**Economics 250 Mid-Term Test 2**
21 March 2019

Instructions: You may use an approved hand calculator. Do not hand in the question sheet. Answer all four questions in the answer booklet provided. Show your work. Formulas and tables are provided at the end of the question pages.

**1.** Suppose that medical researchers know that a treatment has a 60 percent chance of being successful for any patient.

(a) If they treat 8 patients what is the probability that the treatment succeeds for 5 of them? What is the probability that it succeeds for 6 or more?

(b) If they treat 100 patients what is the probability that the treatment succeeds for 50 or more?

**2.** An economist is asked to estimate the average age in a population. From a random sample of 16 people, she finds a sample average age 39. Suppose that she knows the population standard deviation of age is $\sigma = 5$.

(a) Find a 95% confidence interval for the population average age.

(b) Suppose she instead reports the margin of error as 2. What must the confidence level be?

**3.** Economists hypothesize that the average income in a country is 15, measured in thousands of US dollars (so this is the null hypothesis). The alternative hypothesis is that the average income is greater than 15. Suppose that they know that the population standard deviation of income is $\sigma = 3$ thousand dollars. They study a sample of 16 people.

(a) If they find a sample average income of 15.75 then what is the $P$-value?

(b) Suppose they decide in advance that they will reject the null hypothesis if the average income per capita is greater than 16.5. What is the probability of Type I error?

(c) If the true (but unknown) population average income in fact is 17.4 then what is the power of the test in part (b)?

**4.** Suppose that 9 students take a professional exam and receive an average score of 50. They then participate in a coaching session, and retake the exam, now with an average score of 54. The sample standard deviation of the *change* in their scores is 6.

(a) Find a 90% confidence interval for the population average change in score as a result of the course.

(b) Test the null hypothesis that the effect of the course on average is zero against the alternative that it is positive, and report a range for the $P$-value.

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**1.** (a: 2 marks) Here $n = 8$, $p = 0.6$ and so from Table C or the binomial formula for $k = 5$ the probability is 0.2786 or 27.86%. The probability that it succeeds for 6 or more is found by adding up the values for $k = 6, ..., 8$ which gives 0.3154 or 31.54%.

(b: 2 marks) With $n = 100$ we use the normal approximation that $X \sim N(np, \sqrt{np(1-p)})$ so $X \sim N(60, 4.8989)$. Standardizing gives $z = (50 - 60)/4.8989 = -2.04$ so from table A the probability is 1-0.0207 = 0.9793 or 97.93%.

**2.** (a: 2 marks) The 95% confidence interval is:

$$39 \pm 1.96 \cdot \frac{5}{4} = 39 \pm 2.45 = (36.55, 41.45).$$

(b: 2 marks) We have:

$$\text{ME} = 2 = z \cdot \frac{5}{4},$$

so $z = 1.6$. From Table A 0.9452 is to the left so $2 \times (1 - .9452) = 0.1096$ is in the two tails so the confidence level is 0.8904 or 89.04%.

**3.** (a: 2 marks) Under the null the sampling distribution of the sample mean is:

$$\overline{x} \sim N(15, 0.75).$$

Locating $\overline{x} = 15.75$ in this distribution gives $z = 1$. The $P$-value is the area above that which is 0.1587.

(b: 2 marks) That critical value gives $z = (16.5 - 15)/0.75 = 2$ so the probability of Type I error (also labelled $\alpha$) is 0.0228 or 2.28%.

(c: 2 marks) Under the alternative the distribution is:

$$\overline{x} \sim N(17.4, 0.75).$$

Locating the critical value 16.5 in this distribution gives:

$$z = \frac{16.5 - 17.4}{0.75} = -1.2.$$

From Table A the area above that point is 1-0.1151 = 0.8849: That is the power of the test.

**4.** (a: 2 marks) The average change is 4 with sample standard deviation 6. Thus the 90% CI is:

$$4 \pm t_{9-1,0.05}\frac{6}{\sqrt{9}} = 4 \pm 1.86 \times 2 = (0.28, 7.72).$$

(b: 2 marks) The test statistic is:

$$t = \frac{4-0}{2} = 2,$$

with $df = n - 1 = 8$. Reading Table D shows the $P$-value is between 0.05 and 0.025.