Economics 250 Mid-Term Test 1
12 February 2019

Instructions: You may use a hand calculator. Do not hand in the question sheet. Answer all four questions in the answer booklet provided. Show your work. Formulas and tables are provided at the end of the question pages.

1. Suppose that a study records the effect of chronic illness on labour income, an effect labelled x, for eight people. It records three values of -11, four values of -13, and one value of -15.

(a) Find the median and mode.

(b) Find the sample mean.

(c) Find the sample standard deviation.

(d) If an insurance program pays sick workers a value $y = -25 - 2x$ then what is the mean of y? What is the correlation between x and y?

2. Suppose that the rate of depreciation of the Turkish lira against the euro (labelled x) has this distribution: $x \sim N(4, 2)$.

(a) Find the frequency with which x is greater than 5.

(b) Find the frequency with which x is between 2 and 6.

(c) Find a range centered at the mean that contains 95% of the values of x.

(d) Find the coefficient of variation.

[continued over]

3. Suppose that we classify adults as having completed college or university (C) or not (NC). We also separately classify them as employed (E) or not (NE). Suppose 60% of adults are in category C. Also suppose 70% are in category E. Conditional on being in category NC the probability of being in category NE is 40%.

(a) What is $P(NC \cap NE)$?

(b) What is $P(E|C)$?

(c) Given that an adult is in category E what is the probability that they are in category C?

(d) Are the events E and C independent?

4. An analyst believes that the number of seats a political party will win in the next election, labelled s, is a discrete random variable. The possible outcomes are $\{20, 40, 70\}$ with probabilities $\{0.4, 0.4, 0.2\}$.

(a) Find the expected value of s.

(b) Find the standard deviation of s.

(c) Suppose that the public funding the party receives, labelled f, depends on the number of seats it wins according to this formula: $f = 0.10s$. What are the expected value and standard deviation of the party's funding f?

Economics 250 Midterm Test 1: Answer Guide

1. (a: 2 marks) The median and mode are both -13.

(b: 2 marks) The sample mean is -12.5.

(c: 2 marks) The sample variance is:

$$s^2 = \frac{3(1.5)^2 + 4(-0.5)^2 + 1(2.5)^2}{7} = 2$$

so the sample standard deviation is $s = \sqrt{2} \approx 1.414$.

(d: 2 marks) The mean of y is $\overline{y} = -25 - 2\overline{x} = 0$. The correlation is -1 because there is a perfect negative relationship between x and y.

2. (a: 2 marks) Standardizing 5 gives $z = 0.5$. From Table A the probability is $P(z > 1) = 1 - .6915 = 0.3085$.

(b: 2 marks) The corresponding values of z are ± 1 so from Table A there is 0.1587 in each tail so there is 0.6826 frequency between 2 and 6.

(c: 2 marks) To include 95% we go out plus and minus 1.96 standard deviations so that is $1.96 \times 2 = 3.92$. Thus the range is (0.08, 7.92).

(d: 1 mark) The CV is $100 \times 2/4 = 50\%$.

3. (a: 2 marks) Clearly $P(NE) = 0.30$. The multiplication rule gives:

$$P(NC \cap NE) = P(NE|NC)P(NC) = 0.4 \times 0.4 = 0.16$$

(b: 2 marks) Completing the two-way table using the multiplication rule:

$$P(E|C) = \frac{P(E \cap C)}{P(C)} = 0.46/0.6 = 0.766.$$

(c: 2 marks) From Bayes rule:

$$P(C|E) = \frac{P(E|C)P(C)}{P(E)} = \frac{0.766 \times 0.6}{0.7} = 0.6566.$$

(d: 2 marks) No they are not independent. $P(E|C) = 0.766 > P(E) = 0.70$.

4. (a: 2 marks)

$$E(s) = 0.4 \times 20 + 0.4 \times 40 + 0.2 \times 70 = 8 + 16 + 14 = 38.$$

(b: 2 marks) The variance is:

$$\sigma_s^2 = 0.4(-18)^2 + 0.4(2)^2 + 0.2(32)^2 = 129.6 + 1.6 + 204.8 = 336$$

so the standard deviation is:

$$\sigma_s = 18.33$$

(c: 2 marks) You can repeat your exercise from parts (a) and (b) with the new values for each outcome, or simply recall from several points in the course that this has the same effect on the mean and standard deviation, which thus are 3.8 and 1.833.