Economics 250 Mid-Term Test 2
21 March 2025

Instructions: You may use an approved hand calculator. Answer all four questions in the answer booklet provided. Show your work. Formulas and tables are provided at the end of the question pages. Hand in the question sheet at the end of the test.

1. Suppose that medical researchers know that a treatment has a 60 percent chance of being successful for any patient.

(a) If they treat 7 patients what is the probability that the treatment succeeds for 3 of them? What is the probability that it succeeds for 4 or more?

(b) If they treat 100 patients what is the probability that the treatment succeeds for 56 or more?

2. An economist is asked to estimate the average age in a population. From a random sample of 25 people, she finds a sample average age 42. Suppose that she knows the population standard deviation of age is $\sigma = 5$.

(a) Find a 95% confidence interval for the population average age.

(b) Suppose she instead reports the margin of error as 1. What must the confidence level be?

3. Economists hypothesize that the average income in a country is 30, measured in thousands of US dollars (so this is the null hypothesis). The alternative hypothesis is that the average income is less than 30. Suppose that they know that the population standard deviation of income is $\sigma = 6$ thousand dollars. They study a sample of 16 people.

(a) If they find a sample average income of 28.5 then what is the P-value?

(b) Suppose they decide in advance that they will reject the null hypothesis if the average income per capita is less than 27. What is the probability of Type I error?

(c) If the true (but unknown) population average income in fact is 25.2 then what is the power of the test in part (b)?

4. Suppose that 16 students take a professional exam and receive an average score of 100. They then participate in a coaching session, and retake the exam, now with an average score of 104. The sample standard deviation of the *change* in their scores is 7.

(a) Find a 90% confidence interval for the population average change in score as a result of the course.

(b) Test the null hypothesis that the effect of the course on average is zero against the alternative that it is positive, and report a range for the P-value.

Economics 250 Midterm Test 2 Winter 2019: Answer Guide

1. (a: 2 marks) Here $n = 7$, $p = 0.6$ and so from Table C or the binomial formula for $k = 7$ the probability is 0.1935 or 19.35%. The probability that it succeeds for 4 or more is found by adding up the values for $k = 4, ..., 7$ which gives 0.7102 or 71.02%.

(b: 2 marks) With $n = 100$ we use the normal approximation that $X \sim N(np, \sqrt{np(1-p)})$ so $X \sim N(60, 4.8989)$. Standardizing gives $z = (56 - 60)/4.8989 = -0.8165$. I'll use the value $z = -0.82$ in table A. The probability is 1-0.2061 = 0.7939 or 79.39%.

2. (a: 2 marks) The 95% confidence interval is:

$$42 \pm 1.96 \cdot \frac{5}{5} = 42 \pm 1.96 = (40.04, 43.96).$$

(b: 2 marks) We have:

$$\text{ME} = 1.0 = z \cdot \frac{5}{5},$$

so $z = 1.0$. From Table A 0.1587 is to the left of -1 so $2 \times 0.1587 = 0.3174$ is in the two tails so the confidence level is 0.6826 or 68.26%.

3. (a: 2 marks) Under the null the sampling distribution of the sample mean is:

$$\overline{x} \sim N(30, 1.5).$$

because $\sigma/\sqrt{n} = 6/4 = 1.5$. Locating $\overline{x} = 28.5$ in this distribution gives $z = -1$. The P-value is the area below that which is 0.1587.

(b: 2 marks) That critical value gives $z = (27 - 30)/1.5 = -2$ so the probability of Type I error (also labelled α) is 0.0228 or 2.28%.

(c: 2 marks) Under the alternative the distribution is:

$$\overline{x} \sim N(25.2, 1.5).$$

Locating the critical value 27 in this distribution gives:

$$z = \frac{27 - 25.2}{1.5} = 1.2.$$

From Table A the area below that point is 0.8849 or 88.49%. That is the power of the test.

4. (a: 2 marks) The average change is 4 with sample standard deviation 7. Thus the 90% CI is:

$$4 \pm t_{16-1,0.05} \frac{7}{\sqrt{16}} = 4 \pm 1.753 \times 1.75 = (0.93225, 7.06775).$$

(Rounding is allowed of course.)

(b: 2 marks) The test statistic is:

$$t = \frac{4-0}{1.75} = 2.2857,$$

with $df = n - 1 = 15$. Reading Table D shows the P-value is between 0.01 and 0.02. (We do not multiply by 2 because this is a one-tailed test.)