Appendix to *Long-Run Monetary Neutrality and Long-Horizon Regressions*†

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This appendix describes our data, presents background results for the paper, outlines the construction of the standard error necessary to calculate the VAR-long-horizon regression test of LRMN, presents our Monte Carlo bootstrap methods, and reports Monte Carlo results not included in the paper.

Data

We obtain the data from several sources. We wish to thank Nils Olekalns, Alfred Haug, and Neil Ericsson for providing us with all of the Australian data, the Canadian real GDP series, and the U.K. data of Attfield, Demery, and Duck (1995), respectively. The Canadian monetary aggregate series for the period $1872 - 1968$ are from Metcalf, Redish, and Shearer (1998) and from 1968 to 1993 they are the CANSIM series B1646 and B1630. Our U.K. and U.S. data up to and including 1975 are from Friedman and Schwartz (1982). Attfield, Demery, and Duck provides the later U.K. data. We extend the U.S. sample to 1997 following Boschen and Otrok (1994). The net national product series post-1975 is taken from the University of Virginia Library webpage, *http://www.lib.virginia.edu/socsci/nipa/nipa.html*, and the two U.S. monetary aggregates are in the Federal Reserve Board virtual database. This data is available from us on request.

Unit Root and Cointegration Tests

The long-horizon regression test of LRMN rests on output and the monetary aggregate being unit root processes. This part of the appendix presents evidence and details about unit root and cointegration tests of Australian, Canadian, U.K., and U.S. output, the money stock, and the monetary base. We use the augmented Dickey-Fuller (ADF) $t-$ratio to test for unit a unit root in these series. The ADF regression is

$$\Delta w_t \quad = \quad \phi \, + \, \varphi \, t \, + \, \rho w_{t-1} \, + \, \sum_{i=1}^{p} \gamma_i \Delta w_{t-i} \, + \, e_{p,t}. \tag{A.2}$$

Under the null (alternative) hypothesis that the series w is I(1), ρ equals zero ($\rho < 0$). We follow Campbell and Perron (1991) and choose the lag length p moving from the general-to-specific model. In our case, this begins at $p = 8$ and deletes successive lags until a $t-$ratio greater than 1.6 (in absolute value) is encountered. Table A1 provides the values of p used to estimate the ADF regression (A.2) in parentheses next to the ADF $t-$ratio. MacKinnon (1991) gives asymptotic ten percent, five percent, and one percent critical values for the ADF $t-$ratio equal to -3.13, -3.41, and -3.96 respectively.

Stock (1991) asymptotic confidence intervals appear in brackets beneath the ADF $t-$ratios. The 95 percent confidence intervals are constructed as $(1 + c_0/T, \; 1 + c_1/T)$, where T is the sample size and c_0 and c_1 are found in table A.1, part B, of Stock.

Table A1 contain the DF $t-$ratios and Stock 95 percent asymptotic confidence intervals. There are only two possible rejections of the unit root hypothesis. These are the Canadian monetary base for its $1872 - 1994$ sample and the $1896 - 1997$ sample of the U.S. money stock. However, the upper bound of the largest autoregressive root of the latter series is 0.97. Further, the relationship between the monetary base and the money stock implies a unit root in one suggests a unit root in the other. Hence, we feel confident that the data is adequately described by the unit root model.

Berben and van Dijk (1998) argue that tests of long-horizon asset return predictability rely on an error correction model that is part of the maintained hypothesis. When true, the long-horizon regression test of predictability can be interpreted as a test for cointegration. We examine this issue for the long-horizon regression test of LRMN with the Engle and Granger (1987) cointegration test. The Engle and Granger procedure begins with the OLS regression of y on x and a constant, where x_t is either the broad money stock m or the monetary base b. Given unit roots in y and x, the null hypothesis of no cointegration between y and x implies that $\{u_t\}$ is has a unit root process.

A test of this conjecture involves the ADF regression (A.2) with $\phi = \varphi = 0$. The lag order of the ADF regression is chosen in the same manner as the unit root tests and is reported in parentheses next to the ADF $t-$ratio of table A2. The asymptotic ten percent, five percent, and one percent critical values for this $t-$ratio equal -3.04, -3.34, and -3.90, respectively. Once again, the source of these asymptotic critical values is MacKinnon (1991).

Table A2 reports the results of these cointegration tests. The combination of Canadian output and its monetary base and U.S output and its money stock reject the no cointegration hypothesis at the five percent level. Given these monetary aggregates also rejected the unit root tests, this is not a surprise. Nonetheless, we uncover little support for Berben and van Dijk's thesis because of thin evidence of cointegration in our long-annual samples.

Monte Carlo Bootstrap Procedures

Our Monte Carlo experiments rely on our long-annual samples and estimates of the coefficients of the simultaneous equation model (1). Estimates of this bivariate model requires an identification scheme. All of these identifications include assumption (ii), $\sigma_{uv} = 0$, a diagonal covariance matrix and either LREM, $\xi_{x,y,0} + \xi_{x,y}(\mathbf{1}) = 0$, or LRPS, $[\xi_{x,y,0} + \xi_{x,y}(\mathbf{1})]/[1 - \xi_{x,x}(\mathbf{1})] = 1$. When we generate data under LRMN, we also impose $\xi_{y,x,0} + \xi_{y,x}(\mathbf{1}) = 0$. This add a third restriction which over-identifies the simultaneous equations model (1). We adapt the Hausman, Newey, and Taylor (1987) full-information maximum likelihood (FIML) estimator whenever faced with an overidentified structural vector autoregression (SVAR). Under the alternative of no LRMN, only two restrictions are imposed. In this case, the SVAR is just-identified which allows us to estimate it with the instrumental variables (IV) method of King and Watson (1997). All the SVARs are assumed to be first-order to be consistent with the unrestricted VARs estimated to compute the VAR-long-horizon regression estimator.

The Monte Carlo experiments are calibrated to the FIML and IV estimates. We generate 10,000 bootstrap samples of $3T$ observations of SVAR errors $\widetilde{S}_\ell = [\widetilde{u}_\ell \; \widetilde{v}_\ell]'$ by sampling with replacement from estimated SVAR errors, $\{\widehat{S}\}_{t=1}^{T}$, which are conditional on an identification and coefficient estimates. Note that the elements of $\widetilde{S}_\ell$ are orthogonal and serially uncorrelated by construction. Next, synthetic data $\widetilde{W}_\ell \equiv \{\Delta\widetilde{y}_\ell, \Delta\widetilde{x}_\ell\}$ is produced according to the reduced-form law of motion

$$\widetilde{W}_\ell = \widehat{\xi}_0^{-1}\alpha + \widehat{\xi}_0^{-1}\widehat{\xi}_1\widetilde{W}_{\ell-1} + \widehat{\xi}_0^{-1}\widetilde{S}_\ell, \qquad (A.3)$$

for each bootstrap sample, where $\widehat{\xi}_0$ is the estimated structural impact matrix of the simultaneous equations model (1),

$$\xi_0 = \begin{bmatrix} 1 & -\xi_{x,y,0} \\ -\xi_{y,x,0} & 1 \end{bmatrix}, \quad \xi_1 = \begin{bmatrix} \xi_{x,x,1} & \xi_{x,y,1} \\ \xi_{y,x,1} & \xi_{y,y,1} \end{bmatrix}.$$

Since the structural shocks are assumed to be uncorrelated, we draw $\widetilde{u}_\ell$ and $\widetilde{v}_\ell$ independently from $\{\widehat{S}_\ell\}_{\ell=1}^{3T}$. We discard the first $2T$ observations to minimize the impact of starting values.

We use the synthetic data to construct $\beta_{k,OLS}$ and $\beta_{k,VAR}$, their standard errors, and test LRMN, $\beta_{k,OLS} = 0$ and $\beta_{k,VAR} = 0$. Given the 10,000 replications, we calculate the type I error rates reported in tables 2 and 5 and the size-adjusted power of the tests found in tables 3 and 6. At each repetition, we save the point estimates of the numerator of $\beta_{k,VAR}$ to compute the empirical densities of the $Cov(\Delta_k y_t,\, \Delta_k x_t)$ found in figure 2.

The Standard Error of the VAR Estimator

The VAR-long-horizon regression estimator of β_k is a non-linear function the unrestricted VAR parameters $\Gamma(\mathbf{L})$ and the covariance matrix of the VAR errors. Let the vector of the true values of these parameters be denoted $\mathcal{G}_0$ with covariance matrix Ω. The implication is $\widehat{\beta}_{k,VAR} = \beta_{k,VAR}(\widehat{\mathcal{G}})$. The usual arguments yield the standard error of $\beta_{k,VAR}$,

$$S.E.(\widehat{\beta}_{k,VAR}) = \sqrt{\nabla\beta_{k,VAR}(\widehat{\mathcal{G}})\widehat{\Omega}\nabla\beta_{k,VAR}(\widehat{\mathcal{G}})'},$$

where ∇ represents the gradient of $\beta_{k,VAR}$ evaluated at $\widehat{\mathcal{G}}$ which is conformable with Ω. Hodrick (1992) employs this method to test the long-horizon predictability of equity returns.

Additional Results

Tables A3−A12 and figure A1 provide additional information about the behavior of the long-horizon regression tests of LRMN in our Australian, Canadian, U.K., and U.S. samples. The results in these tables and figures are based on the Monte Carlo bootstrap methods discussed in section A3.

Table A3−A8 provide results about the impact of the Andrews and Monahan (1992) pre-whitening procedure for the heteroskedastic, autocorrelation consistent (HAC) standard error estimator of β_k on the long-horizon regression tests of LRMN. We allude to this in footnote 11 of the paper.

There are only two differences between the OLS-long-horizon regression tests of LRMN of table 2 based on a HAC estimator with no pre-whitening correction for the OLS $t-$ratio and table A3. One is a rejection of LRMN at the five percent level using the HAC estimator with a pre-whitening step rather than the ten percent for the no pre-whitening case conditional on the Australian monetary base and $\beta_{25,OLS}$. The other is the U.K. money stock yields a rejection of LRMN at the ten percent level when the standard error computation of $\beta_{k,OLS}$ involves pre-whitening when table 2 shows no rejections of LRMN for the U.K. monetary aggregates. The VAR-long-horizon regression tests of LRMN yields the same inference irrespective of the HAC estimator as shown in table 4 and A4.

The size and power long-horizon regression test of LRMN shows minimal improvement when the standard error of $\beta_{k,OLS}$ and $\beta_{k,VAR}$ is calculated with the pre-whitened HAC estimator. Tables A5 and A6 contain the type I error rates for the alternative HAC estimator. The size distortion of the LRMN test based on $\beta_{k,OLS}$ is not as severe as reported in table 2, but it remains large. The size distortion inherent in the VAR-long-horizon regression test of LRMN is nearly identical to the results of table 5 which depend on the Newey and West (1994) HAC estimator. The same pattern arises in a comparison of size-adjusted power across tables A7-A8 and table 3 and 6. The upshot is that the power of the long-horizon regression tests of LRMN approximates size when we use a HAC estimator with a pre-whitening correction. Thus, we find that operating a pre-whitening procedure to estimate the HAC standard error matters little for the results we report.

Table A9 presents size adjusted power results for the re-scaled $t-$ratio proposed by Valkanov (2003). He argues that dividing the $t-$ratio used to test long-horizon return predictability by the sample size T produces a test which possesses superior small sample properties. However, the results in table A9 show this does not appear to be the case for the long-horizon regression test of LRMN in our long-annual samples because size continues to approximate power.

The bottom row of windows of figures 1-4 of the paper contain empirical confidence bands. These bands rely on empirical $p-$values (some of) which we present in tables A10 and A11 for the OLS and VAR estimators, respectively. In turn, the empirical $p-$values are built on the type I error rates found in tables 2 and 5. The implication of the size distortion of the OLS-long-horizon regression test of LRMN is that it is large enough to produce empirical p-values in table A10 that reverse almost any rejection of LRMN. The exception being for the Canadian monetary base at ten and 15 year horizons. Table A11 contains qualitatively similar empirical $p-$values for the VAR-long-horizon regression test of LRMN, except the rejection of LRMN for the Canadian monetary base occurs at all horizons.

Size-adjusted power for case of a proportional long-run monetary (LRM) non-neutrality appears in table A12. That is, we impose $\lambda = 1$ on the DGP of the Monte Carlo. Given stable velocity, an increase in the money supply leads to a proportionate change in real income while the price level is remains unchanged. This also overidentifies the simultaneous model of equation (1) because LREM and a diagonal covariance matrix is imposed on the DGP. Thus, we engage the FIML estimator to estimate the SVAR.

The experiment of a LRM non-neutrality examines the ability of the long-horizon regression test of LRMN to detect economically large deviations from LRMN. Although table A12 reports the size-adjusted power is general higher across all forecast horizons and long-annual samples, the size-adjusted power results continue to indicate an uninformative test of LRMN. This is especially apparent at the longer horizons of $k = 25$ and $k = 30$.

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Table A1. ADF Statistics and Confidence Intervals for the Largest Autoregressive Root[†]

Sample	y	m	b
Australia			
1901-1994	-1.632 (4)	-0.655 (1)	-2.272 (6)
	$[0.901, 1.057]$	$[1.016, 1.062]$	$[0.825, 1.052]$
Canada	-2.868 (1)	-2.015 (6)	-5.096 (8)
1872-1994	$[0.799, 1.031]$	$[0.890, 1.040]$	$[\cdot, 0.803]$
U.K.	-2.618 (1)	-0.257 (2)	2.281 (6)
1871-1993	$[0.822, 1.034]$	$[1.014, 1.048]$	$[0.869, 1.039]$
U.S.	-3.769 (6)	-3.810 (1)	-2.897 (1)
1869-1997	$[0.659, 1.015]$	$[\cdot, 0.970]$	$[0.809, 1.030]$

[†]The MacKinnon (1991) asymptotic ten percent, five percent, and one percent critical values for the ADF $t-$ratio are -3.13, -3.41, and -3.96 respectively. The lag length, p, of the ADF regression (A.2) appears in parentheses next to the ADF $t-$ratio. The brackets contain Stock 95 percent confidence intervals of the largest autoregressive root.

Table A2. Engle-Granger Tests for Cointegration[⋆]

Sample	(y, m)	(y, b)
Australia 1900 – 1994	-1.122 (3)	-2.360 (0)
Canada 1872 – 1994	-1.388 (3)	-3.513 (1)
U.K. 1872 – 1994	-2.134 (8)	-2.981 (1)
U.S. 1869 – 1997	-3.566 (6)	-2.357 (6)

[⋆]The MacKinnon (1991) asymptotic ten percent, five percent, and one percent critical values for the Granger and Engle (1987) cointegration test $t-$ratio are -3.04, -3.34, and -3.90 respectively. The lag length, p, of the second-stage ADF regression appears in parentheses next to the ADF $t-$ratio.

Table A3. Estimates of $\beta_{k,OLS}$ and Tests of LRMN:
HAC Estimator Employs Pre-whitened

Sample	Monetary Aggregate	Horizon $k = 5$	$k = 10$	$k = 15$	$k = 20$	$k = 25$	$k = 30$
Australia	m :	0.08	0.11	$0.16^{\dagger}$	$0.21^{\dagger}$	$0.30^{\dagger}$	$0.38^{\dagger}$
1900 – 1994	b :	0.06	0.04	0.06	0.09	$0.19^{\dagger}$	$0.33^{\dagger}$
Canada	m :	$0.29^{\dagger}$	$0.24^{\dagger}$	$0.22^{\dagger}$	$0.21^{\dagger}$	$0.21^{\dagger}$	$0.21^{\dagger}$
1872 – 1994	b :	$0.25^{\dagger}$	$0.22^{\dagger}$	$0.23^{\dagger}$	$0.25^{\dagger}$	$0.29^{\dagger}$	$0.35^{\dagger}$
United Kingdom	m :	−0.04	−0.02	−0.02	0.00	0.03	$0.06^{\dagger\dagger}$
1871 – 1993	b :	−0.03	−0.04	−0.03	−0.02	0.02	0.06
United States:	m :	$0.34^{\dagger}$	$0.33^{\dagger}$	$0.32^{\dagger}$	$0.27^{\dagger}$	$0.30^{\dagger}$	$0.42^{\dagger}$
1869 – 1997	b :	$0.20^{\dagger}$	0.14	0.08	−0.03	−0.08	−0.10

Table A4. Estimates of $\beta_{k,VAR}$ and Tests of LRMN:
HAC Estimator Pre-whitened

Sample	Monetary Aggregate	Horizon $k = 5$	$k = 10$	$k = 15$	$k = 20$	$k = 25$	$k = 30$
Australia	m :	0.10	0.10	0.10	0.10	0.10	0.10
1900 – 1994	b :	0.12	0.13	0.13	0.13	0.13	0.13
Canada	m :	$0.28^{\dagger}$	$0.28^{\dagger}$	$0.28^{\dagger}$	$0.28^{\dagger}$	$0.28^{\dagger}$	$0.28^{\dagger\dagger}$
1872 – 1994	b :	$0.27^{\dagger}$	$0.27^{\dagger}$	$0.27^{\dagger}$	$0.27^{\dagger}$	$0.27^{\dagger}$	$0.27^{\dagger}$
United Kingdom	m :	0.03	0.06	0.07	0.07	0.07	0.07
1871 – 1993	b :	0.10	0.12	0.13	0.13	0.13	0.13
United States:	m :	$0.30^{\dagger}$	$0.29^{\dagger\dagger}$	$0.29^{\dagger\dagger}$	$0.29^{\dagger\dagger}$	0.29	0.29
1869 – 1997	b :	$0.14^{\dagger}$	$0.13^{\dagger}$	$0.13^{\dagger\dagger}$	$0.13^{\dagger\dagger}$	$0.13^{\dagger\dagger}$	$0.13^{\dagger\dagger}$

Rejection of LRMN is denoted by † (††) at the five (ten) percent level.

Table A5. Type I Error Rates of the OLS-Long-Horizon Regression Test of LRMN: HAC Estimator Pre-whitened

Sample	Monetary Aggregate	Horizon $k=5$	$k=10$	$k=15$	$k=20$	$k=25$	$k=30$
Australia	m :	0.185	0.175	0.210	0.256	0.295	0.322
1900 – 1994	b :	0.086	0.127	0.175	0.220	0.267	0.300
Canada	m :	0.236	0.181	0.192	0.220	0.242	0.270
1872 – 1994	b :	0.179	0.153	0.176	0.199	0.234	0.272
United Kingdom	m :	0.197	0.163	0.178	0.205	0.238	0.265
1871 – 1993	b :	0.086	0.107	0.143	0.179	0.214	0.245
United States	m :	0.346	0.235	0.227	0.241	0.263	0.283
1869 – 1997	b :	0.441	0.282	0.265	0.272	0.286	0.303

Table A6. Type I Error Rates of the VAR-Long-Horizon Regression Test of LRMN: HAC Estimator Pre-whitened

Sample	Monetary Aggregate	Horizon $k=5$	$k=10$	$k=15$	$k=20$	$k=25$	$k=30$
Australia	m :	0.176	0.135	0.105	0.078	0.056	0.036
1900 – 1994	b :	0.056	0.042	0.029	0.019	0.013	0.008
Canada	m :	0.312	0.260	0.226	0.186	0.152	0.122
1872 – 1994	b :	0.217	0.179	0.148	0.120	0.094	0.075
United Kingdom	m :	0.221	0.188	0.158	0.132	0.108	0.088
1871 – 1993	b :	0.086	0.070	0.057	0.045	0.033	0.025
United States	m :	0.482	0.414	0.366	0.316	0.273	0.227
1869 – 1997	b :	0.582	0.505	0.449	0.397	0.348	0.298

Error rates calculated at nominal size of 0.05.

Table A7. Size-Adjusted Power of the OLS-Long-Horizon:
Regression Test of LRMN HAC Estimator Pre-whitened

Sample	Monetary Aggregate	Horizon $k=5$	$k=10$	$k=15$	$k=20$	$k=25$	$k=30$
Australia	m :	0.072	0.061	0.057	0.053	0.055	0.055
1900 – 1994	b :	0.093	0.061	0.058	0.055	0.054	0.052
Canada	m :	0.125	0.098	0.079	0.074	0.066	0.063
1872 – 1994	b :	0.075	0.066	0.064	0.061	0.064	0.059
United Kingdom	m :	0.044	0.048	0.050	0.051	0.048	0.046
1871 – 1993	b :	0.052	0.058	0.055	0.051	0.055	0.054
United States	m :	0.018	0.030	0.036	0.040	0.042	0.040
1869 – 1997	b :	0.140	0.109	0.090	0.082	0.078	0.073

Table A8. Size-Adjusted Power of the VAR-Long-Horizon:
Regression Test of LRMN HAC Estimator Pre-whitened

Sample	Monetary Aggregate	Horizon $k=5$	$k=10$	$k=15$	$k=20$	$k=25$	$k=30$
Australia	m :	0.082	0.082	0.082	0.081	0.081	0.081
1900 – 1994	b :	0.075	0.073	0.072	0.072	0.072	0.072
Canada	m :	0.147	0.143	0.143	0.142	0.142	0.142
1872 – 1994	b :	0.092	0.091	0.090	0.089	0.089	0.089
United Kingdom	m :	0.042	0.042	0.043	0.042	0.042	0.043
1871 – 1993	b :	0.042	0.042	0.042	0.042	0.042	0.042
United States	m :	0.023	0.025	0.025	0.025	0.025	0.025
1869 – 1997	b :	0.123	0.117	0.114	0.113	0.113	0.112

Table A9. Size-Adjusted Power of the OLS-Long-Horizon Regression Test: Valkanov's Rescaled $t-$ratio

Sample	Monetary Aggregate	Horizon $k=5$	$k=10$	$k=15$	$k=20$	$k=25$	$k=30$
Australia	m :	0.289	0.217	0.144	0.095	0.068	0.050
1900 – 1994	b :	0.381	0.251	0.170	0.108	0.072	0.050
Canada	m :	0.225	0.189	0.140	0.103	0.069	0.050
1872 – 1994	b :	0.269	0.213	0.155	0.102	0.074	0.050
United Kingdom	m :	0.240	0.189	0.141	0.100	0.068	0.050
1871 – 1993	b :	0.331	0.240	0.157	0.100	0.073	0.050
United States	m :	0.205	0.173	0.127	0.097	0.068	0.050
1869 – 1997	b :	0.188	0.167	0.123	0.093	0.069	0.050

Table A10. Empirical $p-$values of the OLS-Long-Horizon Regression Test of LRMN

Sample	Monetary Aggregate	Horizon $k=5$	$k=10$	$k=15$	$k=20$	$k=25$	$k=30$
Australia	m :	0.628	0.442	0.294	0.323	0.243	0.175
1900 – 1994	b :	0.584	0.667	0.633	0.578	0.412	0.093
Canada	m :	0.265	0.176	0.241	0.391	0.430	0.406
1872 – 1994	b :	0.117	0.046	0.023	0.113	0.187	0.361
United Kingdom	m :	1.000	1.000	1.000	0.945	0.676	0.517
1871 – 1993	b :	1.000	1.000	1.000	1.000	0.832	0.673
United States	m :	0.353	0.151	0.115	0.401	0.446	0.259
1869 – 1997	b :	0.536	0.679	0.757	1.000	1.000	1.000

Table A11. Empirical $p-$values of the VAR-Long-Horizon Regression Test of LRMN

Sample	Monetary Aggregate	Horizon $k=5$	$k=10$	$k=15$	$k=20$	$k=25$	$k=30$
Australia	m :	0.462	0.477	0.481	0.483	0.484	0.485
1900 – 1994	b :	0.393	0.416	0.423	0.426	0.428	0.429
Canada	m :	0.124	0.124	0.124	0.124	0.124	0.124
1872 – 1994	b :	0.030	0.029	0.028	0.028	0.028	0.028
United Kingdom	m :	0.762	0.489	0.442	0.424	0.413	0.407
1871 – 1993	b :	0.208	0.165	0.155	0.151	0.149	0.148
United States	m :	0.472	0.488	0.493	0.495	0.496	0.497
1869 – 1997	b :	0.437	0.459	0.466	0.470	0.472	0.474

Table A12. Size-Adjusted Power of OLS-Long-Horizon Regression Test of LRMN: Proportional LRM Non-Neutrality, $\lambda = 1$

Sample	Monetary Aggregate	Horizon $k=5$	$k=10$	$k=15$	$k=20$	$k=25$	$k=30$
Australia	m :	0.390	0.231	0.168	0.144	0.133	0.122
1900 – 1994	b :	0.081	0.062	0.060	0.058	0.057	0.052
Canada	m :	0.669	0.471	0.358	0.289	0.233	0.214
1872 – 1994	b :	0.255	0.187	0.158	0.128	0.114	0.104
United Kingdom	m :	0.696	0.502	0.383	0.315	0.266	0.246
1871 – 1993	b :	0.356	0.244	0.180	0.138	0.129	0.122
United States	m :	0.003	0.009	0.014	0.019	0.020	0.023
1869 – 1997	b :	0.961	0.818	0.660	0.553	0.483	0.428

Figure A.1: Empirical Densities of $Cov\left(\Delta_{30}y, \Delta_{30}b\right)$ under the Null and Alternative Hypotheses