Econ 250 Mid-Term Test 1
24 October, 2011

Instructions. You may use a hand calculator. Do not hand in the question sheet. Answer all four questions in the answer booklet provided. Show your work. Formulas and tables are provided at the end of the question pages.

1. The following data give X, the price charged for a particular item, and Y, the quantity of that item sold (in thousands):

Price per Piece (X)	Thousands of Pieces Sold (Y)
2	80
3	60
4	70
5	40
6	0

   (a) Find the sample means $\overline{X}$ and $\overline{Y}$.

   (b) Compute the correlation coefficient between X and Y.

2. A committee of 4 people is to be selected from a group of 3 men and 4 women. If the selection is made randomly, what is the probability the committee will consist of 2 men and 2 women?

3. Suppose a joint probability distribution for the marital status of adults by gender is given in the table below. "Single" means "Never married".

	Single (M_1)	Married (M_2)	Widowed (M_3)	Divorced (M_4)
Male (S_1)	0.138	0.290	0.012	0.044
Female (S_2)	0.114	0.291	0.051	0.060

   Suppose an adult is selected at random.

   (a) Determine the probability that the adult selected is single.

   (b) Determine the probability that the adult selected is divorced, given that the adult selected is a male.

4. A blood test is 99 percent effective in detecting a certain disease when the disease is present. However, the test also yields a false-positive result for 2 percent of the healthy patients tested. (That is, if a healthy person is tested, then with probability 0.02 the test will say that this person has the disease.) Suppose 0.5 percent of the population has the disease. Let

   H: event a patient is healthy,

   D: event a patient has the disease,

   O: event the test result is positive, and

   N: event the test result is negative.

   Find the conditional probability that a randomly tested individual actually has the disease given that his or her test result is positive.

Econ 250 Mid-Term Test 1
24 October, 2011

Instructions. You may use a hand calculator. Do not hand in the question sheet. Answer all four questions in the answer booklet provided. Show your work. Formulas and tables are provided at the end of the question pages.

1. The following data give X, the price charged for a particular item, and Y, the quantity of that item sold (in thousands):

Price per Piece (X)	Thousands of Pieces Sold (Y)
2	80
3	60
4	70
5	40
6	0

   (a) Find the sample means $\overline{X}$ and $\overline{Y}$.
   $\overline{X} = \frac{2+3+4+5+6}{5} = 4$
   $\overline{Y} = \frac{80+60+70+40+0}{5} = 50$

   (b) Compute the correlation coefficient between X and Y.
   $s_X^2 = \frac{(2-4)^2+(3-4)^2+(4-4)^2+(5-4)^2+(6-4)^2}{5-1} = 2.5$
   $s_Y^2 = \frac{(80-50)^2+(60-50)^2+(70-50)^2+(40-50)^2+(0-50)^2}{5-1} = 1000$
   $s_{XY} = \frac{(2-4)(80-50)+(3-4)(60-50)+(4-4)(70-50)+(5-4)(40-50)+(6-4)(0-50)}{5-1} = -45$
   $r_{XY} = \frac{-45}{\sqrt{2.5}\sqrt{1000}} = -0.9$

2. A committee of 4 people is to be selected from a group of 3 men and 4 women. If the selection is made randomly, what is the probability the committee will consist of 2 men and 2 women?

   $P = \frac{C_2^3 C_2^4}{C_4^7} = 0.51$

3. Suppose a joint probability distribution for the marital status of adults by gender is given in the table below. "Single" means "Never married".

	Single (M_1)	Married (M_2)	Widowed (M_3)	Divorced (M_4)
Male (S_1)	0.138	0.290	0.012	0.044
Female (S_2)	0.114	0.291	0.051	0.060

   Suppose an adult is selected at random.

   (a) Determine the probability that the adult selected is single.
   $P(M_1) = 0.138 + 0.114 = 0.252$

   (b) Determine the probability that the adult selected is divorced, given that the adult selected is a male.
   $P(M_4|S_1) = \frac{P(M_4 \cap S_1)}{P(S_1)} = \frac{0.044}{0.138+0.290+0.012+0.044} = 0.09$

4. A blood test is 99 percent effective in detecting a certain disease when the disease is present. However, the test also yields a false-positive result for 2 percent of the healthy patients tested. (That is, if a healthy person is tested, then with probability 0.02 the test will say that this person has the disease.) Suppose 0.5 percent of the population has the disease. Let

   H: event a patient is healthy,

   D: event a patient has the disease,

   O: event the test result is positive, and

   N: event the test result is negative.

Find the conditional probability that a randomly tested individual actually has the disease given that his or her test result is positive.

$P(O|D) = 0.99$

$P(O|H) = 0.02$

$P(D) = 0.005$

$P(H) = 1 - P(D) = 0.995$

$P(D|O) = \frac{P(D \cap O)}{P(O)} = \frac{P(O|D)P(D)}{P(O|D)P(D)+P(O|H)P(H)} = \frac{0.99 \times 0.005}{0.99 \times 0.005+0.02 \times 0.995} = 0.199$