Economics 250 Mid-Term Test 1
10 February 2020

Instructions: You may use a Casio 991 hand calculator. Do not hand in the question sheet. Answer all four questions in the answer booklet provided. Show your work. Formulas and tables are provided at the end of the question pages.

1. An economist studies prices of a book at different locations and finds three values of 18, two values of 20, and one value of 26.

(a) Find the median and mode.

(b) Find the sample mean.

(c) Find the sample standard deviation.

(d) Suppose that these prices, labelled x, are in Canadian dollars. The prices in US dollars would be $y = 0.75 \cdot x$. Find the mean and standard deviation of y.

2. Suppose that the annual inflation rate (labelled π) across many countries and years has this distribution: $\pi \sim N(3, 1)$.

(a) Find the frequency with which π is less than 0.

(b) Find the frequency with which π is between 2 and 4.

(c) Find a range centered at the mean that contains 90% of the values of π.

(d) Find the coefficient of variation.

[continued over]

3. Suppose at a track-and-field competition 10% of athletes are cheating (C) by taking banned substances. Everyone is tested, and 8% of the athletes test positive (Pos) for those substances. The test sometimes yields 'false positive' results though: Given that someone is not cheating, the test gives a positive result 5% of the time.

(a) What is $P(Not\, C \cap Pos)$?

(b) What is $P(Pos|C)$?

(c) What is $P(C|Pos)$?

(d) Are the events testing positive (Pos) and cheating (C) independent?

4. Suppose that we describe the return on the stock market as a discrete random variable, labelled r_s. Suppose that the possible outcomes are $\{-3, 6, 12\}$ with probabilities $\{0.2, 0.5, 0.3\}$.

(a) Find the expected value of r_s.

(b) Find the standard deviation of r_s.

(c) An investor considers combining this investment with a bond return labelled r_b that has mean 2 and variance 0. The new portfolio has equal amounts in each investment so its return is:

$$r_p = 0.5r_s + 0.5r_b.$$

Find the mean and standard deviation of r_p.

Economics 250 Midterm Test 1: Answer Guide

1. (a: 2 marks) The mode is 18. The median is $(18 + 20)/2 = 19$.

(b: 2 marks) The sample mean is

$$\overline{x} = \frac{3 \times 18 + 2 \times 20 + 26}{6} = 20$$

(c: 2 marks) The sample variance is:

$$s^2 = \frac{3(-2)^2 + 2(0)^2 + 1(6)^2}{5} = 9.6$$

so the sample standard deviation is $s = \sqrt{9.6} \approx 3.098$.

(d: 2 marks) The mean of y is $\overline{y} = 0.75\overline{x} = 15$. The standard deviation is $s_y = 0.75 s_x \approx 2.3235$.

2. (a: 2 marks) Standardizing 0 gives $z = -3$. From Table A the probability is $P(z < -3) = 0.0013$.

(b: 2 marks) The corresponding values of z are ± 1 so from Table A there is 0.1587 in each tail so there is 0.6826 frequency between 2 and 4.

(c: 2 marks) To include 90% we go out plus and minus 1.645 standard deviations so that is $1.645 \times 1 = 1.645$. Thus the range is (1.355, 4.645).

(d: 1 mark) The CV is $100 \times 1/3 = 33.3\%$.

3. (a: 2 marks) Clearly $P(NotC) = 0.90$. The multiplication rule gives:

$$P(NotC \cap Pos) = P(Pos|NotC)P(NotC) = 0.05 \times 0.9 = 0.045$$

(b: 2 marks) Completing the two-way table using the multiplication rule:

$$P(Pos|C) = \frac{P(Pos \cap C)}{P(C)} = 0.035/0.10 = 0.35.$$

(c: 2 marks) From the table or from Bayes rule:

$$P(C|Pos) = \frac{P(Pos|C)P(C)}{P(Pos)} = \frac{0.35 \times 0.10}{0.08} = 0.4375.$$

(d: 2 marks) No they are not independent. $P(C|Pos) = 0.4375 > P(C) = 0.10$.

4. (a: 2 marks)

$$E(s) = 0.2 \times (-3) + 0.5 \times 6 + 0.3 \times 12 = -0.6 + 3.0 + 3.6 = 6.$$

(b: 2 marks) The variance is:

$$\sigma_s^2 = 0.2(-9)^2 + 0.5(0)^2 + 0.3(6)^2 = 16.2 + 0 + 10.8 = 27$$

so the standard deviation is:

$$\sigma_s = 5.196$$

(c: 2 marks) Here r_b is just a constant, so

$$r_p = 1 + 0.5r_b.$$

Thus the mean is $1 + 0.5(6) = 4$. The standard deviation is $0.5(5.196) = 2.598$.