Rolling-Window Sample Covariance



(c) RiskMetrics

Figure 8.1: Regularization Frequencies
BRK estimates are regularized if any correlation eigenvalue is negative or the condition number of the correlation matrix is greater than 10×30. The rolling-window sample covariance of daily returns and RiskMetrics forecasts are regularized if the condition number of the corresponding correlation matrix is greater than the above threshold.

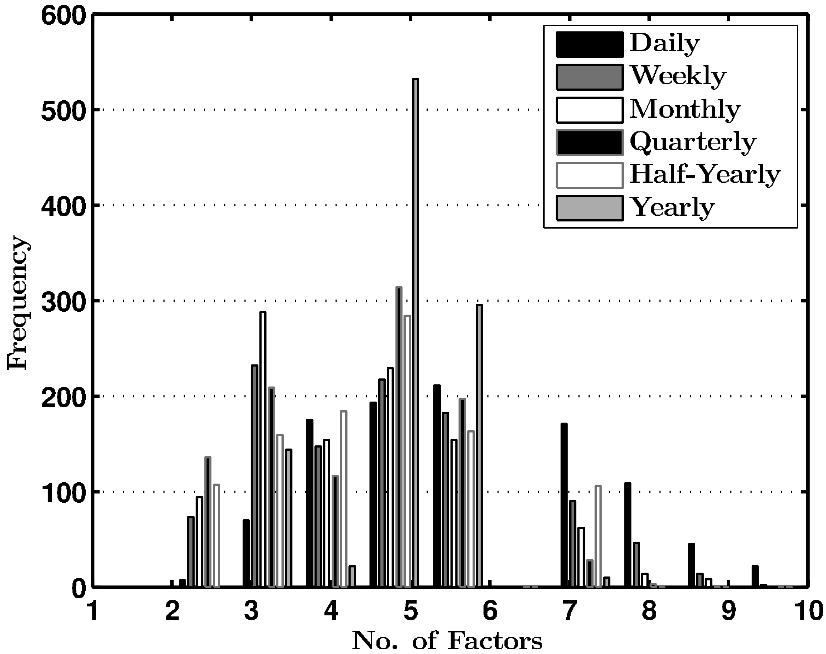


Figure 8.2: Sample Distribution of Number of Factors for FRnB Estimates
Number of factors is determined by applying Bai and Ng (2002) criteria from Section 3 to BRK estimates smoothed over different windows.

Table 8.1: Number of Liquidity Groups G and GMV Portfolio Volatility of ERnB(1) Forecasts
Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights for horizon $h = 1$ in percentage points. Each random sample contains 25 assets out of the entire 30 asset universe. Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	Pre-Crisis		**Crisis**	
G	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$
1	15.43	0.46	23.88	0.62
2	15.41	0.46	23.92	0.61
3	15.40	0.46	23.88	0.62
5	15.44	0.46	23.82	0.63
10	15.45	0.46	23.88	0.62

Table 8.2: GMV Portfolio Performance of Random-Walk-Type BRK Forecasts

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 25 assets out of the entire 30 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. BRK(S) represents random-walk-type forecasts based on unregularized BRK estimates smoothed over S days. Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
BRK(1)	15.43	0.46	136.54	-0.42	0.54	15.82	0.49	30.73	-0.42	0.54	16.56	0.05	8.55	-0.42	0.54
BRK(5)	14.97	0.47	33.12	-0.34	0.51	15.23	0.48	17.60	-0.34	0.51	15.94	0.05	6.12	-0.34	0.51
BRK(20)	15.11	0.49	10.31	-0.31	0.49	15.30	0.50	6.69	-0.31	0.49	15.73	0.05	4.35	-0.31	0.49
BRK(63)	15.39	0.52	4.08	-0.30	0.49	15.50	0.53	2.70	-0.30	0.49	15.80	0.06	2.02	-0.30	0.49
BRK(126)	15.61	0.55	2.44	-0.27	0.48	15.69	0.56	1.51	-0.27	0.48	15.93	0.06	1.15	-0.27	0.48
BRK(252)	16.03	0.63	1.82	-0.21	0.47	16.10	0.64	1.05	-0.21	0.47	16.32	0.07	0.78	-0.21	0.47
Crisis															
BRK(1)	23.88	0.62	132.37	-0.50	0.59	24.58	0.63	29.30	-0.50	0.59	25.56	0.06	8.29	-0.50	0.59
BRK(5)	23.43	0.59	31.79	-0.44	0.56	23.93	0.59	17.12	-0.44	0.56	24.68	0.06	5.80	-0.44	0.56
BRK(20)	23.77	0.58	10.38	-0.43	0.55	24.05	0.59	6.49	-0.43	0.55	24.66	0.06	4.24	-0.43	0.55
BRK(63)	24.35	0.60	5.07	-0.43	0.55	24.53	0.61	3.14	-0.43	0.55	24.95	0.06	2.28	-0.43	0.55
BRK(126)	24.83	0.63	4.05	-0.44	0.55	25.00	0.63	2.40	-0.44	0.55	25.39	0.06	1.70	-0.44	0.55
BRK(252)	25.15	0.67	3.34	-0.46	0.55	25.27	0.67	1.75	-0.46	0.55	25.55	0.07	1.11	-0.46	0.55

Table 8.3: GMV Portfolio Performance of ERnB and FRnB Forecasts Based on Unconditional Regularization

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 25 assets out of the entire 30 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. (Smoothed) BRK estimates are always regularized, which is indicated by an asterisk (*). Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h = 1}$					$\mathbf{h = 5}$					$\mathbf{h = 20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
ERnB(1)*	15.51	0.46	119.46	-0.36	0.54	15.85	0.50	27.12	-0.36	0.54	16.58	0.05	7.70	-0.36	0.54
ERnB(5)*	15.06	0.48	30.61	-0.32	0.53	15.31	0.50	16.30	-0.32	0.53	16.02	0.05	5.80	-0.32	0.53
ERnB(20)*	15.21	0.50	9.86	-0.30	0.52	15.40	0.51	6.45	-0.30	0.52	15.83	0.05	4.20	-0.30	0.52
ERnB(63)*	15.50	0.53	3.88	-0.29	0.53	15.60	0.55	2.68	-0.29	0.53	15.92	0.06	2.02	-0.29	0.53
ERnB(126)*	15.77	0.56	2.34	-0.26	0.52	15.85	0.57	1.48	-0.26	0.52	16.10	0.06	1.16	-0.26	0.52
ERnB(252)*	16.25	0.67	1.86	-0.22	0.52	16.31	0.68	1.11	-0.22	0.52	16.53	0.07	0.81	-0.22	0.52
FRnB(1)*	15.76	0.48	147.92	-0.48	0.58	16.11	0.50	32.88	-0.48	0.58	16.87	0.05	9.23	-0.48	0.58
FRnB(5)*	15.28	0.48	42.95	-0.42	0.55	15.52	0.49	20.66	-0.42	0.55	16.22	0.05	7.08	-0.42	0.55
FRnB(20)*	15.37	0.50	15.95	-0.40	0.54	15.53	0.51	9.41	-0.40	0.54	15.97	0.05	5.38	-0.40	0.54
FRnB(63)*	15.48	0.53	6.03	-0.38	0.54	15.59	0.54	4.27	-0.38	0.54	15.91	0.06	2.97	-0.38	0.54
FRnB(126)*	15.62	0.54	3.38	-0.34	0.52	15.70	0.55	2.17	-0.34	0.52	15.93	0.06	1.58	-0.34	0.52
FRnB(252)*	16.07	0.64	3.29	-0.28	0.51	16.14	0.65	1.69	-0.28	0.51	16.37	0.07	1.14	-0.28	0.51
Crisis															
ERnB(1)*	24.03	0.61	117.74	-0.46	0.59	24.76	0.62	26.37	-0.46	0.59	25.69	0.06	7.59	-0.46	0.59
ERnB(5)*	23.69	0.59	31.03	-0.44	0.58	24.14	0.60	16.31	-0.44	0.58	24.83	0.06	5.62	-0.44	0.58
ERnB(20)*	23.99	0.59	10.59	-0.44	0.57	24.22	0.60	6.49	-0.44	0.57	24.79	0.06	4.18	-0.44	0.57
ERnB(63)*	24.53	0.61	5.68	-0.44	0.57	24.71	0.61	3.30	-0.44	0.57	25.14	0.06	2.32	-0.44	0.57
ERnB(126)*	24.99	0.62	4.02	-0.45	0.56	25.13	0.63	2.40	-0.45	0.56	25.51	0.06	1.70	-0.45	0.56
ERnB(252)*	25.19	0.66	3.31	-0.46	0.56	25.31	0.67	1.74	-0.46	0.56	25.59	0.07	1.12	-0.46	0.56
FRnB(1)*	24.77	0.59	147.44	-0.57	0.62	25.46	0.59	32.56	-0.57	0.62	26.32	0.06	9.17	-0.57	0.62
FRnB(5)*	24.17	0.56	44.79	-0.53	0.61	24.58	0.57	20.77	-0.53	0.61	25.28	0.06	6.80	-0.53	0.61
FRnB(20)*	24.13	0.55	15.22	-0.53	0.60	24.40	0.56	8.58	-0.53	0.60	25.02	0.06	5.04	-0.53	0.60
FRnB(63)*	24.73	0.56	7.04	-0.53	0.60	24.91	0.57	4.29	-0.53	0.60	25.32	0.06	2.87	-0.53	0.60
FRnB(126)*	25.22	0.59	5.86	-0.53	0.59	25.39	0.60	3.43	-0.53	0.59	25.79	0.06	2.30	-0.53	0.59
FRnB(252)*	25.59	0.65	4.06	-0.53	0.58	25.71	0.65	2.23	-0.53	0.58	26.07	0.07	1.40	-0.53	0.58

Table 8.4: GMV Portfolio Performance of 3FRnB and 1FRnB Forecasts Based on Unconditional Regularization

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 25 assets out of the entire 30 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. (Smoothed) BRK estimates are always regularized, which is indicated by an asterisk (*). Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h = 1}$					$\mathbf{h = 5}$					$\mathbf{h = 20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
3FRnB(1)*	15.53	0.49	129.86	-0.45	0.56	15.90	0.51	29.55	-0.45	0.56	16.63	0.05	8.41	-0.45	0.56
3FRnB(5)*	15.20	0.50	36.09	-0.42	0.55	15.47	0.51	19.06	-0.42	0.55	16.16	0.05	6.77	-0.42	0.55
3FRnB(20)*	15.34	0.51	11.62	-0.41	0.54	15.51	0.52	7.69	-0.41	0.54	15.92	0.06	5.12	-0.41	0.54
3FRnB(63)*	15.53	0.53	5.13	-0.39	0.54	15.63	0.54	3.51	-0.39	0.54	15.93	0.06	2.66	-0.39	0.54
3FRnB(126)*	15.65	0.54	3.05	-0.36	0.53	15.73	0.55	1.93	-0.36	0.53	15.98	0.06	1.57	-0.36	0.53
3FRnB(252)*	16.10	0.65	2.19	-0.30	0.52	16.16	0.66	1.30	-0.30	0.52	16.39	0.07	1.01	-0.30	0.52
1FRnB(1)*	16.21	0.44	99.72	-0.38	0.51	16.61	0.47	23.20	-0.38	0.51	17.39	0.05	6.88	-0.38	0.51
1FRnB(5)*	16.16	0.43	25.93	-0.38	0.51	16.38	0.43	14.59	-0.38	0.51	17.14	0.05	5.47	-0.38	0.51
1FRnB(20)*	16.45	0.44	8.70	-0.38	0.51	16.60	0.44	6.03	-0.38	0.51	17.20	0.05	4.08	-0.38	0.51
1FRnB(63)*	16.82	0.44	3.83	-0.37	0.52	16.91	0.45	2.66	-0.37	0.52	17.39	0.05	2.09	-0.37	0.52
1FRnB(126)*	16.98	0.45	2.50	-0.34	0.51	17.05	0.46	1.55	-0.34	0.51	17.41	0.05	1.19	-0.34	0.51
1FRnB(252)*	17.96	0.54	1.97	-0.32	0.50	18.03	0.55	1.11	-0.32	0.50	18.40	0.06	0.83	-0.32	0.50
Crisis															
3FRnB(1)*	24.49	0.61	124.71	-0.53	0.61	25.21	0.62	28.14	-0.53	0.61	26.03	0.06	8.08	-0.53	0.61
3FRnB(5)*	24.20	0.59	34.73	-0.52	0.60	24.58	0.58	18.50	-0.52	0.60	25.22	0.06	6.35	-0.52	0.60
3FRnB(20)*	24.58	0.58	11.73	-0.52	0.60	24.80	0.58	7.40	-0.52	0.60	25.27	0.06	4.76	-0.52	0.60
3FRnB(63)*	25.28	0.61	6.02	-0.52	0.60	25.43	0.62	3.86	-0.52	0.60	25.83	0.06	2.82	-0.52	0.60
3FRnB(126)*	25.66	0.66	4.76	-0.52	0.59	25.82	0.66	2.93	-0.52	0.59	26.23	0.07	2.15	-0.52	0.59
3FRnB(252)*	25.69	0.67	3.81	-0.52	0.57	25.82	0.68	2.11	-0.52	0.57	26.12	0.07	1.41	-0.52	0.57
1FRnB(1)*	24.35	0.57	79.00	-0.47	0.57	24.95	0.59	18.85	-0.47	0.57	25.77	0.06	5.66	-0.47	0.57
1FRnB(5)*	24.57	0.52	20.99	-0.48	0.57	25.00	0.54	12.34	-0.48	0.57	25.55	0.06	4.49	-0.48	0.57
1FRnB(20)*	25.10	0.52	7.79	-0.50	0.57	25.37	0.54	5.22	-0.50	0.57	25.87	0.06	3.47	-0.50	0.57
1FRnB(63)*	26.05	0.58	4.43	-0.51	0.58	26.22	0.59	2.67	-0.51	0.58	26.51	0.06	1.88	-0.51	0.58
1FRnB(126)*	26.68	0.59	3.94	-0.53	0.58	26.80	0.60	2.25	-0.53	0.58	26.99	0.06	1.58	-0.53	0.58
1FRnB(252)*	26.72	0.57	3.60	-0.54	0.59	26.78	0.58	1.81	-0.54	0.59	26.80	0.06	1.12	-0.54	0.59

Table 8.5: GMV Portfolio Performance of Covariance Matrix Forecasts Employing Daily Returns

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 25 assets out of the entire 30 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on S-VEC and DCC models, regularized RiskMetrics1994 and RiskMetrics2006 estimators (RM$_{1994}$ and RM$_{2006}$), as well as the rolling window sample covariance matrix of daily returns over 252 days regularized by a one- or three-factor structure (1F or 3F), a factor structure based on the Bai and Ng (2002) criteria (BN-F), eigenvalue cleaning (S-EvCl) and shrinkage towards an equicorrelation or one-factor model (SHRK$_{EC}$ or SHRK$_{SF}$). In addition, results for the equally-weighted portfolio (EQW) are reported. Rolling-window sample covariance and RiskMetrics forecasts are regularized if the condition number of the corresponding correlation matrix is greater than 10×30. SCOV denotes the unregularized rolling window sample covariance. Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h = 1}$					$\mathbf{h = 5}$					$\mathbf{h = 20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
EQW	20.44	0.37	1.00	0.00	0.20	20.50	0.38	0.43	0.00	0.20	20.71	0.04	0.22	0.00	0.20
SCOV	16.32	0.54	9.38	-0.60	0.66	16.38	0.54	4.61	-0.60	0.66	16.54	0.06	2.42	-0.60	0.66
RM$_{1994}$	18.56	0.59	50.79	-0.95	0.79	18.72	0.58	26.08	-0.95	0.79	19.08	0.06	12.35	-0.95	0.79
RM$_{2006}$	17.31	0.54	51.26	-0.82	0.74	17.41	0.54	23.79	-0.82	0.74	17.57	0.06	9.36	-0.82	0.74
S-VECH	16.28	0.55	6.81	-0.55	0.67	16.35	0.55	3.47	-0.55	0.67	16.49	0.06	1.71	-0.55	0.67
DCC	16.08	0.55	39.66	-0.59	0.64	16.13	0.56	14.65	-0.59	0.64	16.34	0.06	5.17	-0.58	0.64
Crisis															
EQW	39.66	0.88	1.45	0.00	0.20	39.62	0.88	0.63	0.00	0.20	39.45	0.09	0.35	0.00	0.20
1F	26.31	0.65	9.24	-0.70	0.67	26.47	0.68	4.55	-0.70	0.67	26.69	0.07	2.37	-0.70	0.67
3F	26.72	0.74	10.85	-0.78	0.69	26.82	0.77	5.40	-0.78	0.69	26.94	0.08	2.78	-0.78	0.69
BN-F	28.74	0.84	16.93	-0.96	0.79	28.78	0.85	8.52	-0.96	0.79	28.69	0.09	4.33	-0.96	0.79
S-EvCl	27.43	0.72	11.92	-0.81	0.75	27.54	0.75	5.71	-0.81	0.75	27.60	0.08	3.03	-0.81	0.75
SHRK$_{EC}$	26.52	0.67	10.52	-0.76	0.72	26.63	0.70	5.08	-0.76	0.72	26.73	0.07	2.67	-0.76	0.72
SHRK$_{SF}$	26.83	0.68	11.80	-0.83	0.74	26.95	0.70	5.69	-0.83	0.74	27.06	0.07	2.98	-0.83	0.74
RM$_{1994}$	29.12	0.68	46.63	-0.89	0.77	29.58	0.75	23.70	-0.89	0.77	29.79	0.08	10.80	-0.89	0.77
RM$_{2006}$	28.02	0.65	45.57	-0.83	0.74	28.37	0.69	21.03	-0.82	0.74	28.43	0.07	7.98	-0.82	0.74
S-VECH	27.59	0.54	14.77	-0.80	0.75	27.70	0.57	7.31	-0.80	0.75	27.61	0.06	3.68	-0.79	0.75
DCC	25.68	0.66	39.40	-0.57	0.65	26.01	0.67	17.99	-0.57	0.65	26.67	0.07	6.52	-0.57	0.65

Table 8.6: GMV Portfolio Performance of Rolling Window Sample Covariance Matrix of Daily Returns and RiskMetrics2006 Regularized Unconditionally

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 25 assets out of the entire 30 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on the RiskMetrics2006 estimator (RM$^*_{2006}$), as well as the rolling window sample covariance matrix of daily returns over 252 days regularized by a one- or three-factor structure (1F* or 3F*), a factor structure based on the Bai and Ng (2002) criteria (BN-F*), eigenvalue cleaning (S-EvCl*) and shrinkage towards an equicorrelation or one-factor model (SHRK$^*_{EC}$ or SHRK$^*_{SF}$). Regularization is always carried out, which is indicated by an asterisk (*). Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
1F*	16.68	0.59	4.59	-0.49	0.56	16.73	0.59	2.36	-0.49	0.56	16.90	0.06	1.39	-0.49	0.56
3F*	16.54	0.67	7.26	-0.57	0.61	16.59	0.68	3.66	-0.57	0.61	16.72	0.07	1.99	-0.57	0.61
BN-F*	16.85	0.65	11.19	-0.65	0.68	16.90	0.66	5.46	-0.65	0.68	17.08	0.07	2.88	-0.65	0.68
S-EvCl*	16.64	0.53	7.17	-0.50	0.67	16.69	0.52	3.58	-0.50	0.67	16.90	0.05	1.96	-0.50	0.67
SHRK$^*_{EC}$	16.03	0.53	7.03	-0.52	0.63	16.09	0.54	3.47	-0.52	0.63	16.26	0.05	1.89	-0.52	0.63
SHRK$^*_{SF}$	16.04	0.53	7.73	-0.53	0.63	16.11	0.54	3.82	-0.53	0.63	16.27	0.05	2.04	-0.53	0.63
RM$^*_{2006}$	17.01	0.52	45.25	-0.74	0.70	17.07	0.52	21.63	-0.73	0.70	17.10	0.05	8.28	-0.71	0.69
Crisis															
1F*	26.28	0.66	6.11	-0.63	0.65	26.37	0.67	2.86	-0.63	0.65	26.41	0.07	1.60	-0.63	0.65
3F*	26.90	0.79	8.88	-0.75	0.68	27.07	0.81	4.25	-0.75	0.68	27.30	0.08	2.27	-0.75	0.68
BN-F*	29.10	0.88	19.05	-0.96	0.79	29.20	0.88	9.40	-0.96	0.79	29.17	0.09	4.49	-0.96	0.79
S-EvCl*	27.54	0.69	11.59	-0.76	0.74	27.66	0.70	5.22	-0.76	0.74	27.77	0.07	2.69	-0.76	0.74
SHRK$^*_{EC}$	26.36	0.66	9.00	-0.72	0.71	26.47	0.68	4.33	-0.72	0.71	26.56	0.07	2.37	-0.72	0.71
SHRK$^*_{SF}$	26.67	0.67	11.04	-0.81	0.74	26.80	0.70	5.34	-0.81	0.74	26.90	0.07	2.84	-0.81	0.74
RM$^*_{2006}$	27.92	0.64	42.35	-0.77	0.72	28.26	0.69	19.69	-0.77	0.71	28.29	0.07	7.45	-0.76	0.71

Table 8.7: Basis Point Fees for Switching from Low-Frequency to High-Frequency Forecasts (Pre-Crisis, $\mu^{\mathbf{id}} = \mathbf{0.05}$)

Medians ($m(\cdot)$) across 1,000 random samples of the annualized basis point fee (Δ_γ^a) a risk-averse investor with quadratic utility and relative risk aversion γ would pay to switch from covariance forecasts using daily data to high-frequency-based forecasts. We assume that the constant conditional mean return is identical across all stocks and set it to $\mu^{id} = 0.05$ (annualized). Also reported are break-even transaction costs (c_γ^*) in percentage points, defined as the ratio of Δ_γ^a and the difference of average portfolio turnovers. Each random sample contains 25 assets out of the 30 asset universe. Evaluation is conducted for the pre-crisis period, 01/2007 to 06/2008. For each length of the smoothing window applied to BRK estimates, the regularization method (conditional or unconditional(*)) that minimizes median realized portfolio volatility is chosen. BRK(S) represents random-walk-type forecasts based on unregularized BRK estimates smoothed over S days. The low-frequency benchmarks are the sample covariance computed over 252 days and unconditionally regularized by shrinkage towards an equicorrelation model (SHRK$^*_{EC}$) or by imposing a one factor model (1F*).

	$\mathbf{h=1}$				$\mathbf{h=5}$				$\mathbf{h=20}$			
	$m(\Delta_1^a)$	$m(c_1^*)$	$m(\Delta_{10}^a)$	$m(c_{10}^*)$	$m(\Delta_1^a)$	$m(c_1^*)$	$m(\Delta_{10}^a)$	$m(c_{10}^*)$	$m(\Delta_1^a)$	$m(c_1^*)$	$m(\Delta_{10}^a)$	$m(c_{10}^*)$
vs. SHRK$^*_{EC}$												
BRK(1)	8.85	0.08	88.62	0.75	4.05	0.03	40.52	0.28	-5.34	-0.04	-53.53	-0.45
BRK(5)	16.02	0.61	160.43	6.07	13.30	0.18	133.24	1.85	4.87	0.05	48.74	0.53
BRK(20)	14.19	4.32	142.16	43.24	12.39	0.75	124.11	7.54	8.15	0.16	81.58	1.62
BRK(63)	9.98	-3.12	99.91	-31.26	9.16	-2.17	91.78	-21.71	6.66	2.33	66.71	23.36
BRK(126)	6.13	-1.23	61.35	-12.37	5.75	-0.55	57.65	-5.49	4.35	-0.28	43.60	-2.78
BRK(252)	-0.92	0.26	-9.26	2.60	-1.15	0.13	-11.55	1.31	-2.26	0.13	-22.62	1.27
vs. 1F*												
BRK(1)	19.98	0.16	200.10	1.57	14.94	0.10	149.62	1.04	5.96	0.04	59.73	0.38
BRK(5)	27.08	0.94	271.17	9.42	24.17	0.31	242.00	3.14	16.11	0.17	161.29	1.67
BRK(20)	25.16	4.33	251.97	43.38	23.17	1.05	232.03	10.54	19.34	0.32	193.67	3.21
BRK(63)	20.94	-35.55	209.67	-355.95	19.93	11.45	199.54	114.63	17.86	1.40	178.87	13.99
BRK(126)	17.21	-7.45	172.32	-74.65	16.60	-3.72	166.27	-37.22	15.52	-3.10	155.43	-31.03
BRK(252)	9.99	-3.31	100.07	-33.20	9.55	-1.37	95.66	-13.70	8.83	-0.67	88.46	-6.73

Table 8.8: GMV Portfolio Performance of Rolling Window Sample Covariance Matrix of Daily Returns over 378 Days Regularized Conditionally or Unconditionally

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 25 assets out of the entire 30 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on the rolling window sample covariance matrix of daily returns over 378 days regularized by a one- or three-factor structure (1F or 3F), a factor structure based on the Bai and Ng (2002) criteria (BN-F), eigenvalue cleaning (S-EvCl) and shrinkage towards an equicorrelation or one-factor model (SHRK$_{EC}$ or SHRK$_{SF}$). Regularization is carried out if the condition number of the corresponding correlation matrix is greater than 10×100. In addition, unconditional regularization is considered and indicated by an asterisk (*). SCOV denotes the unregularized rolling window sample covariance. Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
SCOV	17.78	0.57	7.12	-0.61	0.66	17.89	0.58	3.59	-0.61	0.66	18.12	0.06	1.93	-0.61	0.66
1F*	18.47	0.68	3.73	-0.49	0.57	18.58	0.69	1.89	-0.49	0.57	18.90	0.07	1.13	-0.49	0.57
3F*	18.00	0.79	5.96	-0.57	0.62	18.08	0.80	2.96	-0.57	0.62	18.25	0.08	1.64	-0.57	0.62
BN-F*	18.25	0.74	9.80	-0.62	0.67	18.38	0.75	4.81	-0.62	0.67	18.63	0.08	2.36	-0.62	0.67
S-EvCl*	18.14	0.48	7.12	-0.49	0.67	18.25	0.50	3.39	-0.49	0.67	18.53	0.05	1.80	-0.49	0.67
SHRK$^*_{EC}$	17.59	0.57	5.92	-0.54	0.64	17.68	0.58	3.01	-0.54	0.64	17.92	0.06	1.65	-0.54	0.64
SHRK$^*_{SF}$	17.60	0.58	6.33	-0.56	0.64	17.70	0.59	3.21	-0.56	0.64	17.94	0.06	1.74	-0.56	0.64
Crisis															
1F	26.53	0.61	16.03	-0.69	0.68	26.63	0.62	6.29	-0.69	0.68	26.75	0.06	3.40	-0.69	0.68
3F	26.60	0.68	16.34	-0.77	0.69	26.77	0.69	6.53	-0.77	0.69	26.84	0.07	3.57	-0.77	0.69
BN-F	27.51	0.76	16.64	-0.87	0.76	27.60	0.77	7.07	-0.87	0.76	27.55	0.08	3.86	-0.87	0.76
S-EvCl	26.93	0.71	12.74	-0.78	0.74	27.02	0.73	5.36	-0.78	0.74	27.05	0.07	2.86	-0.78	0.74
SHRK$_{EC}$	26.35	0.65	10.37	-0.75	0.72	26.46	0.67	4.44	-0.75	0.72	26.54	0.07	2.35	-0.75	0.72
SHRK$_{SF}$	26.56	0.65	9.04	-0.79	0.73	26.66	0.67	4.11	-0.79	0.73	26.74	0.07	2.19	-0.79	0.73
1F*	26.10	0.63	4.93	-0.61	0.65	26.17	0.64	2.31	-0.61	0.65	26.16	0.07	1.34	-0.61	0.65
3F*	26.81	0.81	6.70	-0.74	0.67	26.96	0.83	3.17	-0.74	0.67	27.08	0.08	1.77	-0.74	0.67
BN-F*	28.06	0.84	13.20	-0.89	0.76	28.16	0.85	6.13	-0.89	0.76	28.20	0.08	3.40	-0.89	0.76
S-EvCl*	27.28	0.69	6.98	-0.74	0.74	27.36	0.70	3.35	-0.74	0.74	27.41	0.07	1.90	-0.74	0.74
SHRK$^*_{EC}$	26.10	0.66	6.62	-0.71	0.70	26.18	0.67	3.13	-0.71	0.70	26.25	0.07	1.77	-0.71	0.70
SHRK$^*_{SF}$	26.32	0.66	7.69	-0.77	0.72	26.41	0.67	3.68	-0.77	0.72	26.48	0.07	2.03	-0.77	0.72

Table 8.9: GMV Portfolio Performance of Rolling Window Sample Covariance Matrix of Daily Returns over 252 Days Regularized Conditionally or Unconditionally

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 25 assets out of the entire 30 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on the rolling window sample covariance matrix of daily returns over 252 days regularized by a one- or three-factor structure (1F or 3F), a factor structure based on the Bai and Ng (2002) criteria (BN-F), eigenvalue cleaning (S-EvCl) and shrinkage towards an equicorrelation or one-factor model (SHRK$_{EC}$ or SHRK$_{SF}$). Regularization is carried out if the condition number of the corresponding correlation matrix is greater than 10×100. In addition, unconditional regularization is considered and indicated by an asterisk (*). SCOV denotes the unregularized rolling window sample covariance. Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
SCOV	17.77	0.55	9.84	-0.65	0.66	17.89	0.56	4.90	-0.65	0.66	18.06	0.06	2.60	-0.65	0.66
1F*	18.05	0.61	5.17	-0.52	0.58	18.16	0.62	2.67	-0.52	0.58	18.38	0.06	1.62	-0.52	0.58
3F*	17.95	0.69	7.59	-0.61	0.62	18.05	0.70	3.88	-0.61	0.62	18.18	0.07	2.21	-0.61	0.62
BN-F*	18.22	0.65	11.86	-0.66	0.67	18.35	0.66	5.86	-0.66	0.67	18.55	0.07	3.13	-0.66	0.67
S-EvCl*	17.99	0.54	7.68	-0.53	0.64	18.11	0.54	3.88	-0.53	0.64	18.35	0.05	2.17	-0.53	0.64
SHRK$^*_{EC}$	17.43	0.55	7.15	-0.56	0.62	17.54	0.55	3.59	-0.56	0.62	17.72	0.06	1.98	-0.56	0.62
SHRK$^*_{SF}$	17.45	0.55	8.01	-0.57	0.62	17.56	0.55	4.01	-0.57	0.62	17.75	0.06	2.17	-0.57	0.62
Crisis															
1F	26.31	0.65	9.24	-0.70	0.67	26.47	0.68	4.55	-0.70	0.67	26.69	0.07	2.37	-0.70	0.67
3F	26.72	0.74	10.85	-0.78	0.69	26.82	0.77	5.40	-0.78	0.69	26.94	0.08	2.78	-0.78	0.69
BN-F	28.74	0.84	16.93	-0.96	0.79	28.78	0.85	8.52	-0.96	0.79	28.69	0.09	4.33	-0.96	0.79
S-EvCl	27.43	0.72	11.92	-0.81	0.75	27.54	0.75	5.71	-0.81	0.75	27.60	0.08	3.03	-0.81	0.75
SHRK$_{EC}$	26.52	0.67	10.52	-0.76	0.72	26.63	0.70	5.08	-0.76	0.72	26.73	0.07	2.67	-0.76	0.72
SHRK$_{SF}$	26.83	0.68	11.80	-0.83	0.74	26.95	0.70	5.69	-0.83	0.74	27.06	0.07	2.98	-0.83	0.74
1F*	26.28	0.66	6.11	-0.63	0.65	26.37	0.67	2.86	-0.63	0.65	26.41	0.07	1.60	-0.63	0.65
3F*	26.90	0.79	8.88	-0.75	0.68	27.07	0.81	4.25	-0.75	0.68	27.30	0.08	2.27	-0.75	0.68
BN-F*	29.10	0.88	19.05	-0.96	0.79	29.20	0.88	9.40	-0.96	0.79	29.17	0.09	4.49	-0.96	0.79
S-EvCl*	27.54	0.69	11.59	-0.76	0.74	27.66	0.70	5.22	-0.76	0.74	27.77	0.07	2.69	-0.76	0.74
SHRK$^*_{EC}$	26.36	0.66	9.00	-0.72	0.71	26.47	0.68	4.33	-0.72	0.71	26.56	0.07	2.37	-0.72	0.71
SHRK$^*_{SF}$	26.67	0.67	11.04	-0.81	0.74	26.80	0.70	5.34	-0.81	0.74	26.90	0.07	2.84	-0.81	0.74

Table 8.10: GMV Portfolio Performance of Rolling Window Sample Covariance Matrix of Daily Returns over 126 Days Regularized Conditionally or Unconditionally

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 25 assets out of the entire 30 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on the rolling window sample covariance matrix of daily returns over 126 days regularized by a one- or three-factor structure (1F or 3F), a factor structure based on the Bai and Ng (2002) criteria (BN-F), eigenvalue cleaning (S-EvCl) and shrinkage towards an equicorrelation or one-factor model (SHRK$_{EC}$ or SHRK$_{SF}$). Regularization is carried out if the condition number of the corresponding correlation matrix is greater than 10×100. In addition, unconditional regularization is considered and indicated by an asterisk (*). Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h = 1}$					$\mathbf{h = 5}$					$\mathbf{h = 20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
1F	18.70	0.69	26.06	-0.85	0.74	18.84	0.69	12.05	-0.85	0.74	19.16	0.07	6.35	-0.85	0.74
3F	18.70	0.69	25.63	-0.86	0.74	18.82	0.70	11.89	-0.86	0.74	19.15	0.07	6.24	-0.86	0.74
BN-F	18.76	0.68	25.42	-0.88	0.75	18.89	0.69	11.92	-0.88	0.75	19.22	0.07	6.17	-0.88	0.75
S-EvCl	18.68	0.69	25.13	-0.85	0.74	18.82	0.69	11.67	-0.85	0.74	19.14	0.07	6.12	-0.85	0.74
SHRK$_{EC}$	18.68	0.69	23.51	-0.86	0.74	18.82	0.70	11.07	-0.86	0.74	19.14	0.07	5.87	-0.86	0.74
SHRK$_{SF}$	18.67	0.69	22.83	-0.86	0.74	18.81	0.69	10.79	-0.86	0.74	19.13	0.07	5.77	-0.86	0.74
1F*	17.74	0.65	8.44	-0.57	0.60	17.86	0.66	4.33	-0.57	0.60	18.19	0.07	2.52	-0.57	0.60
3F*	17.76	0.68	12.26	-0.68	0.63	17.85	0.69	6.15	-0.68	0.63	18.17	0.07	3.41	-0.68	0.63
BN-F*	18.97	0.75	22.69	-0.86	0.73	19.13	0.76	10.40	-0.86	0.73	19.48	0.08	5.23	-0.86	0.73
S-EvCl*	17.83	0.63	13.24	-0.63	0.64	17.97	0.64	6.51	-0.63	0.64	18.34	0.07	3.59	-0.63	0.64
SHRK$^*_{EC}$	17.43	0.61	10.79	-0.63	0.64	17.56	0.63	5.50	-0.63	0.64	17.88	0.07	3.16	-0.63	0.64
SHRK$^*_{SF}$	17.45	0.62	12.96	-0.66	0.64	17.59	0.62	6.57	-0.66	0.64	17.90	0.06	3.69	-0.66	0.64
Crisis															
1F	26.55	0.60	19.04	-0.76	0.71	26.85	0.61	9.17	-0.76	0.71	27.39	0.07	4.77	-0.76	0.71
3F	27.07	0.68	21.55	-0.81	0.73	27.33	0.71	10.33	-0.81	0.73	27.74	0.07	5.21	-0.81	0.73
BN-F	30.37	0.97	29.81	-1.00	0.81	30.55	0.99	13.66	-1.00	0.81	30.47	0.10	7.00	-1.00	0.81
S-EvCl	28.29	0.59	23.13	-0.84	0.76	28.54	0.62	11.09	-0.84	0.76	28.79	0.07	5.68	-0.84	0.76
SHRK$_{EC}$	27.25	0.56	20.45	-0.80	0.75	27.48	0.60	9.81	-0.80	0.75	27.78	0.07	5.14	-0.80	0.75
SHRK$_{SF}$	28.16	0.59	22.80	-0.87	0.77	28.39	0.65	10.94	-0.87	0.77	28.54	0.07	5.63	-0.87	0.77
1F*	25.89	0.60	8.40	-0.56	0.61	26.01	0.62	4.18	-0.56	0.61	26.14	0.06	2.39	-0.56	0.61
3F*	26.49	0.70	13.23	-0.67	0.65	26.68	0.72	6.55	-0.67	0.65	26.93	0.07	3.44	-0.67	0.65
BN-F*	30.53	1.03	32.21	-0.98	0.79	30.68	1.05	14.15	-0.98	0.79	30.63	0.10	6.76	-0.98	0.79
S-EvCl*	27.92	0.57	17.66	-0.68	0.73	28.14	0.59	8.61	-0.68	0.73	28.43	0.06	4.31	-0.68	0.73
SHRK$^*_{EC}$	26.70	0.54	13.71	-0.65	0.69	26.89	0.56	6.72	-0.65	0.69	27.08	0.06	3.61	-0.65	0.69
SHRK$^*_{SF}$	27.70	0.58	18.02	-0.76	0.72	27.88	0.63	8.78	-0.76	0.72	27.98	0.07	4.61	-0.76	0.72

Table 8.11: GMV Portfolio Performance of Rolling Window Sample Covariance Matrix of Daily Returns over 63 Days Regularized Conditionally or Unconditionally

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 25 assets out of the entire 30 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on the rolling window sample covariance matrix of daily returns over 63 days regularized by a one- or three-factor structure (1F or 3F), a factor structure based on the Bai and Ng (2002) criteria (BN-F), eigenvalue cleaning (S-EvCl) and shrinkage towards an equicorrelation or one-factor model (SHRK$_{EC}$ or SHRK$_{SF}$). Regularization is carried out if the condition number of the corresponding correlation matrix is greater than 10×100. In addition, unconditional regularization is considered and indicated by an asterisk (*). Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
1F	17.91	0.61	18.07	-0.56	0.61	18.04	0.62	8.62	-0.56	0.61	18.56	0.06	4.57	-0.56	0.61
3F	18.02	0.69	25.75	-0.68	0.65	18.13	0.70	11.87	-0.68	0.65	18.54	0.07	5.85	-0.68	0.65
BN-F	19.97	0.82	45.47	-0.96	0.78	20.15	0.83	20.13	-0.96	0.78	20.62	0.09	9.50	-0.96	0.78
S-EvCl	18.20	0.66	25.24	-0.61	0.68	18.34	0.67	11.76	-0.61	0.68	18.80	0.07	5.90	-0.61	0.68
SHRK$_{EC}$	17.89	0.61	20.26	-0.63	0.66	18.04	0.63	9.84	-0.63	0.66	18.44	0.06	5.51	-0.63	0.66
SHRK$_{SF}$	17.85	0.61	24.59	-0.67	0.67	18.01	0.63	11.84	-0.67	0.67	18.49	0.07	6.37	-0.67	0.67
1F*	17.89	0.61	15.03	-0.56	0.60	18.02	0.62	7.86	-0.56	0.60	18.53	0.06	4.41	-0.56	0.60
3F*	18.00	0.69	22.87	-0.67	0.65	18.10	0.70	11.14	-0.67	0.65	18.51	0.07	5.74	-0.67	0.65
BN-F*	19.97	0.82	43.64	-0.96	0.78	20.15	0.82	19.75	-0.96	0.78	20.61	0.09	9.42	-0.96	0.78
S-EvCl*	18.17	0.66	22.42	-0.61	0.67	18.32	0.67	11.13	-0.61	0.67	18.77	0.07	5.77	-0.61	0.67
SHRK$_{EC}^*$	17.90	0.62	17.03	-0.62	0.66	18.03	0.63	9.01	-0.62	0.66	18.43	0.06	5.35	-0.62	0.66
SHRK$_{SF}^*$	17.82	0.61	22.38	-0.67	0.67	17.98	0.62	11.34	-0.67	0.67	18.46	0.06	6.27	-0.67	0.67
Crisis															
1F	25.58	0.63	13.54	-0.51	0.59	25.77	0.65	6.51	-0.51	0.59	25.92	0.06	3.40	-0.51	0.59
3F	26.31	0.69	21.43	-0.64	0.63	26.51	0.72	10.25	-0.64	0.63	26.74	0.07	5.02	-0.64	0.63
BN-F	30.97	1.11	49.51	-1.03	0.79	31.23	1.15	21.74	-1.03	0.79	31.30	0.12	9.69	-1.03	0.79
S-EvCl	27.34	0.57	27.09	-0.64	0.68	27.50	0.59	12.47	-0.64	0.68	27.75	0.06	5.93	-0.64	0.68
SHRK$_{EC}$	26.20	0.56	18.82	-0.61	0.68	26.39	0.59	9.23	-0.61	0.68	26.72	0.06	4.76	-0.61	0.68
SHRK$_{SF}$	27.32	0.65	25.17	-0.73	0.70	27.56	0.68	12.26	-0.73	0.70	27.88	0.07	6.27	-0.73	0.70
1F*	25.57	0.63	12.45	-0.51	0.58	25.77	0.65	6.30	-0.51	0.58	25.91	0.06	3.36	-0.51	0.58
3F*	26.31	0.69	20.44	-0.64	0.63	26.50	0.72	10.05	-0.64	0.63	26.74	0.07	4.98	-0.64	0.63
BN-F*	30.97	1.11	48.67	-1.03	0.79	31.23	1.15	21.59	-1.03	0.79	31.30	0.12	9.67	-1.03	0.79
S-EvCl*	27.34	0.57	26.19	-0.64	0.68	27.50	0.59	12.30	-0.64	0.68	27.74	0.06	5.90	-0.64	0.68
SHRK$_{EC}^*$	26.20	0.56	17.83	-0.61	0.68	26.39	0.59	9.04	-0.61	0.68	26.71	0.06	4.72	-0.61	0.68
SHRK$_{SF}^*$	27.32	0.65	24.35	-0.73	0.70	27.56	0.68	12.11	-0.73	0.70	27.88	0.07	6.24	-0.73	0.70

Table 8.12: GMV Portfolio Performance of Regularized Rolling Window Sample Covariance Matrix of Daily Returns over 20 Days

Medians ($m(\cdot)$) and standard deviations ($s(\cdot)$) across 1,000 random samples of the square root of the annualized average realized portfolio variance ($\bar{\sigma}_p^a$) using predicted GMV weights (in percentage points). Each random sample contains 25 assets out of the entire 30 asset universe. $\overline{\text{po}}$ is the average turnover as defined in (25*) expressed in percentage points. $\overline{\text{pc}}$ denotes the sample average of the portfolio concentration measure (26*). $\overline{\text{sp}}$ is the sample average of the sum of negative portfolio weights. Forecasts are based on the rolling window sample covariance matrix of daily returns over 20 days regularized by a one- or three-factor structure (1F or 3F), a factor structure based on the Bai and Ng (2002) criteria (BN-F), eigenvalue cleaning (S-EvCl) and shrinkage towards an equicorrelation or one-factor model (SHRK$_{EC}$ or SHRK$_{SF}$). Regularization is carried out if the condition number of the corresponding correlation matrix is greater than 10×100. Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008, and the period including the crisis, 07/2008 to 12/2009.

	$\mathbf{h=1}$					$\mathbf{h=5}$					$\mathbf{h=20}$				
	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$	$m(\bar{\sigma}_p^a)$	$s(\bar{\sigma}_p^a)$	$m(\overline{po})$	$m(\overline{sp})$	$m(\overline{pc})$
Pre-Crisis															
1F	18.28	0.63	38.01	-0.50	0.58	18.47	0.65	18.39	-0.50	0.58	19.11	0.06	8.99	-0.50	0.58
3F	18.71	0.67	53.71	-0.66	0.65	18.86	0.70	25.20	-0.66	0.65	19.26	0.07	11.40	-0.66	0.65
BN-F	23.20	0.79	116.77	-1.15	0.87	23.31	0.79	47.32	-1.15	0.87	23.67	0.09	18.83	-1.15	0.87
S-EvCl	18.27	0.66	45.10	-0.55	0.66	18.57	0.68	21.83	-0.55	0.66	19.22	0.07	10.64	-0.55	0.66
SHRK$_{EC}$	18.45	0.62	39.72	-0.55	0.68	18.65	0.64	19.70	-0.55	0.68	19.14	0.06	9.80	-0.55	0.68
SHRK$_{SF}$	18.61	0.63	50.23	-0.64	0.67	18.84	0.64	24.10	-0.64	0.67	19.35	0.06	11.33	-0.64	0.67
Crisis															
1F	25.70	0.82	31.47	-0.48	0.57	26.17	0.83	15.35	-0.48	0.57	26.94	0.09	7.54	-0.48	0.57
3F	26.85	0.79	48.47	-0.63	0.63	27.35	0.79	22.28	-0.63	0.63	28.00	0.08	10.23	-0.63	0.63
BN-F	34.21	1.24	122.82	-1.15	0.85	35.04	1.31	47.95	-1.15	0.85	35.18	0.13	19.31	-1.15	0.85
S-EvCl	26.75	0.75	43.41	-0.54	0.63	27.00	0.79	20.04	-0.54	0.63	27.75	0.08	9.58	-0.54	0.63
SHRK$_{EC}$	25.73	0.74	35.05	-0.55	0.67	26.04	0.79	17.04	-0.55	0.67	26.79	0.08	8.29	-0.55	0.67
SHRK$_{SF}$	27.08	0.80	50.42	-0.66	0.67	27.65	0.82	23.78	-0.66	0.67	28.63	0.08	11.48	-0.66	0.67

Table 8.13: Impact of Estimation Window on Basis Point Fees for Switching from Low-Frequency to High-Frequency Forecasts (Pre-Crisis, $\mu^{\mathbf{id}} = \mathbf{0.05}$) Medians ($m(\cdot)$) across 1,000 random samples of the annualized basis point fee (Δ_γ^a) a risk-averse investor with quadratic utility and relative risk aversion γ would pay to switch from covariance forecasts using regularized sample covariances of daily data computed over different windows to high-frequency-based forecasts. We assume that the constant conditional mean return is identical across all stocks and set it to $\mu^{id} = 0.05$ (annualized). Also reported are break-even transaction costs (c_γ^*) in percentage points, defined as the ratio of Δ_γ^a and the difference of average portfolio turnovers. Each random sample contains 25 assets out of the entire 30 asset universe. Evaluation is conducted for the pre-crisis period, 07/2007 to 06/2008. For each window length of the sample covariance, we consider the (conditional or unconditional) regularization yielding the lowest median realized portfolio volatility ("low volatility") or median portfolio turnover ("low turnover"). Low-volatility benchmarks: unconditional shrinkage towards equicorrelation model (378, 252 and 126 days), unconditional shrinkage towards one-factor model (63) and imposing one-factor model (20). Low-turnover benchmarks: imposing one-factor model unconditionally (378, 252, 126 and 63 days) and imposing one-factor model (20).

	$\mathbf{h = 1}$				$\mathbf{h = 5}$				$\mathbf{h = 20}$			
Wind.	$m(\Delta_1^a)$	$m(c_1^*)$	$m(\Delta_{10}^a)$	$m(c_{10}^*)$	$m(\Delta_1^a)$	$m(c_1^*)$	$m(\Delta_{10}^a)$	$m(c_{10}^*)$	$m(\Delta_1^a)$	$m(c_1^*)$	$m(\Delta_{10}^a)$	$m(c_{10}^*)$
ERnB$(5)^*$ **vs. low volatility (top) & low turnover benchmarks (bottom)**												
378	21.60	0.80	219.85	8.10	18.19	0.24	185.13	2.46	8.62	0.06	87.76	0.60
252	18.30	0.65	186.23	6.65	15.10	0.19	153.72	1.92	4.44	0.02	45.18	0.25
126	18.41	0.86	187.43	8.74	15.52	0.26	157.97	2.67	7.55	0.12	76.84	1.22
63	25.47	2.51	259.21	25.51	23.20	0.77	236.07	7.82	18.31	-3.80	186.36	-38.71
20	33.89	-7.82	344.84	-79.54	32.19	-8.71	327.53	-88.60	30.35	-0.58	308.83	-5.86
378	37.68	1.29	383.33	13.09	34.55	0.43	351.53	4.35	26.86	0.22	273.39	2.24
252	30.21	1.04	307.45	10.59	27.13	0.33	276.13	3.40	17.35	0.15	176.56	1.48
126	24.37	1.03	248.04	10.49	21.47	0.33	218.55	3.34	13.95	0.17	142.00	1.77
63	26.65	1.58	271.23	16.11	23.79	0.52	242.15	5.25	19.49	0.53	198.37	5.44
20	33.89	-7.82	344.84	-79.54	32.19	-8.71	327.53	-88.60	30.35	-0.58	308.83	-5.86
BRK(252) **vs. low volatility (top) & low turnover benchmarks (bottom)**												
378	0.36	-0.15	3.70	-1.55	-0.16	-0.03	-1.58	-0.33	-0.79	0.07	-8.08	0.71
252	-2.61	0.70	-26.55	7.15	-2.94	0.32	-29.93	3.21	-4.60	0.20	-46.87	2.01
126	-2.59	0.20	-26.33	2.02	-2.65	0.07	-26.96	0.72	-1.83	-0.00	-18.63	-0.03
63	4.37	-0.29	44.47	-2.97	5.03	-0.14	51.21	-1.38	9.10	-0.11	92.66	-1.07
20	12.44	-0.44	126.59	-4.52	13.49	-0.20	137.33	-2.03	20.93	-0.16	212.99	-1.61
378	16.36	-8.88	166.52	-90.35	16.09	-3.97	163.80	-40.41	17.04	-2.60	173.40	-26.47
252	8.93	-2.43	90.95	-24.78	8.67	-0.95	88.29	-9.67	7.78	-0.42	79.16	-4.29
126	3.47	-0.72	35.31	-7.34	3.39	-0.29	34.47	-2.91	4.71	-0.17	47.99	-1.77
63	5.79	-0.60	58.96	-6.14	5.92	-0.24	60.31	-2.41	10.31	-0.17	104.91	-1.71
20	12.44	-0.44	126.59	-4.52	13.49	-0.20	137.33	-2.03	20.93	-0.16	212.99	-1.61

Table 8.14: Impact of Estimation Window on Basis Point Fees for Switching from Low-Frequency to High-Frequency Forecasts (Crisis, $\mu^{\mathbf{id}} = \mathbf{0.05}$)
Medians ($m(\cdot)$) across 1,000 random samples of the annualized basis point fee (Δ^a_γ) a risk-averse investor with quadratic utility and relative risk aversion γ would pay to switch from covariance forecasts using regularized sample covariances of daily data computed over different windows to high-frequency-based forecasts. We assume that the constant conditional mean return is identical across all stocks and set it to $\mu^{id} = 0.05$ (annualized). Also reported are break-even transaction costs (c^*_γ) in percentage points, defined as the ratio of Δ^a_γ and the difference of average portfolio turnovers. Each random sample contains 25 assets out of the entire 30 asset universe. Evaluation is conducted for the period including the crisis, 07/2008 to 12/2009. For each window length of the sample covariance, we consider the (conditional or unconditional) regularization yielding the lowest median realized portfolio volatility ("low volatility") or median portfolio turnover ("low turnover"). Low-volatility benchmarks: imposing one-factor model unconditionally (378, 252, 126 and 63 days) and shrinkage towards equicorrelation model (20). Low-turnover benchmarks: imposing one-factor model unconditionally (378, 252, 126 and 63 days) and imposing one-factor model (20).

	$\mathbf{h = 1}$				$\mathbf{h = 5}$				$\mathbf{h = 20}$			
Wind.	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$	$m(\Delta^a_1)$	$m(c^*_1)$	$m(\Delta^a_{10})$	$m(c^*_{10})$
ERnB$(5)^*$ **vs. low volatility (top) & low turnover benchmarks (bottom)**												
378	67.86	2.53	690.02	25.74	58.32	0.79	593.07	8.04	40.04	0.45	407.36	4.56
252	72.77	2.84	739.89	28.90	63.79	0.89	648.70	9.10	46.97	0.56	477.82	5.70
126	61.37	2.63	624.13	26.75	53.19	0.83	541.05	8.41	38.60	0.57	392.77	5.82
63	53.14	2.74	540.47	27.91	46.50	0.87	473.06	8.82	32.13	0.67	326.96	6.80
20	57.51	-17.51	584.91	-178.08	54.68	13.02	556.12	132.45	57.11	-1.16	580.88	-11.75
378	67.86	2.53	690.02	25.74	58.32	0.79	593.07	8.04	40.04	0.45	407.36	4.56
252	72.77	2.84	739.89	28.90	63.79	0.89	648.70	9.10	46.97	0.56	477.82	5.70
126	61.37	2.63	624.13	26.75	53.19	0.83	541.05	8.41	38.60	0.57	392.77	5.82
63	53.14	2.74	540.47	27.91	46.50	0.87	473.06	8.82	32.13	0.67	326.96	6.80
20	57.29	32.98	582.62	335.41	57.63	6.61	586.08	67.27	60.59	-1.74	616.21	-17.72
BRK(252) **vs. low volatility (top) & low turnover benchmarks (bottom)**												
378	26.17	-15.31	266.31	-155.82	25.48	-8.43	259.32	-85.76	18.16	-3.73	184.89	-37.99
252	31.25	-10.80	317.97	-109.93	31.01	-5.32	315.52	-54.16	25.32	-2.46	257.72	-25.08
126	19.25	-3.69	195.95	-37.53	20.12	-1.59	204.76	-16.15	16.39	-0.62	166.85	-6.31
63	11.04	-1.18	112.35	-12.06	13.29	-0.57	135.26	-5.80	9.57	-0.21	97.46	-2.10
20	16.64	-0.53	169.36	-5.36	22.75	-0.30	231.56	-3.04	36.21	-0.25	368.47	-2.56
378	26.17	-15.31	266.31	-155.82	25.48	-8.43	259.32	-85.76	18.16	-3.73	184.89	-37.99
252	31.25	-10.80	317.97	-109.93	31.01	-5.32	315.52	-54.16	25.32	-2.46	257.72	-25.08
126	19.25	-3.69	195.95	-37.53	20.12	-1.59	204.76	-16.15	16.39	-0.62	166.85	-6.31
63	11.04	-1.18	112.35	-12.06	13.29	-0.57	135.26	-5.80	9.57	-0.21	97.46	-2.10
20	15.97	-0.57	162.53	-5.78	25.30	-0.37	257.47	-3.79	38.86	-0.30	395.39	-3.08

References

Bai J, Ng S. 2002. Determining the number of factors in approximate factor models. *Econometrica* **70**: 191–221.

Barndorff-Nielsen O, Hansen P, Lunde A, Shephard N. 2008. Realised kernels in practice: Trades and quotes. *Econometrics Journal* **4**: 1–32.

Hautsch N, Kyj LM, Oomen RCA. 2012. A blocking and regularization approach to high-dimensional realized covariance estimation. *Journal of Applied Econometrics* **27**: 625–645.

Laloux L, Cizeau P, Bouchaud JP, Potters M. 1999. Noise dressing of financial correlation matrices. *Physical Review Letters* **83**: 1467–1470.

Tola V, Lillo F, Gallegati M, Mantegna R. 2008. Cluster analysis for portfolio optimization. *Journal of Economic Dynamics and Control* **32**: 235–258.