Economics 250 Mid-Term Test 1

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Instructions: You may use a hand calculator. Do not hand in the question sheet. Answer all four questions in the answer booklet provided. Show your work. Formulas and tables are provided at the end of the question pages.

1. Suppose that a survey records a person's income in one of two ranges (a) from 10 to 20 or (b) from 20 to 40, measured in thousands of dollars. The survey does not give the specific value of a person's income, so as to protect their privacy. A statistician finds 80 people in the first category and 120 people in the second category.

(a) Find the sample mean and standard deviation of income.

(b) Find the coefficient of variation (as a percent).

2. Suppose that the monthly, percentage rate of change of the Canadian dollar/US Dollar exchange rate is labelled x and is normally distributed with mean 0 and standard deviation 2.

(a) Find the probability that $x > 4$.

(b) Find a value m such that $Prob(-m \leq x \leq m) = 0.90$.

(c) Find a value m such that $Prob(-m \leq x \leq m) = 0.95$.

(d) Find $Prob(-2 < x < 4)$.

3. Suppose that a survey of workers finds their hourly wages w are normally distributed with mean 20 and standard deviation 4 (measured in dollars).

(a) Find the probability that a worker has a wage less than or equal to 16.

(b) The same survey finds that workers' length of experience, measured in years, is continuously and uniformly distributed between 0 and 10. Find the probability that a worker has more than 8 years experience.

(c) Suppose that the probability that a worker has wage greater than 16 *and* experience more than 8 years is 0.1. Find the probability that a worker has wage less than 16 *or* experience more than 8 years.

4. Suppose an investor invests and earns at an interest rate r_1 which is a discrete random variable distributed like this:

Outcome	Probability
-1	0.3
2	0.4
6	0.3

(a) Find the mean and standard deviation of r_1.

(b) The same investor borrows at interest rate r_2 which has mean 1 and standard deviation 1. The correlation between the two interest rates is $\rho(r_1, r_2) = 0.5$. The return on a dollar borrowed and invested thus is the difference between the two returns or interest rates:

$$r_1 - r_2.$$

Find the mean and standard deviation of this combination.

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1. (a) The mean is
$$\overline{x} = \frac{80 \cdot 15 + 120 \cdot 30}{200} = 24$$
The variance is
$$s^2 = \frac{80 \times (15 - 24)^2 + 120 \times (30 - 24)^2)}{199} = 54.2713$$
so the standard deviation is
$$s = 7.367.$$

(b) The CV is $100 \times 7.367/24 = 30.69\%$

2. (a) Standardizing, this means $z > 2$ so the probability is 1-0.9772 = 0.0228 or 2.28%.

(b) This leaves 5% in each tail so for z the value is 1.645. Thus for x it is 3.29.

(c) This leaves 2.5% in each tail so for z the value is 1.96 so for x it is 3.92.

(d) This is equivalent to $-1 < z < 2$ which leaves 0.0228 in the right tail and 0.1587 in the left tail, so $1 - 0.0228 - 0.1578 = 0.8185$ or 81.85%.

3. (a) Standardize to get $z = (16 - 20)/4 = -1$ so from Table A the probability is 0.1587.

(b) The probability of more than 8 years experience is 0.20.

(c) From part (b) we know 20% of workers have high experience (more than 8 years). Now we also know 10% of workers have high experience and high wages (above 16). That means 10% of workers have high experience (more than 8 years) and low wages (less than 16). From the addition rule the probability of low wage *or* high experience is the probability of low wage (0.1587) plus the probability of high experience (0.20) minus the probability of both (0.10) which is 0.2587.

4. (a)
$$E(r_1) = 0.3(-1) + 0.4(2) + 0.3(6) = 2.3$$
and
$$\sigma_1^2 = 0.3(-1 - 2.3)^2 + 0.4(2 - 2.3)^2 + 0.3(6 - 2.3)^2 = 3.267 + 0.036 + 4.107 = 7.41$$
so $\sigma_1 = 2.722$.

(b) Call $w = r_1 - r_2$, so
$$E(w) = 2.3 - 1 = 1.3.$$
Then
$$\sigma_w^2 = 7.41 + (-1)^2 1 + 2(1)(-1)(1)(2.722)(0.5) = 7.41 + 1 - 2.722 = 5.688$$
so
$$\sigma_w = 2.385.$$
(Notice that the investment strategy has a high expected return but also a high variance or risk.)