Economics 250 Mid-Term Test 1
11 February 2013

Instructions: You may use a hand calculator. Do not hand in the question sheet. Answer all **four** questions in the answer booklet provided. Show your work. Formulas and tables are provided after the question pages.

1. Suppose that you have a sample of 4 workers' periods of unemployment (denoted x) with the following durations in months: 3, 4, 4, 9.

(a) Find the sample mean, mode, sample variance, and coefficient of variation.

(b) Suppose that unemployed workers run down their savings by an amount y that depends on the duration of unemployment x, like this:

$$y = -10x.$$

Find the mean and variance of y.

(c) Find the covariance and correlation between x and y.

2. Suppose the exchange rate between the Canadian and US dollars is normally distributed with mean 1.00 and variance 0.000144.

(a) What is the probability that the exchange rate lies in the interval between 1.00 and 1.012?

(b) What is the probability that the exchange rate is greater than 1.012?

(c) A researcher measures the average of the exchange rates over 5 trading days each week. What is the probability that the average is greater than 1.012?

3. A researcher studies information on 60 households and classifies them according to whether they have low household income or not and, separately, whether they have a minimum-wage earner or not. Thirty have neither low household income nor a minimum wage earner. Twelve have a minimum wage earner but do not have low household income. Six have both a minimum wage earner and low household income.

(a) Form a two-way table of frequencies that summarizes the data.

(b) Find the distribution of the minimum wage categorical variable conditional on low household income and, separately, the distribution conditional on not having low household income. Do your findings suggest that there is any relationship between receiving a minimum wage and having a low household income?

4. Suppose that the return, labelled r, on a domestic bond can take on the following values, with these probabilities:

r	Prob
2	0.4
3	0.3
4	0.3

(a) Find the expected value or mean (μ_r) and variance (σ_r^2) of r.

(b) A foreign bond has a return, labelled r^*, with mean 2 and variance 2. The covariance between the two returns, r and r^*, is 1. Suppose investors hold 75% of their portfolios in the domestic bond and 25% in the foreign bond. Find the expected value (mean) and variance of the portfolio return.

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1. (a) The mean is 5 and the mode is 4. The sample variance is 22/3 or 7.33. The coefficient of variation is 2.71/5 or 0.542.

(b) The mean of y is -50 and the variance is 733.

(c) The correlation is -1, since there is a perfect, linear, negative relationship. We know that the two standard deviations are 2.71 and 27.1 so the covariance must be -2.71(27.1) = -73.44.

2. (a) The standard deviation is 0.012 so this cutoff point is one standard deviation above the mean. From table A there is 0.8413 below that point. There is probability 0.5 below 1 since that is the mean. So the difference is 0.3413 or 34.13%

(b) The probability is 0.1587 or 15.87%

(c) Now the variance is 0.000144/5 so the standard deviation is 0.00537. Standardizing gives:
$$z = \frac{1.012 - 1.00}{0.00537} = 2.23.$$
From table A, the probability of exceeding this value is 1-0.9871 = 0.0129 or 1.29%.

3. (a) Here is the two-way table of frequencies:

	MinWage	No MinWage
Low Income	0.1	0.2
Not Low Income	0.2	0.5

Notice that the frequencies add up to 1, which allows us to find the frequency of the combination not described in the question.

(b) Yes, it does differ. For low-income households the conditional frequencies are 0.33 minimum wage and 0.67 not minimum wage. For non-low-income households the corresponding frequencies are 0.286 and 0.714. So there is some association between minimum-wage earning and low income.

4. (a) The mean is 2.9. The variance is 0.69.

(b) The mean of the portfolio return is 0.75(2.9)+0.25(2) = 2.675. The variance is:
$$0.75^2(0.69) + 0.25^2(2) + 2(0.75)(0.25)(1) = 0.388 + 0.125 + 0.375 = 0.888$$