Online Appendix to "When Does Government Debt Crowd Out Investment?"

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This appendix includes the equilibrium system of the model, estimation details, and additional results from sensitivity analysis not included in the paper.

1 Model

The equations that characterize the equilibrium systems are listed. The steady state of the model economy and the log-linearized system are also presented.

1.1 The Equilibrium System

We define $q_t \equiv \frac{\xi_t}{\lambda_t}$ where λ_t and ξ_t are the Lagrangian multipliers for the saver's budget constraint and capital accumulation equations respectively. We let $r_t^K = \frac{R_t^K}{P_t}$, $w_t = \frac{W_t}{P_t}$, and $mc_t = \frac{MC_t}{P_t}$.

Since the economy features a permanent shock to technology, several variables are not stationary along the balanced-growth path. In order to induce stationarity, we perform a change of variables and define: $\tilde{Y}_t \equiv \frac{Y_t}{A_t}$, $\tilde{C}_t \equiv \frac{C_t}{A_t}$, $\tilde{C}_t^S \equiv \frac{C_t^S}{A_t}$, $\tilde{C}_t^N \equiv \frac{C_t^N}{A_t}$, $\tilde{K}_t \equiv \frac{K_t}{A_t}$, $\tilde{K}_t^G \equiv \frac{K_t^G}{A_t}$, $\tilde{I}_t \equiv \frac{I_t}{A_t}$, $\tilde{G}_t^C \equiv \frac{G_t^C}{A_t}$, $\tilde{G}_t^I \equiv \frac{G_t^I}{A_t}$, $\tilde{Z}_t \equiv \frac{Z_t}{A_t}$, $\tilde{w}_t \equiv \frac{w_t}{A_t}$, and $\lambda_t^S \equiv \Lambda_t^S A_t$. The equilibrium system written in the stationary form consists of the following equations.

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Savers' Consumption FOC:

$$(1+\tau_t^C)\lambda_t^S = \frac{u_t^b}{\tilde{C}_t^S - \theta \tilde{C}_{t-1}^S e^{-u_t^a}} \tag{1}$$

Euler Equation:

$$\lambda_t^S = \beta R_t E_t \frac{\lambda_{t+1}^S e^{-u_{t+1}^a}}{\pi_{t+1}} \tag{2}$$

Saver's FOC for capacity utilization:

$$(1-\tau_t^K) r_t^k = q_t[\delta_1 + \delta_2(v_t - 1)] \tag{3}$$

Saver's FOC for capital:

$$q_t = \beta E_t \frac{\lambda_{t+1}^S e^{-u_{t+1}^a}}{\lambda_t^S} \left\{(1-\tau_{t+1}^K) r_{t+1}^K v_{t+1} + [1-\delta(v_{t+1})] q_{t+1}\right\} \tag{4}$$

Saver's FOC for investment:

$$\begin{aligned} 1 = & q_t \left[1 - u_t^i s\left(\frac{\tilde{I}_t e^{u_t^a}}{\tilde{I}_{t-1}}\right) - u_t^i s'\left(\frac{\tilde{I}_t e^{u_t^a}}{\tilde{I}_{t-1}}\right) \frac{\tilde{I}_t e^{u_t^a}}{\tilde{I}_{t-1}}\right] \\ & + E_t \left[q_{t+1} \frac{\lambda_{t+1}^S e^{-u_{t+1}^a}}{\lambda_t^S} u_{t+1}^i s'\left(\frac{\tilde{I}_{t+1} e^{u_{t+1}^a}}{\tilde{I}_t}\right) \left(\frac{\tilde{I}_{t+1} e^{u_{t+1}^a}}{\tilde{I}_t}\right)^2\right] \end{aligned} \tag{5}$$

Law of motion for capital:

$$\tilde{K}_t = (1-\delta[v_t]) e^{-u_t^a} \tilde{K}_{t-1} + u_t^i \left[1 - s\left(\frac{\tilde{I}_t e^{u_t^a}}{\tilde{I}_{t-1}}\right)\right] \tilde{I}_t \tag{6}$$

Non-saver's budget constraint:

$$(1+\tau_t^C)\tilde{C}_t^N = (1-\tau_t^L)\tilde{w}_t L_t + \tilde{Z}_t \tag{7}$$

Union FOC for wage:

$$E_t \sum_{s=0}^{\infty} (\beta \omega_w)^s \frac{u^b_{t+s}(1-\tau^L_{t+s})\tilde{L}_{t+s}\tilde{\tilde{w}}_t}{(1+\tau^C_{t+s})\left[\frac{(1-\mu)}{\tilde{C}^S_{t+s}-\theta \tilde{C}^S_{t+s-1}e^{-u^a_{t+s}}} + \frac{\mu}{\tilde{C}^N_{t+s}-\theta \tilde{C}^N_{t+s-1}e^{-u^a_{t+s}}}\right]}$$

$$\times \prod_{k=1}^{s}\left[\left(\frac{\pi_{t+k-1}e^{u^a_{t+k-1}}}{\pi e^{\gamma}}\right)^{\chi_w}\left(\frac{\pi e^{\gamma}}{\pi_{t+k}e^{u^a_{t+k}}}\right)\right] = E_t \sum_{s=0}^{\infty}(\beta\omega_w)^s(1+\eta^w_{t+s})u^b_{t+s}\tilde{L}^{1+\nu}_{t+s} \quad (8)$$

where $\tilde{\tilde{w}}$ is the real wage given for individual labor services and

$$\tilde{L}_{t+s} = \left\{\frac{\tilde{\tilde{w}}_t}{\tilde{w}_{t+s}}\prod_{k=1}^{s}\left[\left(\frac{\pi_{t+k-1}e^{u^a_{t+k-1}}}{\pi e^{\gamma}}\right)^{\chi_w}\left(\frac{\pi e^{\gamma}}{\pi_{t+k}e^{u^a_{t+k}}}\right)\right]\right\}^{-\frac{1+\eta^w_{t+s}}{\eta^w_{t+s}}} L_{t+s} \quad (9)$$

Aggregate wage index:

$$1 = (1-\omega_w)\left(\frac{\tilde{\tilde{w}}_t}{\tilde{w}_t}\right)^{\frac{1}{\eta^w_t}} + \omega_w\left[\left(\frac{\pi_{t-1}e^{u^a_{t-1}}}{\pi e^{\gamma}}\right)^{\chi_w}\left(\frac{\pi e^{\gamma}}{\pi_t e^{u^a_t}}\right)\frac{\tilde{w}_{t-1}}{\tilde{w}_t}\right]^{\frac{1}{\eta^w_t}} \quad (10)$$

Production function:

$$\tilde{Y}_t pd_t = (v_t e^{-u^a_t}\tilde{K}_{t-1})^{\alpha}(L_t)^{1-\alpha}\left(\frac{\tilde{K}^G_{t-1}e^{-u^a_t}}{\int_0^1 \tilde{y}_t(i)di + \Omega}\right)^{\frac{\alpha^G}{1-\alpha^G}} - \Omega \quad (11)$$

where pd_t stands for price dispersion. Capital-labor ratio:

$$\frac{v_t e^{-u^a_t}\tilde{K}_{t-1}}{L_t} = \frac{\tilde{w}_t}{r^k_t}\frac{\alpha}{1-\alpha} \quad (12)$$

Nominal marginal cost:

$$mc_t = (1-\alpha)^{\alpha-1}\alpha^{-\alpha}(r^k_t)^{\alpha}\tilde{w}_t^{1-\alpha}\left(\frac{\tilde{K}^G_{t-1}e^{-u^a_t}}{\int_0^1 \tilde{y}_t(i)di + \Omega}\right)^{\frac{-\alpha^G}{1-\alpha^G}} \quad (13)$$

Intermediate firm FOC for price level:

$$0 = E_t \left\{ \sum_{s=0}^{\infty} (\beta\omega_p)^s u^b_{t+1} (C^S_{t+s})^{-\gamma} \bar{y}_{t+s} \left[\frac{p_t}{P_t} \prod_{k=1}^{s} \left[\left(\frac{\pi_{t+k-1}}{\pi} \right)^{\chi_p} \left(\frac{\pi}{\pi_{t+k}} \right) \right] - (1+\eta^p_{t+s}) mc_{t+s} \right] \right\} \tag{14}$$

where

$$\bar{y}_{t+s} = \left(\frac{p_t}{P_t} \prod_{k=1}^{s} \left[\left(\frac{\pi_{t+k-1}}{\pi} \right)^{\chi_p} \left(\frac{\pi}{\pi_{t+k}} \right) \right] \right)^{-\frac{1+\eta^p_{t+s}}{\eta^p_{t+s}}} Y_{t+s} \tag{15}$$

Aggregate price index:

$$1 = \left\{ (1-\omega_p) \left(\frac{p_t}{P_t} \right)^{\frac{1}{\eta^p_t}} + \omega_p \left[\left(\frac{\pi_{t-1}}{\pi} \right)^{\chi_p} \left(\frac{\pi}{\pi_t} \right) \right]^{\frac{1}{\eta^p_t}} \right\}^{\eta^p_t} \tag{16}$$

Aggregate consumption:

$$\tilde{C}_t = (1-\mu)\tilde{C}^S_t + \mu \tilde{C}^N_t \tag{17}$$

Aggregate resource constraint:

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t + \tilde{G}^C_t + \tilde{G}^I_t \tag{18}$$

Relation between consumer and producer inflation:

$$\pi^c_t = \pi_t \frac{1+\tau^C_t}{1+\tau^C_{t-1}} \tag{19}$$

Government budget constraint:

$$\tilde{B}_t + \tau^K_t r^K_t v_t e^{-u^a_t} \tilde{K}_{t-1} + \tau^L_t \tilde{w}_t L_t + \tau^C_t \tilde{C}_t = \frac{R_{t-1}\tilde{B}_{t-1}}{\pi_t e^{u^a_t}} + \tilde{G}^C_t + \tilde{G}^I_t + \tilde{Z}_t \tag{20}$$

Law of motion for public capital:

$$\tilde{K}^G_t = (1-\delta^G) e^{-u^a_t} \tilde{K}^G_{t-1} + \tilde{G}^I_t \tag{21}$$

1.2 Steady State

By assumption, in the steady state $v = 1$, $s(e^\gamma) = s'(e^\gamma) = 0$. In addition, we assume that $\pi = 1$, implying $R = e^\gamma/\beta$.

$$r^k = \frac{e^\gamma/\beta - (1-\delta_0)}{1-\tau^K}$$

$$\delta_1 = (1-\tau^K)r^K$$

$$mc = \frac{1}{1+\eta^p}$$

$$\tilde{K} = \frac{1}{1-(1-\delta_0)e^{-\gamma}}\tilde{I}$$

$$\tilde{K}^G = \frac{1}{1-(1-\delta^G)e^{-\gamma}}\tilde{G}^I$$

$$\frac{\Omega}{\tilde{Y}} = \eta^p, \text{ (assuming zero profits)}$$

$$\tilde{w} = \left\{ \frac{1}{1+\eta^p}\alpha^\alpha(1-\alpha)^{1-\alpha}(r^k)^{-\alpha}\left[\frac{\frac{e^{-\gamma}}{1-(1-\delta^G)e^{-\gamma}}s^{G^I}}{1+\frac{\Omega}{\tilde{Y}}}\right]^{\frac{\alpha^G}{1-\alpha^G}} \right\}^{\frac{1}{1-\alpha}}$$

$$\frac{\tilde{K}}{L} = \frac{e^\gamma \tilde{w}}{r^k}\frac{\alpha}{1-\alpha}$$

$$\frac{\tilde{K}}{\tilde{Y}} = e^\gamma\left[\left(1+\frac{\Omega}{\tilde{Y}}\right)\left[e^{-\gamma}\frac{\tilde{K}}{L}\right]^{(1-\alpha)(1-\alpha^G)}\left(\frac{e^{-\gamma}}{1-(1-\delta^G)e^{-\gamma}}s^{G^I}\right)^{-\alpha^G}\right]^{\frac{1}{1-\alpha^G}}$$

$$\frac{L}{\tilde{Y}} = \left[\left(1+\frac{\Omega}{\tilde{Y}}\right)\left[e^{-\gamma}\frac{\tilde{K}}{L}\right]^{-\alpha(1-\alpha^G)}\left(\frac{e^{-\gamma}}{1-(1-\delta^G)e^{-\gamma}}s^{G^I}\right)^{-\alpha^G}\right]^{\frac{1}{1-\alpha^G}}$$

$\frac{\tilde{C}}{\tilde{Y}}$, $\frac{\tilde{C}^S}{\tilde{Y}}$, $\frac{\tilde{C}^N}{\tilde{Y}}$, $\frac{\tilde{Z}}{\tilde{Y}}$, and $\tilde{Y}$ can be solved from the following set of equations:

$$\frac{\tilde{C}}{\tilde{Y}} = 1 - s^{G^C} - s^{G^I} - [1-(1-\delta_0)e^{-\gamma}]\frac{\tilde{K}}{\tilde{Y}}$$

$$\frac{\tilde{C}^N}{\tilde{Y}} = \frac{1}{1+\tau^C}\left[(1-\tau^L)\tilde{w}\frac{L}{\tilde{Y}} + \frac{\tilde{Z}}{\tilde{Y}}\right]$$

$$\frac{\tilde{C}^S}{\tilde{Y}} = \frac{1}{1-\mu}\left[\frac{\tilde{C}}{\tilde{Y}} - \mu\frac{\tilde{C}^N}{\tilde{Y}}\right]$$

$$\frac{\tilde{Z}}{\tilde{Y}} = \left(1 - Re^{-\gamma}\right)s^B - s^{G^C} - s^{G^I} + \tau^C + \frac{\tilde{C}}{\tilde{Y}} + \tau^K r^k e^{-\gamma}\frac{\tilde{K}}{\tilde{Y}} + \tau^L \tilde{w}\frac{L}{\tilde{Y}}$$

$$\tilde{Y}^{1+\frac{1}{\nu}} = \left(\frac{L}{\tilde{Y}}\right)^{-1}\left[\frac{w(1-\tau^L)}{(1+\tau^C)(1+\eta^w)\left(\frac{e^\gamma}{e^\gamma-\theta}\right)\left[\frac{(1-\mu)}{\frac{\tilde{C}^S}{\tilde{Y}}} + \frac{\mu}{\frac{\tilde{C}^N}{\tilde{Y}}}\right]}\right]$$

Given the solution to $\tilde{Y}$, all other level steady state variables can be backed out using the steady state ratios.

1.3 The Log-Linearized System

We define the log deviations of a variable X from its steady state as $\hat{X}_t = \ln X_t - \ln X$, except for $\hat{u}_t^a \equiv u_t^a - \gamma$, $\hat{\eta}_t^p = \ln(1+\eta_t^p) - \ln(1+\eta^p)$, and $\hat{\eta}_t^w = \ln(1+\eta_t^w) - \ln(1+\eta^w)$. The equilibrium system in the log-linearized form consists of the following equations:

Savers' Consumption FOC:

$$\frac{\tau^C}{1+\tau^C}\hat{\tau}_t^C + \hat{\lambda}_t^S = \hat{u}_t^b - \frac{e^\gamma}{e^\gamma-\theta}\hat{\tilde{C}}_t^S + \frac{\theta}{e^\gamma-\theta}\hat{\tilde{C}}_{t-1}^S - \left[1 - \frac{e^\gamma}{e^\gamma-\theta}\right]\hat{u}_t^a \tag{22}$$

Euler Equation:

$$\hat{\lambda}_t^S = \hat{R}_t - E_t\hat{u}_{t+1}^a - E_t\hat{\pi}_{t+1} + E_t\hat{\lambda}_{t+1}^S \tag{23}$$

Savers' FOC for capacity utilization:

$$\hat{r}_t^k - \frac{\tau^K}{1-\tau^K}\hat{\tau}_t^K = \hat{q}_t + \frac{\delta_2}{\delta_1}\hat{v}_t \tag{24}$$

Savers' FOC for capital:

$$\hat{q}_t = E_t\hat{\pi}_{t+1} - \hat{R}_t + \beta e^{-\gamma}(1-\tau^K)r^K E_t\hat{r}_{t+1}^K - \beta e^{-\gamma}r^K\tau^K E_t\hat{\tau}_{t+1}^K + \beta e^{-\gamma}(1-\delta_0)E_t\hat{q}_{t+1} \tag{25}$$

Savers' FOC for investment:

$$(1+\beta)\hat{\tilde{I}}_t + \hat{u}_t^a - \frac{1}{se^{2\gamma}}\left[\hat{q}_t + \hat{u}_t^i\right] - \beta E_t\hat{\tilde{I}}_{t+1} - \beta E_t\hat{u}_{t+1}^a = \hat{\tilde{I}}_{t-1} \tag{26}$$

Law of motion for capital:

$$\hat{\tilde{K}}_t = (1-\delta_0)e^{-\gamma}(\hat{\tilde{K}}_{t-1} - \hat{u}_t^a) - \delta_1 e^{-\gamma}\hat{v}_t + [1-(1-\delta_0)e^{-\gamma}](\hat{u}_t^i + \hat{\tilde{I}}_t) \tag{27}$$

Non-savers' budget constraint:

$$\tau^C \tilde{C}^N[\hat{\tau}_t^C + \hat{\tilde{C}}_t^N] = (1-\tau^L)\tilde{w}L[\hat{\tilde{w}}_t + \hat{L}_t] - \tau^L \tilde{w} L \hat{\tau}_t^L + \tilde{Z}\hat{\tilde{Z}}_t \tag{28}$$

Wage equation:

$$\begin{aligned}
\hat{\tilde{w}}_t =& \frac{1}{1+\beta}\hat{\tilde{w}}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\tilde{w}}_{t+1} + \frac{\chi^w}{1+\beta}\hat{\pi}_{t-1} - \frac{1+\beta\chi^w}{1+\beta}\hat{\pi}_t + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} \\
& - \kappa_w\left[\hat{\tilde{w}}_t - \nu\hat{L}_t - \hat{\eta}_t^w - \frac{\tau^L}{1-\tau^L}\hat{\tau}_t^L - \frac{\tau^C}{1+\tau^C}\hat{\tau}_t^C\right] + \frac{\chi^w}{1+\beta}\hat{u}_{t-1}^a \\
& + \frac{e^\gamma\kappa_w}{e^\gamma-\theta}\left[\frac{\frac{1-\mu}{\tilde{C}^S}}{\frac{1-\mu}{\tilde{C}^S}+\frac{\mu}{\tilde{C}^N}}\right]\hat{\tilde{C}}_t^S + \frac{e^\gamma\kappa_w}{e^\gamma-\theta}\left[\frac{\frac{\mu}{\tilde{C}^N}}{\frac{1-\mu}{\tilde{C}^S}+\frac{\mu}{\tilde{C}^N}}\right]\hat{\tilde{C}}_t^N \\
& - \frac{\theta\kappa_w}{e^\gamma-\theta}\left[\frac{\frac{1-\mu}{\tilde{C}^S}}{\frac{1-\mu}{\tilde{C}^S}+\frac{\mu}{\tilde{C}^N}}\right]\hat{\tilde{C}}_{t-1}^S + \frac{\theta\kappa_w}{e^\gamma-\theta}\left[\frac{\frac{\mu}{\tilde{C}^N}}{\frac{1-\mu}{\tilde{C}^S}-\frac{\mu}{\tilde{C}^N}}\right]\hat{\tilde{C}}_{t-1}^N \\
& - \left\{\frac{1+\beta\chi^w-\rho_a\beta}{1+\beta} - \frac{\theta\kappa_w}{e^\gamma-\theta}\right\}\hat{u}_t^a
\end{aligned} \tag{29}$$

where $\kappa_w \equiv [(1-\beta\omega_w)(1-\omega_w)]/[\omega_w(1+\beta)\left(1+\frac{(1+\eta^w)\nu}{\eta^w}\right)]$.

Production function:

$$\hat{\tilde{Y}}_t = \frac{\tilde{Y}+\Omega}{\tilde{Y}}\left[\alpha(1-\alpha^G)(\hat{v}_t + \hat{\tilde{K}}_{t-1} - \hat{u}_t^a) + (1-\alpha)(1-\alpha^G)\hat{L}_t + \alpha^G(\hat{\tilde{K}}_{t-1}^G - \hat{u}_t^a)\right] \tag{30}$$

Capital-labor ratio:

$$\hat{r}_t^k - \hat{\tilde{w}}_t = \hat{L}_t - \hat{\tilde{K}}_{t-1} + \hat{v}_t - \hat{u}_t^a \tag{31}$$

Marginal cost:

$$\hat{mc}_t = \alpha\hat{r}_t^k + (1-\alpha)\hat{\tilde{w}}_t - \frac{\alpha^G}{1-\alpha^G}\hat{\tilde{K}}_{t-1}^G + \frac{\alpha^G}{1-\alpha^G}\hat{u}_t^a + \frac{\alpha^G}{1-\alpha^G}\frac{\tilde{Y}}{\tilde{Y}+\Omega}\hat{\tilde{Y}}_t \tag{32}$$

Phillips equation:

$$\hat{\pi}_t = \frac{\beta}{1+\chi^p\beta}E_t\hat{\pi}_{t+1} + \frac{\chi^p}{1+\chi^p\beta}\hat{\pi}_{t-1} + \kappa_p\left(\hat{mc}_t + \hat{\eta}_t^p\right) \tag{33}$$

where $\kappa_p = [(1-\beta\omega_p)(1-\omega_p)]/[\omega_p(1+\beta\chi^p)]$.

Aggregate consumption:

$$\tilde{C}\hat{\tilde{C}}_t = (1-\mu)\tilde{C}^S\hat{\tilde{C}}_t^S + \mu\tilde{C}^N\hat{\tilde{C}}_t^N \tag{34}$$

Aggregate resource constraint:

$$\tilde{Y}\hat{\tilde{Y}}_t = \tilde{C}\hat{\tilde{C}}_t + \tilde{I}\hat{\tilde{I}}_t + \tilde{G}^C\hat{\tilde{G}}_t^C + \tilde{G}^I\hat{\tilde{G}}_t^I \tag{35}$$

Relation between consumer and producer inflation:

$$\hat{\pi}_t^c = \hat{\pi}_t + \frac{\tau^C}{1+\tau^C}\hat{\tau}_t^C - \frac{\tau^C}{1+\tau^C}\hat{\tau}_{t-1}^C \tag{36}$$

Government budget constraint:

$$\begin{aligned}&\frac{\tilde{B}}{\tilde{Y}}\hat{\tilde{B}}_t + \tau^K r^k e^{-\gamma}\frac{\tilde{K}}{\tilde{Y}}\left[\hat{\tau}_t^K + \hat{r}_t^k + \hat{\tilde{K}}_{t-1} + \hat{v}_t - \hat{u}_t^a\right] + \tau^L\tilde{w}\frac{L}{\tilde{Y}}[\hat{\tau}_t^L + \hat{\tilde{w}}_t + \hat{L}_t] + \tau^C\tilde{C}[\hat{\tau}_t^C + \hat{\tilde{C}}_t]\\ &= \frac{R}{e^\gamma}\frac{\tilde{B}}{\tilde{Y}}[\hat{R}_{t-1} + \hat{\tilde{B}}_{t-1} - \hat{\pi}_t - \hat{u}_t^a] + \frac{\tilde{G}^C}{\tilde{Y}}\hat{\tilde{G}}_t^C + \frac{\tilde{G}^I}{\tilde{Y}}\hat{\tilde{G}}_t^I + \frac{\tilde{Z}}{\tilde{Y}}\hat{\tilde{Z}}_t\end{aligned} \tag{37}$$

Law of motion for public capital:

$$\hat{\tilde{K}}_t^G = (1-\delta^G)e^{-\gamma}(\hat{\tilde{K}}_{t-1}^G - \hat{u}_t^a) + [1-(1-\delta^G)e^{-\gamma}]\hat{\tilde{G}}_t^I \tag{38}$$

2 Estimation

This section includes data description, estimation procedures, and the comparison of prior-predictive and posterior analysis.

2.1 Data Description

Unless otherwise noted, the following data are from the National Income and Product Accounts Tables released by the Bureau of Economic Analysis. All data in levels are nominal values. Nominal data are converted to real values by dividing by the GDP deflator (Table 1.1.4, line 1).

Consumption. Consumption, C, is defined as total personal consumption expenditures (Table 1.1.5, line 2).

Investment. Investment, I, is defined as gross private domestic investment (Table 1.1.5, line 7).

Consumption Tax Revenues. The consumption tax revenues, T^c, include excise taxes and customs duties (Table 3.2, lines 5 and 6).

Capital and Labor Tax Revenues. Following Jones (2002), first the average personal income tax rate is computed:

$$\tau^p = \frac{IT}{W + PRI/2 + CI},$$

where IT is personal current tax revenues (Table 3.2, line 3), W is wage and salary accruals (Table 1.12 line 3), PRI is proprietors' income (Table 1.12, line 3), and CI is capital income. Capital income is defined as rental income (Table 1.12, line 12), corporate profits (Table 1.12, line 13), interest income (Table 1.12 line 18), and PRI/2.

The average labor income tax revenue, T^l, is computed as:

$$\tau^p(W + PRI/2) + CSI,$$

where CSI is contributions for government social insurance (Table 3.2, line 11). The average capital income tax revenue, T^k is calculated as:

$$\tau^p CI + CT,$$

where CT is taxes on corporate income (Table 3.2, line 7).

Government Expenditure. Government expenditure, G^C, is defined as government consumption expenditure (Table 3.2, line 20), government investment for defense

(Table 3.9.5, line 13), and government net purchases of non-produced assets (Table 3.2, line 43), minus government consumption of fixed capital (Table 3.2, line 44).

Government Investment. Government investment, G^I, is defined as government investment for non-defense (Table 3.9.5, line 18).

Transfers. Transfers, TR, are defined as net current transfers, net capital transfers, and subsidies (Table 3.2, line 31), minus the tax residual. Net current transfers are defined as current transfer payments (Table 3.2, line 21) minus current transfer receipts (Table 3.2, line 15). Net capital transfers are defined as capital transfer payments (Table 3.2, line 42) minus capital transfer receipts (Table 3.2, line 38). The tax residual is defined as current tax receipts (Table 3.2, line 2), contributions for government social insurance (Table 3.2, line 11), income receipts on assets (Table 3.2, line 12), and the current surplus of government enterprises (Table 3.2, line 18), minus total tax revenue, T (consumption, labor, and capital tax revenues).

Hours Worked. Hours worked are constructed from the following variables:

H the index for nonfarm business, all persons, average weekly hours duration, 1992 = 100, seasonally adjusted (from the Department of Labor).

Emp civilian employment for sixteen years and over, measured in thousands, seasonally adjusted (from the Department of Labor, Bureau of Labor Statistics, CE16OV). The series is transformed into an index where 1992Q3 = 100.

Hours worked are then defined as

$$N = \frac{H * Emp}{100}.$$

Wage Rate. The wage rate is defined as the index for hourly compensation for nonfarm business, all persons, 1992 = 100, seasonally adjusted (from the U.S. Department of Labor).

Inflation. The gross inflation rate is defined using the GDP deflator (Table 1.1.4, line 1).

Interest Rate. The nominal interest rate is defined as the average of daily figures of the Federal Funds Rate (from the Board of Governors of the Federal Reserve System).

Definitions of Observable Variables

The observable variable X is defined by making the following transformation to variable x:

$$X = \ln\left(\frac{x}{Popindex}\right) * 100\ ,$$

where

Popindex index of Pop, constructed such that 1992Q3 = 1;

Pop Civilian noninstitutional population in thousands, ages 16 years and over, seasonally adjusted (from the Bureau of Labor Statistics).

x = consumption, investment, hours worked, government spending, government investment, capital tax revenues, consumption tax revenues, labor tax revenues, and transfers. The real wage rate is defined in the same way, except that it is not divided by the total population.

Data for the observables and the log-linearized variables are linked by the following equations:

$$\begin{bmatrix} \text{dlCons}_t \\ \text{dlInv}_t \\ \text{dlWage}_t \\ \text{dlGovSpend}_t \\ \text{dlGovInv}_t \\ \text{dlCapTaxRev}_t \\ \text{dlLabTaxRev}_t \\ \text{dlConsTaxRev}_t \\ \text{dlTransfers}_t \\ \text{lHours}_t \\ \text{lInflation}_t \\ \text{lFedFunds}_t \end{bmatrix} = \begin{bmatrix} 100\gamma \\ 100\gamma \\ 100\gamma \\ 100\gamma \\ 100\gamma \\ 100\gamma \\ 100\gamma \\ 100\gamma \\ 100\gamma \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \hat{c}_t - \hat{c}_{t-1} + \hat{u}^a_t \\ \hat{i}_t - \hat{i}_{t-1} + \hat{u}^a_t \\ \hat{w}_t - \hat{w}_{t-1} + \hat{u}^a_t \\ \hat{G}^C_t - \hat{G}^C_{t-1} + \hat{u}^a_t \\ \hat{G}^I_t - \hat{G}^I_{t-1} + \hat{u}^a_t \\ \hat{T}^K_t - \hat{T}^K_{t-1} + \hat{u}^a_t \\ \hat{T}^L_t - \hat{T}^L_{t-1} + \hat{u}^a_t \\ \hat{T}^C_t - \hat{T}^C_{t-1} + \hat{u}^a_t \\ \hat{Z}_t - \hat{Z}_{t-1} + \hat{u}^a_t \\ \hat{L}_t \\ \hat{\pi}_t \\ \hat{R}_t \end{bmatrix}, \tag{39}$$

where l and dl stand for 100 times the log (demeaned) and the log difference of each variable respectively. u^a_t is the percentage deviation of the technology growth rate from the steady state growth rate γ.

2.2 Estimation Methods

We use Bayesian inference methods to construct the parameters' posterior distribution, which is a combination of the likelihood function and prior information (see An and Schorfheide (2007) for a survey). We assume that the parameters are drawn independently and denote $p(\theta)$ as the product of the marginal parameter distributions. Table 1 lists the prior distributions. Given the plausible interactions between monetary and fiscal policies, $p(\theta)$ has a non-zero density outside the determinacy region of the parameter space. We restrict the parameter space to the subspace in which the model has a unique rational expectations equilibrium. We denote this subspace as Θ_D and let $\mathcal{I}\{\theta \in \Theta_D\}$ be an indicator function that is one if θ is in the determinacy region and zero otherwise. Thus, our joint prior distribution is defined as

$$\tilde{p}(\theta) = \frac{1}{c} p(\theta) \mathcal{I}\{\theta \in \Theta_D\}, \quad \text{where}\, c = \int_{\theta \in \Theta_D} p(\theta) d\theta \ .$$

The equilibrium system of the model is written in a state-space form, where observables are linked with other variables in the model. For a given set of structural parameters, we compute the value for the log posterior function, which combines the likelihood of the data, $\mathcal{L}(y|\theta)$, with the probability values of the parameters given the prior distributions. The posterior is proportional to

$$p(Y|\theta) \propto \mathcal{L}(y|\theta)\tilde{p}(\theta) \ .$$

We construct the posterior distribution using the random walk Metropolis-Hastings algorithm. We sample one million draws from the posterior distribution. A step size of 0.3 yields an acceptance ratio of 0.28. We discard the first 200,000 draws and thin every 20 draws. Figures 1-5 display plot slices of the likelihood around the mode. The condition number of the Hessian at the mode is 41,420. Figures 6-9 display trace plots.

2.3 Prior-Posterior Analysis on Crowding Out Effects

A priori, our model does not impose restrictions on whether government debt crowds out or in investment. Table 3 quantifies the extent of crowding out based on 30,000 draws from the prior and posterior distributions. The top rows record the percentage of draws that lead investment to decline on impact following various fiscal shocks. On impact, the priors appear to restrict several fiscal instruments to either crowd in or out investment (a government investment increase and labor tax decrease are notable exceptions). However, this does not necessarily restrict the posterior impact multipliers to have the same sign. This can be seen from comparing the impact prior and posterior range for a consumption tax decrease. While the prior places most mass on investment being crowded out following a consumption tax decrease, the posterior reverses this and places most mass on investment being crowded in.

The bottom rows report the 5th and 95th cumulative present-value investment multipliers following various fiscal shocks.[1] The priors allow the 90 percent interval of the investment multipliers to cover wide ranges, with capital and labor tax decreases and transfer increases including both positive and negative multipliers. In contrast, most of the 90 percent intervals for posterior multipliers are either entirely positive or negative (a capital tax decrease is the sole exception).

2.4 Estimation Results

Table 2 reports the posterior mode used to initialize the random walk Metropolis-Hastings algorithm, along with the estimated posterior means, modes, and the Geweke Separated Partial Means (GSPM) test p-values.[2] Figures 10 and 11 plot priors against posterior distributions. The plots suggest that the data contain information for identifying most parameters. Table 4 gives a comparison of our estimates with others. Table 5 reports the posterior means and 90% credible intervals (in parentheses) for various estimated models. The log-marginal data densities, calculated using Geweke's (1999) modified harmonic mean estimator, are reported along with the Bayes factors relative to the benchmark model. The log-marginal data densities are calculated using a truncation parameter of 0.5. The model comparisons indicate that the data cannot distinguish between the benchmark model and four alternative specifications: (1) when

[1] Investment multipliers are defined as the present-value sum of investment changes in levels divided by the present-value sum of changes in a fiscal variable. Depending on the fiscal shock that triggers debt growth, the denominator can be changes in capital, labor, or consumption tax revenues, government consumption or investment, or transfers. The sums are over 1000 quarters, and present values are discounted by the model-implied interest rate path.

[2] The GSPM test determines whether the mean from the first 20% of the MCMC draws is identical to the mean of the final 50% of the draws. A Z-test of the hypothesis of equality of the two means is carried out and the corresponding chi-squared marginal significance is calculated (see Geweke (2005), pages 149-150, for more details). The reported p-values are based on assuming 4% tapered autocorrelation.

$\alpha_g = 0.0$, (2) when $\alpha_g = 0.1$, (3) when $\varphi_K = \varphi_L = 0$, and (4) when only lump-sum transfers adjust to debt ($\gamma_{GC} = \gamma_{GI} = \gamma_K = \gamma L = 0$).

3 Sensitivity Analysis

We investigate the robustness of the effects of expansionary fiscal policy on investment under several alternative model specifications. The results of these robustness checks are summarized in Table 6. To get a sense of how the investment response varies quantitatively across model specifications, we report the posterior median impact and cumulative present value multipliers for each case. Investment multipliers are defined as the present-value sum of investment changes in levels divided by the present-value sum of changes in a fiscal variable. Depending on the fiscal shock that triggers debt growth, the denominator can be changes in capital, labor, or consumption tax revenues, government consumption or investment, or transfers. The sums are over 1000 quarters, and present values are discounted by the model-implied interest rate path.

3.1 Varying α^G

The elasticity of output to government capital, α^G, cannot be identified from our observables. For the baseline estimation, we calibrate $\alpha^G = 0.05$. To determine how sensitive our estimates and inferences are to this parameter, we estimate the model for two alternative cases where $\alpha^G = 0.0$ and $\alpha^G = 0.1$. We find that the data cannot distinguish between the three values for α^G, as the log marginal data densities in the three cases are virtually identical. The 'Varying α^G' columns of Table 6 show the investment multipliers when $\alpha^G = 0.0$ and $\alpha^G = 0.1$. Varying α^G affects only the multipliers for government investment substantially. When α^G is very small, a government investment shock resembles a non-productive government consumption shock. The more productive government investment is (the larger α^G is), the higher the cumulative present value multiplier is, as the returns to investment rapidly increase. When $\alpha^G = 0.0$, the cumulative present value multiplier is substantially negative, similarly to the response following a government spending shock. The slight difference from the government spending shock multiplier is due to different persistence estimates ρ_{GC} and ρ_{GI}.

3.2 No Automatic Stabilizers

We check whether our results are sensitive to the estimates of automatic stabilizer coefficients, φ_K and φ_L. We estimate a version of the model where these parameters are calibrated to zero. The fifth column of Table 6 shows that this substantially affects only the multipliers for capital and labor tax decreases. Following a capital or labor tax decrease, output rises as productivity increases. Automatic stabilizers cause capital and labor taxes to increase as well, offsetting the initial policy effect and reducing the financing adjustments necessary to pay off increase in debt. Without such automatic adjustments, debt increases more, requiring larger future adjustments and lowering the cumulative present value multipliers.

3.3 Standard Calibration of Labor Supply Elasticity

Our benchmark model estimate of the labor preference parameter (κ) differs from a standard value used in the real business cycle literature. We re-estimate the model when this parameter is calibrated to a more typical value ($\kappa = 1$). Again, this modification has small quantitative effects overall (shown in the last column of Table 6). It slightly lowers the present-value capital tax multiplier.

Table 1: **Prior Distributions for the Estimated Parameters.**

Parameters	**Prior**		
	func.	mean	std.
preference and technology			
100γ, steady state growth	N	0.5	0.03
κ, inverse Frisch labor elast.	G	2	0.5
θ, habit	B	0.5	0.2
μ, fraction of non-Ricar. households	B	0.3	0.1
frictions			
ω_w, wage stickiness	B	0.5	0.1
ω_p, price stickiness	B	0.5	0.1
ψ, capital utilization	B	0.6	0.15
s, investment adj. cost	N	6	1.5
χ^w, wage partial indexation	B	0.5	0.15
χ^p, price partial indexation	B	0.5	0.15
fiscal policy			
γ_{GC}, govt consumption resp to debt	N	0.15	0.1
γ_{GI}, govt investment resp to debt	N	0.15	0.1
γ_K, capital tax resp to debt	N	0.15	0.1
γ_L, labor tax resp to debt	N	0.15	0.1
γ_Z, transfers resp to debt	N	0.15	0.1
φ_K, capital resp. to output	G	0.75	0.35
φ_L, labor resp. to output	G	0.40	0.15
monetary policy			
ϕ_π, interest rate resp. to inflation	N	1.5	0.25
ϕ_y, interest rate resp. to output	N	0.125	0.05
ρ_r, lagged interest rate resp.	B	0.5	0.2
serial correl. in disturbances			
ρ_a, technology	B	0.5	0.2
ρ_b, preference	B	0.5	0.2
ρ_i, investment	B	0.5	0.2
ρ_w, wage markup	B	0.5	0.2
ρ_p, price markup	B	0.5	0.2
ρ_{GC}, government consumption	B	0.5	0.2
ρ_{GI}, government investment	B	0.5	0.2
ρ_K, capital tax	B	0.5	0.2
ρ_L, labor tax	B	0.5	0.2
ρ_C, consumption tax	B	0.5	0.2
ρ_Z, transfer	B	0.5	0.2
std. of shocks			
σ_a, technology	IG^1	0.1	2
σ_b, preference	IG^1	0.1	2
σ_m, monetary policy	IG^1	0.1	2
σ_i, investment	IG^1	0.1	2
σ_w, wage markup	IG^1	0.1	2
σ_p, price markup	IG^1	0.1	2
σ_{GC}, government consumption	IG^1	1	∞
σ_{GI}, government investment	IG^1	1	∞
σ_K, capital tax	IG^1	1	∞
σ_L, labor tax	IG^1	1	∞
σ_C, consumption tax	IG^1	1	∞
σ_Z, transfers	IG^1	1	∞
σ_{KL}, co-movement btw K and L taxes	N	0.2	0.1

1: The Inverse Gamma distribution is given by $f(x|s,\nu) = \nu^s \Gamma^{-1}(s) x^{-s-1} \exp^{\frac{-\nu}{x}}$.

Table 2: **Posterior Estimates.** Reports the posterior mode (used to initialize the random walk Metropolis Hastings algorithm), mean, median, 90% credible interval, and the p-value for Geweke's Separated Partial Means test. Final acceptance rate: 28%. 1,000,000 draws were made, with the first 200,000 used as a burn-in period and every 20th thinned.

Parameters	**Posterior**					
	mode	mean	median	5 %	95%	Geweke Chi-Sq p
preference and technology						
100γ, steady state growth	0.50	0.50	0.50	0.45	0.55	0.09
κ , inverse Frisch labor elast.	2.18	2.41	2.37	1.65	3.30	0.82
θ , habit	0.69	0.70	0.70	0.63	0.77	0.89
μ, fraction of non-Ricar. households	0.10	0.10	0.10	0.06	0.16	0.32
frictions						
ω_w, wage stickiness	0.24	0.28	0.27	0.18	0.43	0.97
ω_p, price stickiness	0.71	0.70	0.70	0.64	0.77	0.84
ψ , capital utilization	0.77	0.75	0.76	0.66	0.84	0.17
s, investment adj. cost	5.89	5.78	5.73	3.65	8.11	0.99
χ^w , wage partial indexation	0.30	0.30	0.30	0.20	0.41	0.73
χ^p , price partial indexation	0.23	0.28	0.27	0.13	0.49	0.60
fiscal policy						
γ_{GC} , govt consumption resp to debt	0.26	0.28	0.28	0.15	0.42	0.07
γ_{GI} , govt investment resp to debt	0.18	0.16	0.17	0.01	0.30	0.75
γ_K , capital tax resp to debt	0.18	0.20	0.20	0.07	0.33	0.98
γ_L , labor tax resp to debt	0.08	0.10	0.10	0.02	0.20	0.86
γ_Z , transfers resp to debt	−0.02	0.02	0.01	−0.09	0.17	0.38
φ_K , capital resp. to output	0.40	0.51	0.48	0.19	0.93	0.73
φ_L , labor resp. to output	0.53	0.73	0.69	0.24	1.37	0.77
monetary policy						
ϕ_π , interest rate resp. to inflation	2.38	2.39	2.39	2.12	2.68	0.79
ϕ_y , interest rate resp. to output	0.03	0.04	0.04	0.004	0.07	0.99
ρ_r , lagged interest rate resp.	0.85	0.85	0.85	0.82	0.88	0.81
serial correl. in disturbances						
ρ_a, technology	0.25	0.24	0.24	0.12	0.35	0.68
ρ_b, preference	0.92	0.91	0.91	0.85	0.95	0.24
ρ_i, investment	0.92	0.90	0.90	0.84	0.95	0.05
ρ_w, wage markup	0.95	0.92	0.93	0.82	0.97	0.91
ρ_p, price markup	0.96	0.95	0.95	0.91	0.98	0.66
ρ_{GC}, government consumption	0.97	0.97	0.97	0.94	0.99	0.24
ρ_{GI}, government investment	0.94	0.94	0.94	0.89	0.98	0.09
ρ_K, capital tax	0.86	0.87	0.87	0.81	0.92	0.39
ρ_L, labor tax	0.92	0.91	0.91	0.83	0.97	0.56
ρ_C, consumption tax	0.99	0.98	0.98	0.97	0.995	0.37
ρ_Z, transfer	0.90	0.91	0.92	0.83	0.98	0.68
std. of shocks						
σ_a , technology	0.95	0.98	0.98	0.86	1.12	0.10
σ_b , preference	2.03	2.26	2.17	1.67	3.18	0.62
σ_m , monetary policy	0.14	0.14	0.14	0.12	0.16	0.25
σ_i , investment	0.37	0.39	0.38	0.32	0.47	0.96
σ_w , wage markup	0.28	0.26	0.26	0.19	0.37	0.63
σ_p , price markup	0.13	0.14	0.13	0.10	0.18	0.84
σ_{GC} , government consumption	1.85	1.89	1.88	1.68	2.12	0.68
σ_{GI} , government investment	4.16	4.23	4.21	3.77	4.75	0.41
σ_K , capital tax	4.41	4.49	4.47	3.99	5.07	0.07
σ_L , labor tax	1.73	1.76	1.76	1.57	1.99	0.19
σ_C , consumption tax	3.19	3.25	3.24	2.89	3.65	0.67
σ_Z , transfers	2.35	2.40	2.38	2.13	2.69	0.27
σ_{KL} , co-movement btw K and L taxes	0.24	0.24	0.24	0.19	0.29	0.39

Table 3: **Prior and posterior analysis.** The top two rows are percentages of prior and posterior draws that lead to crowding out of investment on impact following various fiscal shocks. The bottom two rows are 90-percent intervals of cumulative present value multipliers for prior and posterior draws following various fiscal shocks. Results are based on 30,000 draws from the prior and posterior distributions.

		$G^C \uparrow$	$G^I \uparrow$	$\tau^K \downarrow$	$\tau^L \downarrow$	$\tau^C \downarrow$	$Z \uparrow$
Impact	Prior	99.93%	15.78%	1.01%	55.29%	99.79%	99.02%
	Posterior	100%	95%	0%	1%	2%	100%
PV Inv.	Prior	(-0.76, 0.00)	(2.62, 4.75)	(-0.86, 1.43)	(-0.65, 0.22)	(-0.68, -0.17)	(-0.72, 0.04)
Multiplier	Posterior	(-2.73, -0.52)	(2.95, 5.32)	(-6.26, 2.25)	(-0.87, -0.09)	(-0.97, -0.30)	(-0.91, -0.35)

Table 4: **Comparisons of Estimates with Others.** Estimates under Smets and Wouters (2007) and this paper are posterior means, and estimates under Fernandez-Villaverde et al. (2010) are posterior medians.

	Smets and Wouters		Fernandez-Villaverde et al.		This Paper	
sample period	1966:1-2004:4		1959:1-2007:1		1983:1-2008:1	
κ , inverse Frisch elast.	1.83	$N(2, 0.75)$	1.17	$N(1, 0.25)$	2.41	$G(2, 0.5)$
s, investment adj. cost	5.74	$N(4, 1.5)$	9.74	$N(4, 1.5)$	5.78	$N(6, 1.5)$
ψ , capital utilization	0.54	$B(0.5, 0.15)$	0.001	calibrated	0.75	$B(0.6, 0.15)$
ω_w , wage stickiness	0.70	$B(0.5, 0.1)$	0.68	$B(0.5, 0.1)$	0.28	$B(0.5, 0.1)$
ω_p , price stickiness	0.66	$B(0.5, 0.1)$	0.82	$B(0.5, 0.1)$	0.70	$B(0.5, 0.1)$
χ^w , wage indexation	0.58	$B(0.5, 0.15)$	0.62	$B(0.5, 0.1)$	0.30	$B(0.5, 0.15)$
χ^p , price indexation	0.24	$B(0.5, 0.15)$	0.63	$B(0.5, 0.15)$	0.28	$B(0.5, 0.15)$
ϕ_π , interest to inflation	2.04	$N(1.5, 0.25)$	1.29	$N(1.5, 0.125)$	2.39	$N(1.5, 0.25)$
ϕ_y , interest to output	0.08	$N(0.12, 0.05)$	0.19	$N(0.12, 0.05)$	0.04	$N(0.125, 0.1)$
ρ_m, Taylor persistence	0.81	$B(0.75, 0.1)$	0.77	$B(0.75, 0.1)$	0.85	$B(0.5, 0.2)$

Table 5: **Sensitivity analysis.** The table reports posterior medians and 90% credible intervals (in parentheses) for various models. Log-marginal data densities, calculated using Geweke's (1999) modified harmonic mean estimator, are reported along with Bayes factors relative to the benchmark model.

Key Parameters	**Models**				
	benchmark ($\alpha^G = 0.05$)	$\alpha G = 0.0$	$\alpha^G = 0.1$	$\varphi_K = \varphi_L = 0$	only transfers adjust to B
Preference and technology					
100γ, steady state growth	0.50	0.50	0.50	0.50	0.50
	(0.45, 0.55)	(0.45, 0.55)	(0.45, 0.55)	(0.45, 0.55)	(0.45, 0.55)
κ, inverse Frisch labor elast.	2.37	2.41	2.39	2.34	2.36
	(1.65, 3.30)	(1.70, 3.33)	(1.64, 3.42)	(1.60, 3.30)	(1.65, 3.30)
θ, habit	0.70	0.70	0.70	0.74	0.70
	(0.63, 0.77)	(0.63, 0.77)	(0.63, 0.79)	(0.64, 0.76)	(0.62, 0.78)
μ, fraction of non-Ricar. households	0.10	0.11	0.10	0.10	0.11
	(0.06, 0.16)	(0.06, 0.16)	(0.06, 0.15)	(0.06, 0.16)	(0.06, 0.16)
Frictions					
ω_w, wage stickiness	0.27	0.26	0.28	0.35	0.26
	(0.18, 0.43)	(0.18, 0.41)	(0.19, 0.71)	(0.19, 0.88)	(0.17,0.61)
ω_p, price stickiness	0.70	0.71	0.71	0.74	0.70
	(0.64, 0.77)	(0.64, 0.77)	(0.64, 0.77)	(0.65, 0.95)	(0.63, 0.77)
ψ, capital utilization	0.76	0.77	0.74	0.75	0.75
	(0.66, 0.84)	(0.67, 0.85)	(0.64, 0.83)	(0.66, 0.83)	(0.65, 0.83)
s, investment adj. cost	5.73	5.62	5.98	6.48	5.59
	(3.65, 8.11)	(3.56, 8.00)	(3.77, 8.46)	(3.90, 9.00)	(3.47, 8.03)
χ^w, wage partial indexation	0.30	0.31	0.28	0.25	0.30
	(0.20, 0.41)	(0.21, 0.43)	(0.18, 0.39)	(0.13, 0.39)	(0.20, 0.42)
χ^p, price partial indexation	0.27	0.28	0.26	0.29	0.26
	(0.13, 0.49)	(0.13, 0.49)	(0.12, 0.46)	(0.13, 0.50)	(0.12, 0.48)
Fiscal policy					
γ_{GC}, govt consumption resp to debt	0.28	0.28	0.28	0.28	-
	(0.15, 0.42)	(0.14, 0.41)	(0.14, 0.42)	(0.14, 0.42)	
γ_{GI}, govt investment resp to debt	0.17	0.17	0.16	0.18	-
	(0.01, 0.30)	(-0.03, 0.31)	(-0.02, 0.30)	(0.03, 0.31)	
γ_K, capital tax resp to debt	0.20	0.20	0.20	0.18	-
	(0.07, 0.33)	(0.07, 0.33)	(0.07, 0.32)	(0.07, 0.30)	
γ_L, labor tax resp to debt	0.10	0.10	0.10	0.10	-
	(0.02, 0.20)	(0.02, 0.20)	(0.02, 0.20)	(0.0, 0.24)	
γ_Z, transfers resp to debt	0.01	0.00	0.03	-0.02	0.20
	(-0.09, 0.17)	(-0.09, 0.16)	(-0.08, 0.19)	(-0.11, 0.14)	(0.13, 0.32)
φ_K, capital resp. to output	0.48	0.48	0.48	-	0.56
	(0.19, 0.93)	(0.20, 0.93)	(0.20, 0.94)		(0.23, 1.07)
φ_L, labor resp. to output	0.69	0.68	0.66	-	0.60
	(0.24, 1.37)	(0.24, 1.43)	(0.23, 1.32)		(0.22, 1.34)
Monetary policy					
ϕ_π, interest rate resp. to inflation	2.39	2.38	2.39	2.23	2.41
	(2.12, 2.68)	(2.10, 2.67)	(2.10, 2.68)	(1.63, 2.63)	(2.13, 2.71)
ϕ_y, interest rate resp. to output	0.04	0.04	0.03	0.06	0.03
	(0.0, 0.07)	(0.01, 0.08)	(0.0, 0.07)	(0.01, 0.12)	(0.00, 0.07)
ρ_r, lagged interest rate resp.	0.85	0.85	0.85	0.86	0.86
	(0.82, 0.88)	(0.82, 0.88)	(0.82, 0.88)	(0.82, 0.89)	(0.82, 0.88)
model comparison					
Bayes Factor rel. to benchmark	1	exp[-0.001]	exp[0.002]	exp[0.0]	exp[0.002]

Table 6: **Robustness checks for the short- and long-run effects of expansionary fiscal policy on investment.** The rows display impact and cumulative present value (PV) multipliers for investment following various shocks. Multipliers calculated at posterior medians.

		Investment Multipliers				
		Benchmark $\alpha^G = 0.05$	Varying α^G $\alpha^G = 0.0$	$\alpha^G = 0.1$	No Output Responses	$\kappa = 1$
$G^C \uparrow$	PV	-1.22	-1.23	-1.15	-1.30	-1.23
	impact	-0.07	-0.07	-0.07	-0.07	-0.05
$G^I \uparrow$	PV	4.04	-0.83	8.21	4.03	3.98
	impact	-0.08	-0.08	-0.08	-0.04	-0.07
$\tau^K \downarrow$	PV	-1.97	-1.90	-1.92	-4.28	-2.23
	impact	0.08	0.08	0.08	0.07	0.08
$\tau^L \downarrow$	PV	-0.44	-0.44	-0.43	-0.52	-0.37
	impact	0.02	0.02	0.02	0.02	0.04
$\tau^C \downarrow$	PV	-0.61	-0.62	-0.59	-0.63	-0.59
	impact	0.04	0.04	0.03	0.05	0.05
$Z \uparrow$	PV	-0.61	-0.59	-0.63	-0.58	-0.68
	impact	-0.03	-0.03	-0.03	-0.02	-0.02

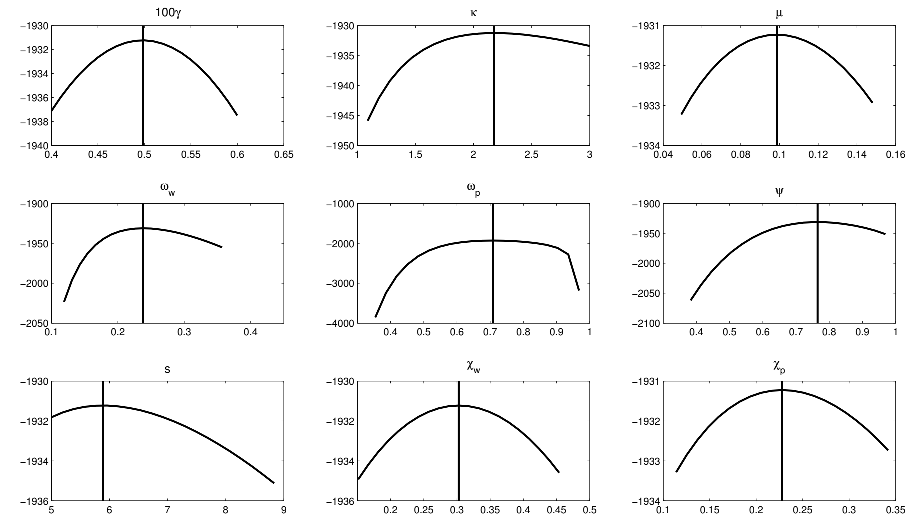

Figure 1: **Plots of slices of the likelihood around the posterior mode I.** The vertical line indicates the posterior mode.

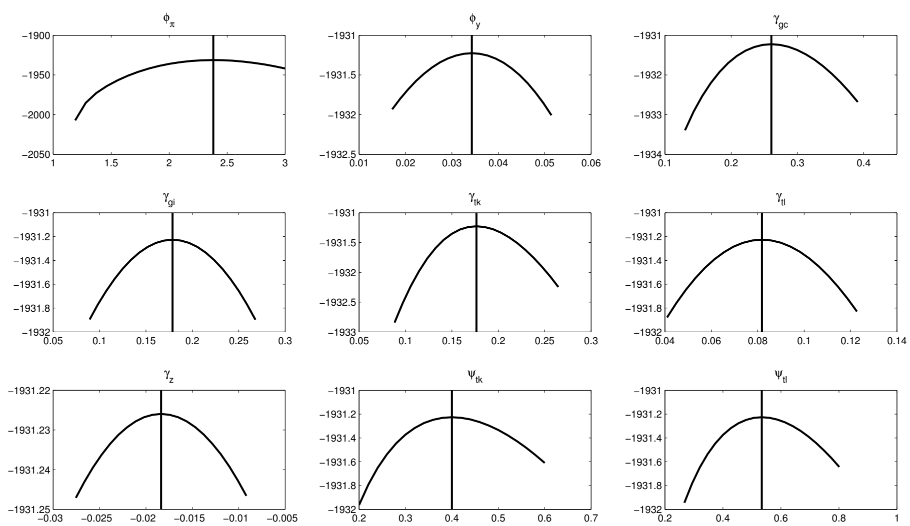

Figure 2: **Plots of slices of the likelihood around the posterior mode II.** The vertical line indicates the posterior mode.

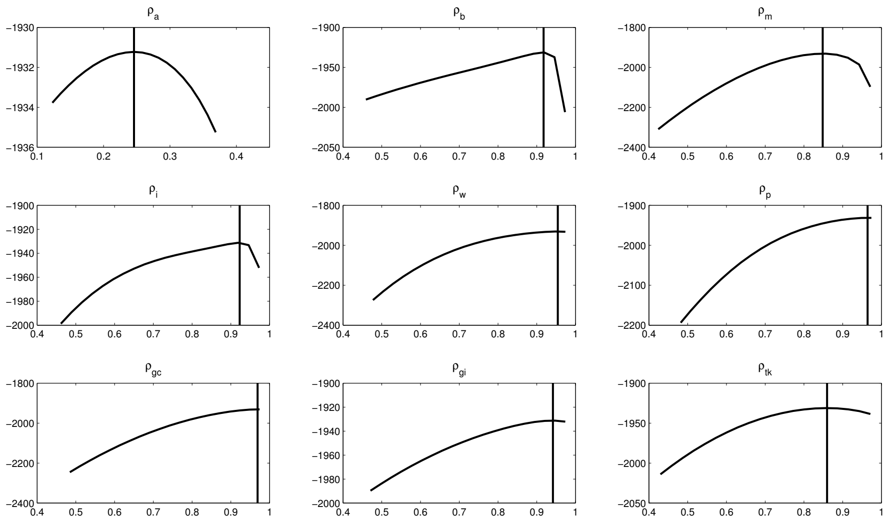

Figure 3: **Plots of slices of the likelihood around the posterior mode III.** The vertical line indicates the posterior mode.

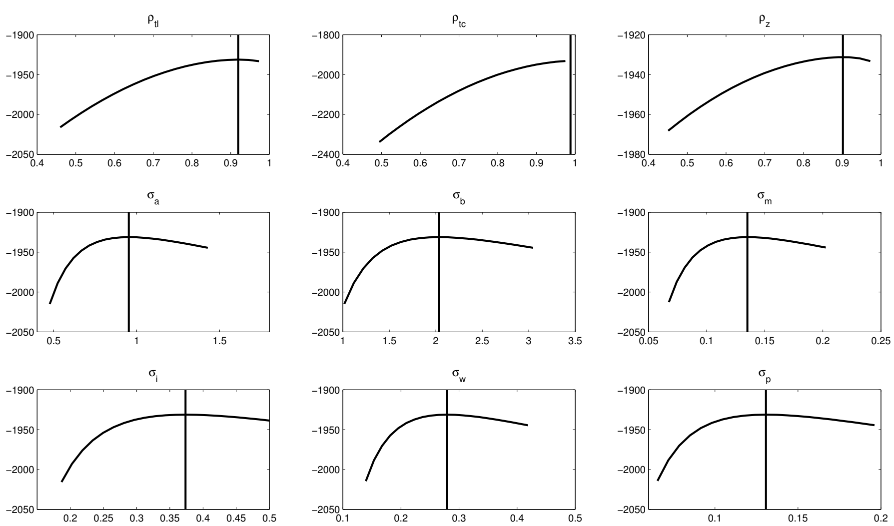

Figure 4: **Plots of slices of the likelihood around the posterior mode IV.** The vertical line indicates the posterior mode.

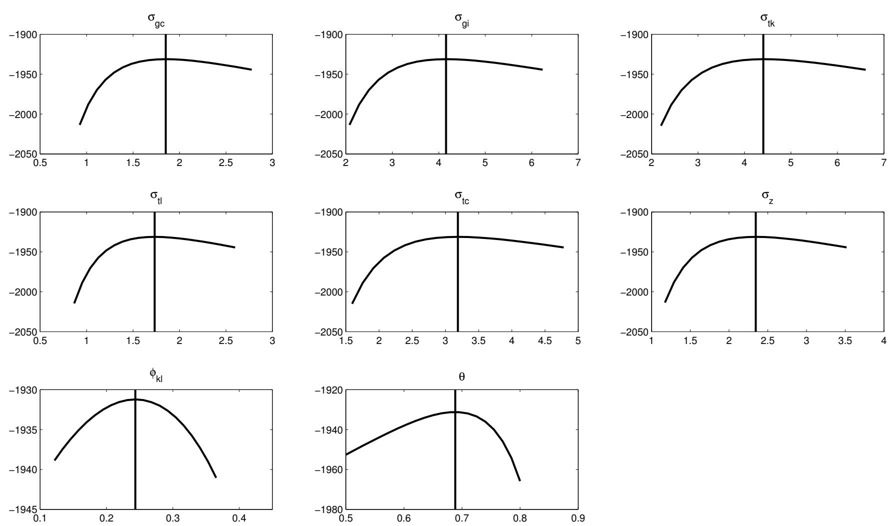

Figure 5: **Plots of slices of the likelihood around the posterior mode V.** The vertical line indicates the posterior mode.

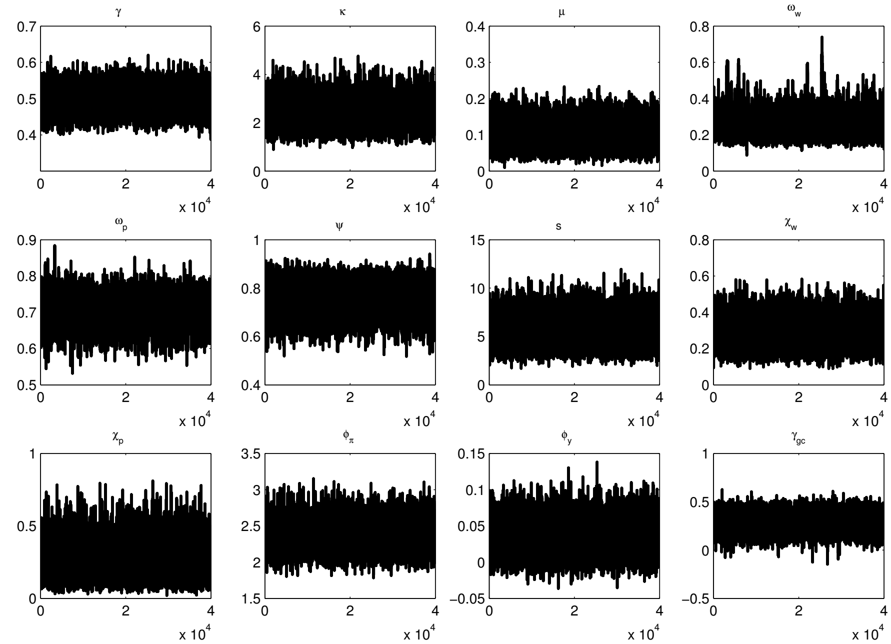

Figure 6: **Trace plots I.**

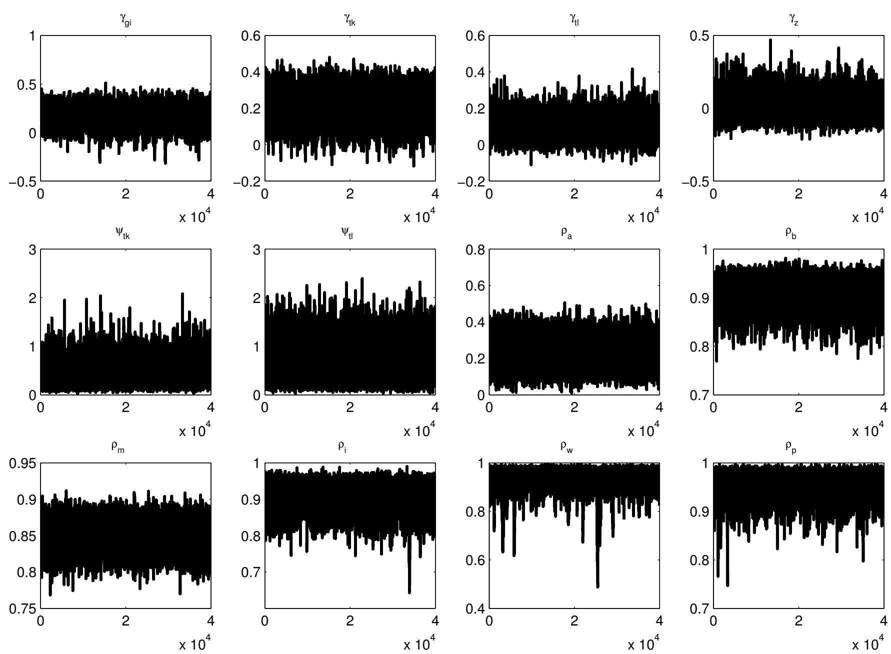

Figure 7: **Trace plots II.**

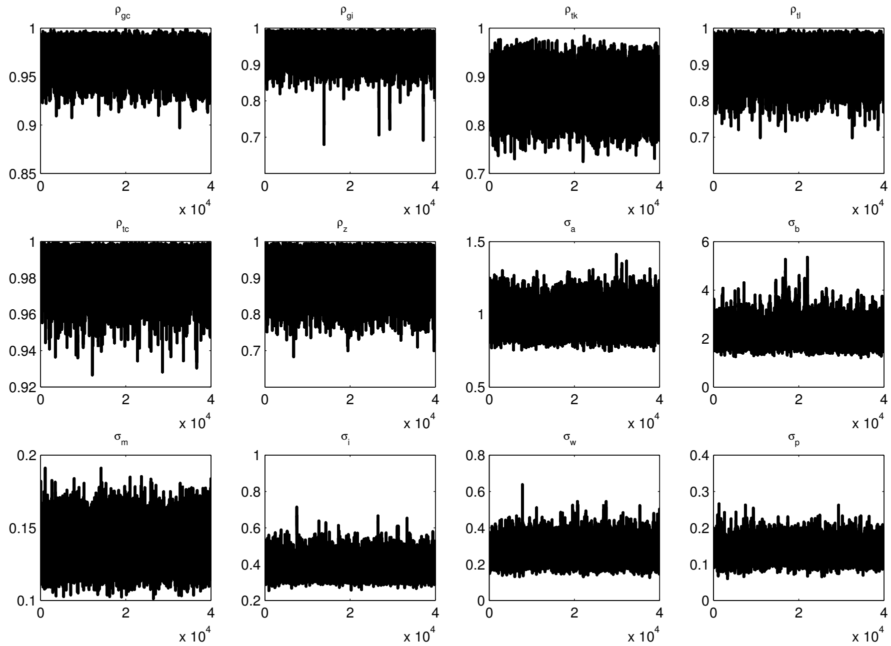

Figure 8: **Trace plots III.**

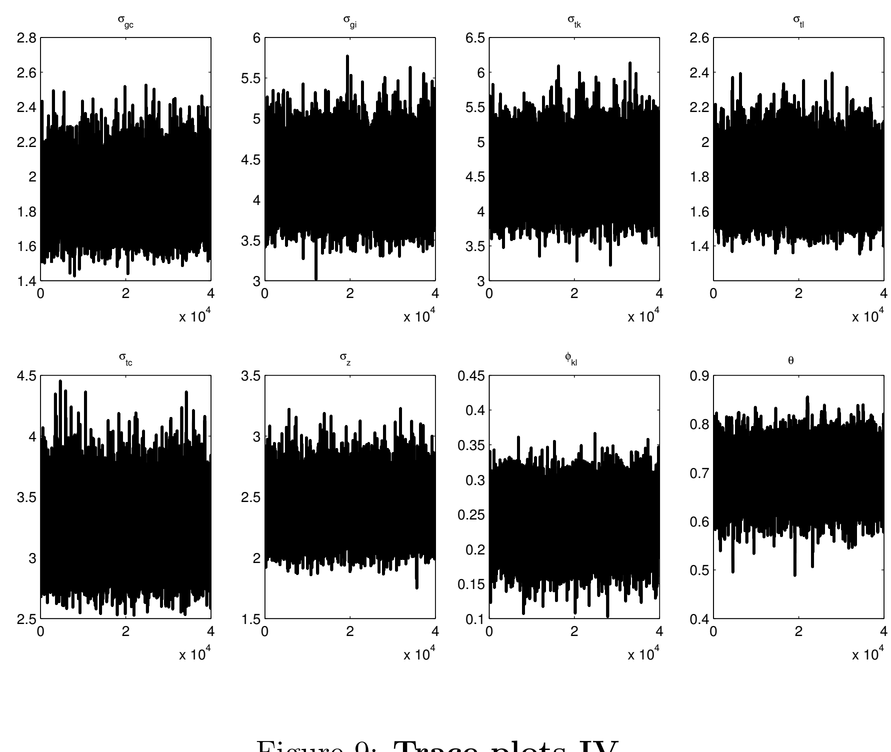

Figure 9: **Trace plots IV.**

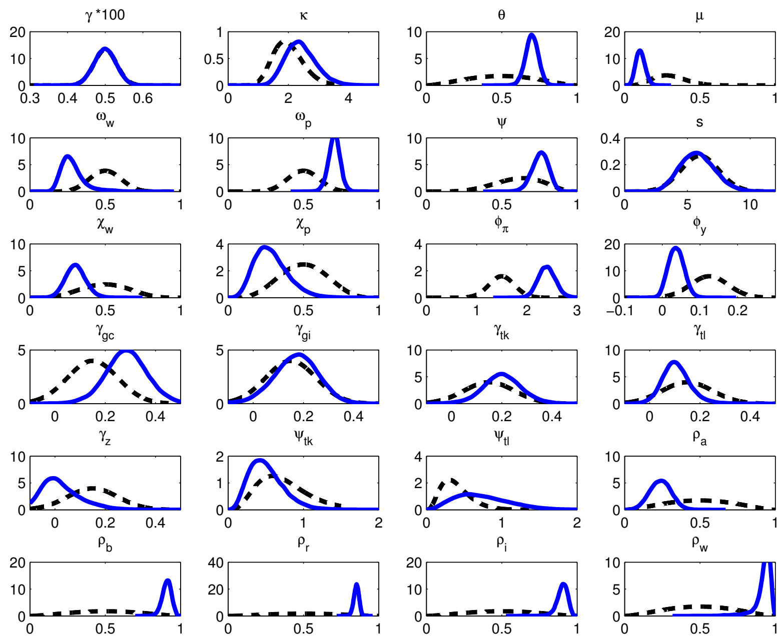

Figure 10: **Prior (dashed lines) vs. posterior (solid lines) distributions I.**

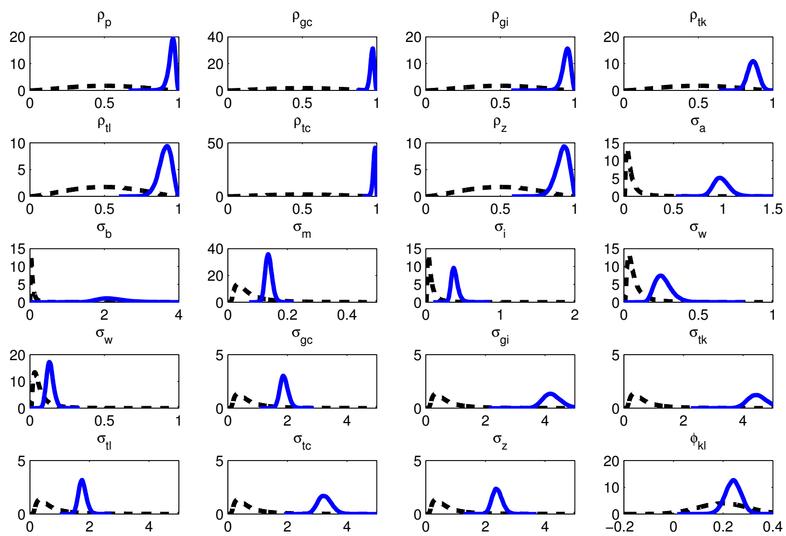

Figure 11: **Prior (dashed lines) vs. posterior (solid lines) distributions II.**

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