Economics 250 Mid-Term Test 1
9 February 2016

Instructions: You may use a hand calculator. Do not hand in the question sheet. Answer all four questions in the answer booklet provided. Show your work. Formulas and tables are provided at the end of the question pages.

1. Suppose that there are 100 students in a class. On a project, 30 students receive a grade of 70, 20 students receive a grade of 75, and 50 students receive a grade of 80.

(a) Find the median and mode of the grades.

(b) Find the sample mean of the grades.

(c) Find the sample standard deviation of the grades.

2. A survey of North American adults classifies them according to whether or not they are smokers (here labelled S) and whether or not they have been diagnosed with heart disease (labelled D). Thus a non-smoker with no diagnosis of heart disease would be labelled non-S and non-D. Here is a two-way table of the survey data:

	S	Non-S
D	100	200
Non-D	600	1500

(a) Find the marginal distribution of smoking status and the marginal distribution of heart disease diagnosis status.

(b) Find the conditional frequency of heart disease diagnosis D first conditional on S then conditional on non-S. (*i.e.* first for smokers then for non-smokers.)

[continued over]

3. Suppose that the monthly, percentage rate of change of the Canadian dollar/US Dollar exchange rate is labelled x and is normally distributed with mean 0 and standard deviation 2.

(a) Find the probability that $x > 4$.

(b) Find the probability that $x = 0$.

(c) Find a value m such that $Prob(-m \leq x \leq m) = 0.90$.

(d) Suppose that an analyst calculates rate of change, averaged over two months, so $n = 2$. Label this sample average $\bar{x}$. Find the probability that $\bar{x} > 4$.

4. Suppose an investor invests and earns at an interest rate r_1 which is a discrete random variable distributed like this:

Outcome	Probability
-1	0.2
2	0.5
6	0.3

(a) Find the mean and standard deviation of r_1.

(b) The same investor then has the opportunity to hold a second investment with return r_2. This return has exactly the same mean and standard deviation as r_1. The two returns have a correlation of 0.5. The investor chooses a 60:40 mix of the two investments, so her/his overall return is a random variable r_p, with

$$r_p = 0.6r_1 + 0.4r_2.$$

Find the mean and standard deviation of r_p.

Economics 250 Midterm Test 1: Answer Guide

1. (a: 2 marks) The mode is 80. The median is 77.5 (the average of 75 and 80).

(b: 2 marks) The mean is 76.

(c: 2 marks) The sample standard deviation is 4.38.

2. (a: 3 marks) There are 2400 people in the survey so the marginal distribution for smoking status is 0.292 (or you could round to 29%) S and 0.708 Non-S (or you could round to 71%). The marginal distribution for heart disease diagnosis status is 0.125 for D and 0.875 for Non-D. Again you can report these as fractions or percentages.

(b: 3 marks) Conditional on S the frequency of D is 0.143. Conditional on Non-S the frequency of D is 0.118.

3. (a: 2 marks) Standardize to get $z = (4-0)/2 = 2$ so from Table A the probability is 1-0.9772 = 0.0228.

(b: 1 mark) The probability that $x = 0$ is 0 because this is a continuous distribution.

(c: 2 marks) To cover 90% we add plus or minus 1.645 times the standard deviation, so that means $m = 3.29$.

(d: 3 marks) The standard deviation for $\bar{x}$ is $2/\sqrt{n} = 1.4142$. Standardize to get $z = (4-0)/1.4142 = 2.828$ so from Table A 1-0.9977 = 0.0023.

4. (a: 3 marks)
$$E(r_1) = 0.2(-1) + 0.5(2) + 0.3(6) = 2.6$$
and
$$\sigma_1^2 = 0.2(-1 - 2.6)^2 + 0.5(2 - 2.6)^2 + 0.3(6 - 2.6)^2 = 2.592 + 0.180 + 3.468 = 6.238$$
so $\sigma_1 = 2.497$.

(b: 3 marks) The two returns have the same mean so their weighted average has that mean also, so the mean of r_p is 2.6. The variance is:
$$\sigma_p^2 = 0.6^2 \sigma_1^2 + 0.4^2 \sigma_2^2 + 2(0.6)(0.4)(0.5)6.238 = 4.74,$$
so the standard deviation is $\sigma_p = 2.177$.