Economics 250 Mid-Term Test 1
13 February 2017

Instructions: You may use a hand calculator. Do not hand in the question sheet. Answer all four questions in the answer booklet provided. Show your work. Formulas and tables are provided at the end of the question pages.

1. Suppose that a survey of 3 people shows that 1 has an income (labelled y) of 10, 1 has an income of 30, and 1 has an income of 50.

(a) Find the sample mean and sample standard deviation.

(b) Suppose that each person's consumption spending, labelled c, is given by:

$$c = 10 + 0.8y.$$

Find the sample mean and sample standard deviation of c.

(c) What is the sample correlation between the two variables y and c?

2. Suppose that scores on an aptitude test are normally distributed with mean 20 and standard deviation 2.

(a) Find the probability that a score is greater than 22.

(b) Find the probability that a score is between 19 and 21.

(c) Find a range centered at the mean that contains 90% of the scores.

[continued over]

3. Suppose that those in the labour force are either young (Y) or old (O). Also, they are classifed a being either employed (E) or unemployed (U).

(a) Suppose 8% of workers are unemployed and 15% are young. Conditional on being young the probability of being unemployed is 14%. What is $P(Y|U)$, the probability of a worker being young given that they are unemployed?

(b) What percentage of those in the labour force are young *and* unemployed?

(c) Given that a worker is old what is the probability they are unemployed?

4. Suppose that tickets in a lottery cost \$10 and 200 tickets are sold. The lottery organizers use the revenue from the ticket sales to donate \$1000 to charity and award two prizes randomly, one of \$800 and one of \$200. Call x the net value of a ticket purchase (*i.e.* the amount won minus the ticket cost). (Note: A single person cannot win both prizes.)

(a) Find the expected value of x.

(b) Find the standard deviation of x.

(c) If instead the organizers donated nothing to charity but used the revenue to award 4 prizes all of the same amount then what would be the expected value of x?

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1. (a: 2 marks) The mean is 30. The sample standard deviation is 20.

(b: 2 marks) You could find the 3 values of y but it is easier to recall that $\overline{c} = 10 + 0.8\overline{y} = 34$ and that $s_c = 0.8 s_y = 16$.

(c: 2 marks) Again one could find the correlation by hand using the 3 values for each variable. But it is easier to note that there is an exact, positive relationship so the correlation coefficient is 1.

2. (a: 2 marks) Standardizing 22 gives $z = 1$. From Table A the probability is $P(z > 1) = 1 - .8413 = 0.1587$.

(b: 2 marks) The corresponding values of z are ± 0.5 so from Table A there is 0.3085 in each tail so there is 0.383 probability between 19 and 21.

(c: 2 marks) To include 90% we go out plus and minus 1.645 standard deviations so that is 1.645 times 2. Thus the range is (16.71, 23.29).

3. (a: 2 marks) This is an application of Bayes's rule:

$$P(Y|U) = \frac{P(U|Y)P(Y)}{P(U)} = \frac{0.14 \times 0.15}{0.08} = 0.2625.$$

(b: 2 marks) From the multiplication rule:

$$P(Y \cap U) = P(U|Y)P(Y) = 0.14 \times 0.15 = 0.021.$$

(c: 2 marks) We see that $P(O) = 0.85$. We also know that 8% are unemployed, which includes 2.1% who are young and unemployed. Therefore 5.9% are old and unemployed, so:

$$P(U|O) = \frac{0.059}{0.85} = 0.0694.$$

4. (a: 2 marks)

$$E(x) = -10 \times 0.99 + 190 \times 0.005 + 790 \times 0.005 = -9.9 + 0.95 + 3.95 = -5.$$

(b: 2 marks) The variance is:

$$\begin{aligned}\sigma_x^2 &= 0.99[-10 - (-5)]^2 + 0.005[190 - (-5)]^2 + 0.005[790 - (-5)]^2 \\ &= 24.75 + 190.125 + 3160.125 = 3375,\end{aligned}$$

so the standard deviation is:

$$\sigma_x = 58.09$$

(which incidentally is measured in dollars).

(c: 2 marks) In this case the prizes would each be \$500. The expected value would be:

$$E(x) = -10 \times 0.98 + 490 \times 0.02 = -9.8 + 9.8 = 0.$$