Economics 250 Mid-Term Test 2
17 March 2016

Instructions: You may use an approved hand calculator. Do not hand in the question sheet. Answer all four questions in the answer booklet provided. Show your work. Formulas and tables are provided at the end of the question pages.

1. Suppose that a political party is running candidates in a civic election where 10 seats are vacant. The opinion polls suggest that the party has a probability 0.3 of winning any specific seat.

(a) What is the probability that the party wins no seats? What is the probability that it wins either one seat or two seats?

(b) The same party also participates in a national election where there are 200 seats. If the probability of it winning any individual seat remains 0.3 then what is the probability it wins 65 or fewer seats?

2. A labour economist hypothesizes that the average duration of unemployment spells is 10 months. The alternative hypothesis is that the average duration is less than 10 months. Suppose that she knows that the population standard deviation of the duration is $\sigma = 2$ months. She studies a sample of 25 unemployment spells.

(a) If the significance level is $\alpha = 0.025$ find the critical value for this test (*i.e.* the value that separates the rejection region from the non-rejection region.)

(b) Suppose that the true but unknown average duration in the population is 9 months. What is the probability of type II error? What is the power of the test?

3. A development economist studying Indonesia interviews 10 households and finds in this sample an average income (converted to thousands of US dollars) of 8 with a sample standard deviation of 1.

(a) Find a 90% confidence interval for the population, average household income.

(b) The researcher wishes to conduct a test of the hypothesis that the population, average household income is 10 against the alternative hypothesis that it is not equal to 10. If $\alpha = 0.10$ find the non-rejection region for this test. Does the researcher reject the null hypothesis?

4. Nine runners have an average time over 400 metres of 60 seconds. They then adopt a training program and find that, after that program, their average time is 57 seconds. The sample standard deviation of the change in their times is 3 seconds.

(a) Find a 95% confidence interval for the population average change in 400-metre time.

(b) A researcher argues that the margin of error must be 2 seconds or less for the evidence to be convincing. With the same sample mean and standard deviation, how large must the sample size be to deliver this margin of error?

Economics 250 Midterm Test 2 Winter 2016: Answer Guide

1. (a: 2 marks) Here $n = 10$, $p = 0.3$ and so from Table C or the binomial formula for $k = 0$ the probability is 0.0282 or 2.82%. The probability that they win either 1 or two seats is the sum of the two associated probabilities which is 0.1211+0.2335 = 0.3546 or 35.46%.

(b: 2 marks) With $n = 200$ we use the normal approximation that $X \sim N(np, \sqrt{np(1-p)})$ so $X \sim N(60, 6.480)$. Standardizing gives $z = (65 - 60)/6.480 = 0.7716$ so from table A the probability is 0.7794 or 77.94%.

2. (a: 2 marks) This is a lower one-tailed test so to leave 2.5% there implies $z = -1.96$. Then from standardization:

$$-1.96 = \frac{\overline{x}_c - 10}{2/5},$$

so $\overline{x}_c = 9.216$ is the critical value.

(b: 2 marks) Now we standardize the old critical value in the new distribution that is centred at 9:

$$z = \frac{9.216 - 9}{2/5} = .54.$$

The power is 0.7054 and the probability of type II error is 0.2946.

3. (a: 2 marks) From table D with $df = 9$ and 0.05 in each tail we find $t = 1.833$. Thus the 90% CI is

$$8 \pm 1.833\frac{1}{3.166} = 8 \pm 0.579 = (7.421, 8.579).$$

(b: 2 marks) The upper critical value leaves 5% in that tail so:

$$1.833 = \frac{\overline{x}_c - 10}{0.3159}$$

so that $\overline{x}_c = 10.579$. The lower value is thus 9.421. So finding a sample value of 8 means that the researcher would reject the null of 10 at this significance level.

4. (a: 2 marks) The improvement is 3 (or you could record the change which is -3; either is fine). The $df = 8$. The confidence level using the change is:

$$-3 \pm 2.306\frac{3}{\sqrt{9}} = (-5.306, -0.694).$$

(Or you could report the improvement in time rather than the decrease, in other words, positive values.)

(b: 2 marks) For the margin of error to be 2 seconds means we choose n so that:

$$t_{0.025,n-1}\frac{3}{\sqrt{n}} = 2.$$

We cannot solve this directly for n but we can look up and down the relevant column of Table D and experiment. At $n = 11$ so $df = 10$ we get $ME = 2.015$. At $n = 12$ so $df = 11$ we get $ME = 1.906$. So we need at least 12 participants in the study.