Economics 250 Mid-Term Test 2
22 March 2017

Instructions: You may use an approved hand calculator. Do not hand in the question sheet. Answer all four questions in the answer booklet provided. Show your work. Formulas and tables are provided at the end of the question pages.

1. Suppose that medical researchers know that a treatment has a 50 percent chance of being successful for any patient.

(a) If they treat 10 patients what is the probability that the treatment succeeds for 5 of them? What is the probability that it succeeds for 6 or more?

(b) If they treat 100 patients what is the probability that the treatment succeeds for 60 or more?

2. An economist is asked to estimate the average income in a population. From a random sample of 16 people, she finds a sample average income of 30. Suppose that she knows the population standard deviation of income is $\sigma = 3$.

(a) Find a 95% confidence interval for the population average income.

(b) Suppose she wants the margin of error for the 95% confidence interval to be no greater than 1. How large must the sample size be?

3. Labour economists hypothesize that the average duration of unemployment spells is 10 months (so this is the null hypothesis). The alternative hypothesis is that the average duration is greater than 10 months. Suppose that they know that the population standard deviation of the duration is $\sigma = 2$ months. They study a sample of 25 unemployment spells.

(a) If they find a sample average duration of 11 months then what is the P-value?

(b) Suppose they decide in advance that they will reject the null hypothesis if the average duration is greater than 11 months. If the true (but unknown) population average duration is 11.4 months then what is the power of this test?

4. Suppose that you measure spending on food for three households and find that the values are 10, 30, and 50.

(a) Find the sample mean and sample standard deviation.

(b) Construct a 60% confidence interval for the population average.

(c) Suppose that someone proposes the null hypothesis that the population average is 33. Conduct a two-tailed test of this hypothesis with $\alpha = 0.10$ (*i.e.* would you reject or not?)

Economics 250 Midterm Test 2 Winter 2017: Answer Guide

1. (a: 2 marks) Here $n = 10$, $p = 0.5$ and so from Table C or the binomial formula for $k = 5$ the probability is 0.2461 or 24.61%. The probability that it succeeds for 6 or more is found by adding up the values for $k = 6, ..., 10$ which gives 0.377 or 37.7%.

(b: 2 marks) With $n = 100$ we use the normal approximation that $X \sim N(np, \sqrt{np(1-p)})$ so $X \sim N(50, 5)$. Standardizing gives $z = (60-50)/5 = 2.0$ so from table A the probability is 1-0.9772 = 0.0228 or 2.28%.

2. (a: 2 marks) The 95% confidence interval is:

$$30 \pm 1.96 \cdot \frac{3}{4} = 30 \pm 1.47 = (28.53, 31.47).$$

(b: 2 marks) We have:

$$\text{ME} = 1 = 1.96 \cdot \frac{3}{\sqrt{n}},$$

so $\sqrt{n} = 5.88$ and $n = 34.5744$. So she sets $n = 35$.

3. (a: 2 marks) Under the null the sampling distribution of the sample mean is:

$$\overline{x} \sim N(10, 0.4).$$

Locating $\overline{x} = 11$ in this distribution gives $z = 1/0.4 = 2.5$. The P-value is the area above that which is $1 - 0.9938 = 0.0062$.

(b: 2 marks) Under the alternative the distribution is:

$$\overline{x} \sim N(11.4, 0.4).$$

Locating the critical value in this distribution gives:

$$z = \frac{11 - 11.4}{0.4} = -1.$$

From Table A the area below that point is 0.1587. The area above that point is 0.8413 and that is the power of the test.

4. (a: 2 marks) The sample mean is 30 and the sample standard deviation is 20.

(b: 2 marks) We use Table D to find the appropriate t-statistic with $n - 1 = 2$ degrees of freedom. That is 1.061. Thus our confidence interval is:

$$30 \pm 1.061 \cdot \frac{20}{\sqrt{3}} = 30 \pm 12.25 = (17.75, 42.25).$$

(b: 2 marks) Form the t-statistic:

$$t = \frac{30 - 33}{20/\sqrt{3}} = -0.2598.$$

The corresponding critical value for t is -2.920. Thus we do not reject the null.