Economics 250 Mid-Term Test 1
13 February 2018

Instructions: You may use a hand calculator. Do not hand in the question sheet. Answer all four questions in the answer booklet provided. Show your work. Formulas and tables are provided at the end of the question pages.

1. Suppose that writing a test results in a score of 1, 3, or 5. A group of 13 people write the test. Three people receive a grade of 1, seven people receive a grade of 3, and three people receive a grade of 5.

(a) Find the sample mean and mode.

(b) Find the median and the interquartile range.

(c) Find the sample standard deviation.

2. Suppose that the price of shares in Apple (labelled x) varies from minute to minute. It has this distribution: $x \sim N(156, 4)$.

(a) Find the probability that the price is greater than 159.

(b) Find the probability that the price is between 152 and 160.

(c) Find a range centered at the mean that contains 95% of the prices.

(d) Find the coefficient of variation.

[continued over]

3. Suppose that we classify adults as having completed college or university (C) or not (NC). We also separately classify them as employed (E) or not (NE).

(a) Suppose 40% of adults are in category C. Also suppose 60% are in category E. Conditional on being in category C the probability of being employed is 70%. What is $P(C|E)$?

(b) What is $P(NC \cap NE)$?

(c) Given that an adult is in category NE what is the probability that they are in category NC?

(d) Are the events E and C independent?

4. Suppose that an investment earns a rate of return r that is a discrete random variable. The possible outcomes are $\{-1, 1, 3\}$ with probabilities $\{0.2, 0.4, 0.4\}$

(a) Find the expected value of r.

(b) Find the standard deviation of r.

(c) Suppose that a new investment strategy offers a return of $2r$. In other words, the probabilities are the same but the outcomes now are $\{-2, 2, 6\}$ What would the expected value and standard deviation of the return on this strategy be?

Economics 250 Midterm Test 1: Answer Guide

1. (a: 2 marks) The mean is 3. The mode is 3.

(b: 2 marks) The median is 3. The other two quartiles are 2 and 4 so the IQR is [2,4] (or reported as a span of 2 around the median of 3.)

(c: 2 marks) The sample variance is:

$$s^2 = \frac{3(-2)^2 + 7(0)^2 + 3(2)^2}{13 - 1} = 2$$

so the sample standard deviation is $s = \sqrt{2} \approx 1.414$.

2. (a: 2 marks) Standardizing 159 gives $z = 0.75$. From Table A the probability is $P(z > 1) = 1 - .7734 = 0.2266$.

(b: 2 marks) The corresponding values of z are ± 1 so from Table A there is 0.1587 in each tail so there is 0.6826 probability between 152 and 160.

(c: 2 marks) To include 95% we go out plus and minus 1.96 standard deviations so that is 1.96 times 4=7.84. Thus the range is (148.16, 163.84).

(d: 1 mark) The CV is $100 \times 4/156 = 2.56$.

3. (a: 2 marks) This is an application of Bayes's rule:

$$P(C|E) = \frac{P(E|C)P(C)}{P(E)} = \frac{0.70 \times 0.4}{0.60} = 0.466.$$

(b: 2 marks) Completing the two-way table using the multiplication rule:

$$P(NC \cap NE) = 0.28.$$

(c: 2 marks) From the multiplication rule:

$$P(NC|NE) = \frac{P(NC \cap NE)}{P(NE)} = \frac{0.28}{0.4} = 0.7.$$

(d: 2 marks) No they are not independent. $P(E|C) = 0.70 > P(E) = 0.60$.

4. (a: 2 marks)

$$E(r) = -1 \times 0.2 + 1 \times 0.4 + 3 \times 0.4 = -0.2 + 0.4 + 1.2 = 1.4.$$

(b: 2 marks) The variance is:

$$\sigma_r^2 = 0.2(-2.4)^2 + 0.4(-0.4)^2 + 0.4(1.6)^2 = 1.152 + 0.064 + 1.024 = 2.24$$

so the standard deviation is:

$$\sigma_r = 1.497$$

(c: 2 marks) You can repeat your exercise from parts (a) and (b) with the new values for each outcome, or simply recall from several points in the course that this has the same effect on the mean and standard deviation, which thus are 2.8 and 2.994.