1 Smets and Wouters (2007, SW): the model equations

We list here the equations which describe the dynamics of the SW model.

The model variables for the sticky wage and price economy are: output \( (y_t) \), consumption \( (c_t) \), investment \( (i_t) \), Tobin’s q \( (q_t) \), utilized capital \( (k^*_t) \), installed capital \( (k_t) \), capacity utilization \( (z_t) \), rental rate of capital \( (r_t^\ell) \), price markup \( (\mu^p_t) \), real wage \( (w_t) \), total hours worked \( (l_t) \), and nominal interest rate \( (r_t) \). For the corresponding flexible economy: output \( (y_t') \), consumption \( (c_t') \), investment \( (i_t') \), Tobin’s q \( (q_t') \), utilized capital \( (k^{\ast\ast}_t) \), installed capital \( (k_t') \), capacity utilization \( (z_t') \), rental rate of capital \( (r_t'^\ell) \), price markup \( (\mu_t'^p) \), wage markup \( (\mu_t'^w) \), real wage \( (w_t') \), and total hours worked \( (l_t') \), for the corresponding flexible economy.

The shocks are: total factor productivity \( (\varepsilon_t^\ell) \), investment-specific technology \( (\varepsilon_t^i) \), government purchases \( (\varepsilon_t^y) \), Tobin’s q \( (\varepsilon_t^q) \), risk premium \( (\varepsilon_t^r) \), monetary policy \( (\varepsilon_t^m) \), wage markup \( (\varepsilon_t^w) \) and price markup \( (\varepsilon_t^p) \).

Flexible economy:

\[
\begin{align*}
\varepsilon_t^\ell &= \alpha z_t^k + (1 - \alpha) w^*_t \\
\varepsilon_t^i &= \frac{1}{1 - \gamma} r_t^k \\
k_t^{\ast\ast} &= z_t^k + k_{t-1}^k \\
r_t^{k*} &= w_t^* + l_t^* - k_t^{\ast\ast} \\
y_t^\ell &= c_y c_t^i + i_y l_t^* + r^{k_{ss}} k_y z_t^\gamma + \varepsilon_t^q \\
y_t^i &= \Phi (\alpha k_t^{\ast\ast} + (1 - \alpha) l_t^* + \varepsilon_t^i) \\
i_t^\ell &= \frac{1}{1 + \beta \gamma (1 - \sigma_c)} \left( i_{t-1}^\ell + \beta \gamma (1 - \sigma_c) E_t i_{t+1}^\ell + \frac{1}{\gamma^2} \frac{c_t^i}{\gamma} + \varepsilon_t^i \right) \\
q_t^\ell &= -r_t^\ell + (1 - \beta (1 - \delta) \gamma^{-\sigma_c}) E_t r_t^{k_{ss}} + \beta (1 - \delta) \gamma^{-\sigma_c} E_t q_{t+1}^\ell - \varepsilon_t^r \\
c_t^i &= \frac{h}{1 + \frac{\sigma_c}{\gamma}} c_{t-1}^i + \frac{1}{1 + \frac{\sigma_c}{\gamma}} E_t c_{t+1}^i + \frac{\sigma_c - 1}{\sigma_c \gamma} (1 + \frac{b}{\gamma}) \left( l_t^* - E_t l_{t+1}^* \right) \\
&\quad - \frac{1 - h}{\sigma_c \gamma} l_t^* + \varepsilon_t^b \\
w_t^\ell &= \sigma i_t^\ell + \frac{1}{1 - \frac{b}{\gamma}} c_t^i - \frac{h}{1 - \frac{b}{\gamma}} c_{t-1}^i \\
k_t^i &= \frac{1 - \delta}{\gamma} k_{t-1}^i + \left( 1 - \frac{(1 - \delta)}{\gamma} \right) i_t^\ell \\
&\quad + \gamma^2 \varphi \left( 1 - \frac{(1 - \delta)}{\gamma} \right) \left( 1 + \beta \gamma (1 - \sigma_c) \right) \varepsilon_t^i
\end{align*}
\]
Sticky Wage economy:

\[ \mu_t^p = \alpha r_t^k + (1 - \alpha) w_t - \varepsilon_t^p \]  
\[ z_t = \frac{1}{\pi} r_t^k \]  
\[ r_t^k = w_t + l_t - k_t^* \]  
\[ k_t^* = z_t + k_{t-1} \]  
\[ i_t = \frac{1}{1 + \beta \gamma (1 - \sigma_c)} \left( i_{t-1} + \beta \gamma (1 - \sigma_c) E_t \pi_{t+1} + \frac{1}{\gamma^2} \varphi q_t \right) + \varepsilon_t^i \]  
\[ q_t = -r_t + E_t \pi_{t+1} + (1 - \beta (1 - \delta) \gamma^{-\sigma_c}) E_t r_{t+1} \]  
\[ + \beta (1 - \delta) \gamma^{-\sigma_c} E_t q_{t+1} - \varepsilon_t^b \]  
\[ c_t = \frac{1}{1 + \frac{\beta}{\gamma} c_{t-1} + \frac{1}{1 + \frac{\beta}{\gamma}} E_t c_{t+1} + \frac{(\sigma_c - 1) w^{**} l^{**}}{c^{**} \sigma_c} (l_t - E_t l_{t+1})}{1 + \frac{\beta}{\gamma}} \]  
\[ y_t = c_y c_t + i_t + k^{**} h_y z_t + \varepsilon_t^y \]  
\[ y_t = \Phi (\alpha k_t^* + (1 - \alpha) l_t + \varepsilon_t^y) \]  
\[ r_t = r_\pi (1 - \rho) \pi_t + (1 - \rho) r_y (y_t - y_t^*) \]  
\[ + r_{\Delta y} (y_t - y_t^* - y_{t-1} + y_{t-1}^*) + \rho r_{t-1} + \varepsilon_t^r \]  
\[ \pi_t = \frac{1}{1 + \beta \gamma (1 - \sigma_c) l_p} \]  
\[ \left( \beta \gamma (1 - \sigma_c) E_t \pi_{t+1} + i_p \pi_{t-1} + \frac{(1 - \xi_p) (1 - \beta \gamma (1 - \sigma_c) \xi_p)}{1 + (\Phi - 1) \varepsilon_p} \mu_t^p \right) + \varepsilon_t^p \]  
\[ w_t = \frac{1}{1 + \beta \gamma (1 - \sigma_c)} w_{t-1} + \frac{\beta \gamma (1 - \sigma_c)}{1 + \beta \gamma (1 - \sigma_c)} E_t w_{t+1} + \frac{\xi_w}{1 + \beta \gamma (1 - \sigma_c)} \pi_{t-1} \]  
\[ + \frac{1}{1 + \beta \gamma (1 - \sigma_c)} \]  
\[ + \frac{1}{1 + \beta \gamma (1 - \sigma_c)} \pi_{t+1} + \frac{1 - \xi_w}{1 + \beta \gamma (1 - \sigma_c)} \]  
\[ \frac{1}{1 + (\lambda_w - 1) \varepsilon_w} \]  
\[ \left( \sigma_l l_t + \frac{1}{1 - \frac{\beta}{\gamma}} c_t - \frac{1}{1 - \frac{\beta}{\gamma}} c_{t-1} - w_t \right) + \varepsilon_t^w \]  
\[ k_t = \frac{(1 - \delta)}{\gamma} k_{t-1} + \left( 1 - \frac{(1 - \delta)}{\gamma} \right) i_t \]  
\[ + \varphi \gamma^2 \left( 1 - \frac{(1 - \delta)}{\gamma} \right) \left( 1 + \beta \gamma (1 - \sigma_c) \right) \varepsilon_t^i \]
Dynamics of the shocks:

\[ \epsilon_t^a = \rho_a \epsilon_{t-1}^a + \eta_t^a \]  
(25)

\[ \epsilon_t^b = \rho_b \epsilon_{t-1}^b + \eta_t^b \]  
(26)

\[ \epsilon_t^g = \rho_g \epsilon_{t-1}^g + \eta_t^g + \rho_g \eta_t^a \]  
(27)

\[ \epsilon_t^i = \rho_i \epsilon_{t-1}^i + \eta_t^i \]  
(28)

\[ \epsilon_t^r = \rho_r \epsilon_{t-1}^r + \epsilon_t^r \]  
(29)

\[ \epsilon_t^p = \rho_p \epsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p \]  
(30)

\[ \epsilon_t^w = \rho_w \epsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w \]  
(31)

Measurement equations:

\[ \Delta GDP_t = y_t - y_{t-1} + \bar{\gamma} \]  
(32)

\[ \Delta CONS_t = c_t - c_{t-1} \]  
(33)

\[ \Delta INV_t = i_t - i_{t-1} \]  
(34)

\[ \Delta WAG_t = w_t - w_{t-1} \]  
(35)

\[ \Delta P_t = \pi_t + \bar{\pi} \]  
(36)

\[ FEDFUNDS_t = r_t + \bar{r} \]  
(37)

\[ lHOURS_t = l_t + \bar{l} \]  
(38)
2 Smets and Wouters (2007): comparison between IRFs obtained with Bayesian and Maximum Likelihood estimation

In the following figures we report the impulses responses by Smets and Wouters (in solid line), the ones we obtained by estimating the model with ML on the same sample (in dashed line), and on the sample extended to the end of 2007 (in x-marked line). The graphs are meant to show that the results obtained with ML are fairly similar to the original ones with Bayesian techniques. Moreover, our ML results on the original sample are very similar to the ones obtained by Iskrev (2008) in a similar experiment.

Figure 1: Impulse Responses to a risk-premium shock

Notes: The impulse responses in solid lines are those obtained by Smets and Wouters (2007). The dashed ones, are those obtained with ML estimation on the same sample. The x-marked ones are those obtained with ML on the sample extended to 2007.
Figure 2: Impulse Responses to an exogenous spending shock

Notes: See Notes at Figure 1.

Figure 3: Impulse Responses to an investment-specific technology shock

Notes: See Notes at Figure 1.
Figure 4: Impulse Responses to a wage markup shock

Notes: See Notes at Figure 1.

Figure 5: Impulse Responses to a productivity shock

Notes: See Notes at Figure 1.
Figure 6: Impulse Responses to a monetary policy shock

Notes: See Notes at Figure 1.