Queen's University
Faculty of Arts and Sciences
Department of Economics
Economics 250 A & B: Fall 2000 Final

Instructions:
READ CAREFULLY. Calculators are permitted. At the end of the exam are several formulae and tables for the binomial, normal and t – distributions. Answers are to be written in the examination booklet. Do not hand in the question sheet.

You are to answer **ALL** questions. **SHOW ALL YOUR WORK**. There are a total of 100 possible marks to be obtained. **Part A** has 8 questions worth 5 marks each for a total of 40 marks. **Part B** has 4 questions worth 15 marks each for a total of 60.

PART A: Answer all 8 questions (5 Marks Each)

A1. Suppose we assume $X_i \sim NID(\mu, \sigma^2)$ with $i = 1, \ldots 5$ and someone calculates the following estimator of the population mean, μ

$$\ddot{X} = \frac{2}{5}X_2 + \frac{4}{5}X_3 - \frac{1}{5}X_5$$

Is this estimator biased? What is its sampling distribution and compare to the sample mean that uses all 5 observations?

A2. A researcher is interested in performing a hypothesis test about the difference population means of blood flow $(\mu_1 - \mu_2)$ using two independent samples. The test statistic is 1.78 and the number of observations for the two samples was over 1000. However, this is a medical experiment for a new drug given to the first sample that is suppose to increase blood flow. Show at the 5% level, how the rejection or retention of the null hypothesis depends on whether a one-sided or two-sided hypothesis test is conducted.

A3. Republicans often do not wish to reveal that they are such in exit polls. When Republicans are asked whether they are Republicans, 10% will say they are Democrats and 90% will say they are Republicans. Democrats always tell the truth. Suppose we know that Republicans make up 60% of all voters. A sample of 5 voters are asked who they support and 3 reply Republicans. What is the probability that they are Republicans.

PART A CONTINUES ON NEXT PAGE

A4. In Prince Edward Island an employment survey of 100 people found that 90 out 100 people are employed. Calculate the 99% confidence interval and interpret. Without doing the calculation, state whether the following test of the population proportion employed (*p*)

$$H_0 : p = .92$$

would be rejected or retained at the 1% level of significance.

A5. Suppose that victim's rights advocates are successful in lowering the burden of proof in criminal trials. Discuss the consequences in terms of probability of *TypeI* and *TypeII* errors.

A6. Explain exactly what is meant by approximating the binomial distribution by a normal distribution. Illustrate this approximation when we wish to know the probability of at least 8 people arriving late to class out of 10 when the probability of anyone being late is $p = .6$

A7. On a certain island in the Pacific, people are equally likely to have heights ranging from 5 feet to 7 feet. When five people are chosen at random, what is the probability of at least 3 of them being greater than 6.6 feet.

A8. Income distributions are known to be quite skewed with some people having very high incomes. You are given the following sample information about the incomes in Canada and want to test at the 10% level of significance whether the average income is $18,000.

$$\sum_{i=1}^{25} X_i = 500,000 \qquad s^2 = 9000$$

State all the assumptions you are making in conducting this test.

PART B IS ON THE NEXT PAGE

PART B : Answer all 4 questions (15 Marks each)

B1. The following information is obtained from a sample of 1000 Florida voters. 550 voters say they support the Democrat Presidential Candidate and 450 voters say they support the Republican Presidential Candidate.

a. The Democrat and Republican parties both make the claim that they have at least 50% support from the voters in Florida. Test the Republican claim at the 5% significance level. Explicitly state your null and alternative hypotheses and the conclusion you find.

b. What is the probability of a Type I error for the test in part **a**?

c. What is the probability of a Type II error for the test in part **a** when the true proportion of voters in Florida that support the Republican Presidential candidate is 49%?

d. Would the probability of a Type II error for the test in part **a** increase or decrease if the significance level of the test was reduced from 5% to 1%?

B2. A sample of 400 residents in Ontario was taken. Each resident was asked their opinion on the level of business confidence in Ontario. The resident had to provide a number between 0 and 100 which indicated their opinion on the level of business confidence. 0 was the lowest possible level of business confidence and 100 was the highest possible level. The mean from this sample was 80 and the standard deviation was 6.

a. Construct a 95% confidence interval for the level of business confidence in Ontario. Interpret this confidence interval.

b. Construct a 99% confidence interval for the level of business confidence in Ontario. Interpret this confidence interval.

c. Explain how the Central Limit Theorem is used to derive the formula for the confidence interval that was constructed in part **a.**

d. We are now told that the distribution of business confidence in Ontario is in fact uniformly distributed between 70 and 90. Given this information, calculate the actual mean and standard deviation of the level of business confidence in Ontario.

e. Given the information from part **d**, calculate the probability of a randomly chosen Ontario resident having a level of business confidence less than 75. What would this probability be if we mistakenly used the normal distribution instead of the uniform distribution?

B3. An investor holds three assets. A stock, a bond, and a unit of cash. The return on the stock is normally distributed with a mean of 10 and a standard deviation of 3. The return on the bond is normally distributed with a mean of 5 and a standard deviation of 1. The return on the unit of cash is not a random variable as it is always 0. For all three assets the returns are independent of each other.

a. What is the expected return from holding these three assets?

b. What is the standard deviation of the return from holding these three assets?

c. What is the distribution of the return from holding these three assets?

d. What is the probability of realising a return between 10 and 20 from holding these three assets?

e. If the investor decided to now hold two units of the stock, one unit of the bond, and no cash, instead of one unit of each asset, is the expected return going to increase or decrease, and is the standard deviation of the return going to increase or decrease, relative to the case of holding one unit of each asset? Explain why.

B4. Suppose an investor buys an asset. The asset each week provides a return of either 1 or 0. The returns on each week are independent of each other. The probability of the investor receiving a return of 1 in any week is 0.5 and the investor holds this asset for 10 weeks. The ten week return for this investor is the sum of the returns received on each of the ten weeks.

a. What is the probability that the investor receives a return of 4 when she holds this asset for 10 weeks?

b. What is the probability that the investor receives a return of between 3 and 7, inclusive, when she holds this asset for 10 weeks?

c. What is the expected return that the investor receives by holding this asset for 10 weeks?

d. What is the standard deviation of the return that the investor faces by holding this asset for 10 weeks?

e. Suppose the investor decides to invest in another asset. The 10 week return on this asset is uniformly distributed between 1 and 12. What is the expected 10 week return on this asset?

Statistics Formulas

1. **Notation:**
2. All summations are for $i = 1, \ldots, n$ unless otherwise stated
3. ~ means ‘distributed as’.

1. Counting Formulae

$$_NP_R = N!/(N-R)!$$

$$_NC_R = \binom{N}{R} = N!/(N-R)!R!$$

2. Probability Theory

$$P[A \cup B] = P[A] + P[B] - P[A \cap B] \text{(additive law)}$$

$$P[A \cap B] = P[A]P[B|A] = P[B]P[A|B] \text{(multiplicative law)}$$

$$P[A] = \sum P[A \cap E_i] = \sum P[E_i]P[A|E_i]$$

$$P[E_i|A] = \frac{\mathrm{P[E_i]P[A|E_i]}}{\mathrm{P[A]}} \text{(Bayes's theorem)}$$

3. Random Variables

Let X and Y be discrete random variables, then:

$$\mu_X = E[X] = \sum_x xP[X = x]$$

$$\sigma_X^2 = V[X] = E[(X - \mu_X)^2] = \sum_x (x - \mu_X)^2 P[X = x]$$

$$\sigma_X^2 = V[X] = E[X^2] - \mu_X^2 = \sum_x x^2 P[X = x] - \mu_X^2$$

If X and Y are independent random variables and a, b, and c are constants, then:

$$E[a + bX + cY] = a + b\mu_X + c\mu_Y$$

$$V[a + bX + cY] = b^2\sigma_X^2 + c^2\sigma_Y^2$$

4. Univariate Probability Distributions

Binomial Distribution: For $x = 0, 1, 2, \ldots, n$ and :

$$Pr[X = x] = \binom{n}{x} p^x (1-p)^{n-x}$$

$$E[X] = np$$

$$V[X] = np(1-p)$$

Uniform Distribution: For $a < x < b$:

$$f(x) = \frac{1}{b-a}$$

$$E[X] = \frac{a+b}{2}$$

$$V[X] = \frac{(b-a)^2}{12}$$

Normal Distribution:For $-\infty < x < \infty$:

$$f(x) = (2\pi\sigma^2)^{-1/2} exp[\frac{-(x-\mu)^2}{2\sigma^2}]$$

$$E[X] = \mu$$

$$V[X] = \sigma^2$$

$$X \sim N(\mu_X, \sigma_X^2)$$

$$Z = \frac{X - \mu_X}{\sigma_X} \sim N(0,1)$$

5. Estimators in General

If $\hat{\theta} \sim N(\theta, V[\hat{\theta}])$, say, then under appropriate conditions:

$$Z = \frac{\hat{\theta} - \theta}{SD[\hat{\theta}]} \sim N(0,1)$$

Confidence Intervals: (a) $100(1-\alpha)\%$ confidence interval for $\theta : \hat{\theta} \pm Z_{\alpha/2} SD[\hat{\theta}]$, with known variance.

(b) I f $V[\hat{\theta}]$ is unknown, then

$$t = \frac{\hat{\theta} - \theta}{SE[\hat{\theta}]} \sim t$$

with appropriate degrees of freedom. $100(1-\alpha)\%$ confidence interval for $\theta : \hat{\theta} \pm t_{\alpha/2} SE[\hat{\theta}]$, where $SE[\hat{\theta}]$ is an estimator of $SD[\hat{\theta}]$.

6. Estimating Means and Proportions

$$\bar{X} = \frac{1}{n}\sum X_i; \quad s^2 = \frac{1}{n-1}\sum (X_i - \bar{X})^2$$

$$E[\bar{X}] = \mu; \quad V[\bar{X}] = \frac{\sigma^2}{n}$$

$$f = \frac{X}{n}; \quad E[f] = p; \quad V[f] = \frac{p(1-p)}{n}$$

7. Differences of Means and Proportions

$$s_{pool}^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$E[\bar{X}_1 - \bar{X}_2] = \mu_1 - \mu_2; \quad V[\bar{X}_1 - \bar{X}_2] = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

$$s_{\bar{X}_1 - \bar{X}_2} = \sqrt{s_{pool}^2(\frac{1}{n_1} + \frac{1}{n_2})}$$

$$f_{pool} = \frac{X_1 + X_2}{n_1 + n_2};$$

$$E[f_1 - f_2] = p_1 - p_2;$$

$$V[f_1 - f_2] = \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}$$