Queen's University at Kingston

Faculty of Arts and Science

Department of Economics

ECON 250

Winter 2009

FINAL EXAMINATION

April 9, 2009

Instructor: A. Mirza

Time: 3 hours

Instructions:

READ CAREFULLY.

Calculators permitted: blue or gold stickers or Casio 991

At the end of the exam are several formulae and tables for the binomial, normal and *t* distributions. Answers are to be written in the examination booklet provided.
Remember most of the grades are awarded for how you set up the problem and NOT for the calculation itself.

You are to answer **ALL** questions. **SHOW ALL YOUR WORK.**
There are a total of 100 possible marks to be obtained and marks are indicated for each question.

Note: Proctors are unable to respond to queries about the interpretation of exam questions.
Do your best to answer exam questions as written.

Answer all 10 questions.

1. **(10 Marks)** What is meant by the statement that the sample mean has a sampling distribution?

2. **(10 Marks)** An investor is considering six different money market funds. The average number of days to maturity for each of these funds is

$$41 \quad 39 \quad 35 \quad 35 \quad 33 \quad 38$$

   (a) How many possible samples of two funds are there?

   (b) List all possible samples.

   (c) Find the probability distribution function of the sampling distribution of the sample means.

   (d) Verify directly that the mean of the sampling distribution of the sample means is equal to the population mean.

3. **(10 Marks)** A medical researcher wants to investigate the amount of time it takes for patients' headache pain to be relieved after taking a new prescription painkiller. She plans to use statistical methods to estimate the mean of the population of relief times. She believes that the population is normally distributed with a standard deviation of 20 minutes. How large a sample should she take to estimate the mean time to within 1 minute with 90% confidence?

4. **(10 Marks)** A study of the pay of corporate chief executive officers (CEOs) examined the increase in cash compensation of the CEOs of 104 companies, adjusted for inflation, in a recent year. The mean increase in real compensation was $\bar{x} = 6.9\%$, and the standard deviation of the increases was $s = 55\%$.

   (a) Write the null and alternative hypotheses.

   (b) Sketch the Normal curve for the sampling distribution of $\bar{x}$ when H_0 is true. Shade the area that represents the P-value for the observed outcome $\bar{x} = 6.9\%$.

   (c) Approximate the P-value.

   (d) Is the result significant at the $\alpha = 0.05$ level? Do you think the study gives strong evidence that the mean compensation of all CEOs went up?

5. **(10 Marks)** Tom wants to compare the cost of one- and two-bedroom apartments. He collects data for a random sample of 10 ads of each type. The data are as follows:

1 BR	520	645	600	505	450	550	515	495	650	385
2 BR	605	510	580	650	675	675	750	500	495	675

(a) Find a 95% confidence interval for the additional cost of a second bedroom.

(b) State the null and alternative hypothesis to test if two-bedroom apartments rent for more than one-bedroom apartments.

(c) Test the hypothesis in b) and find its p-value. What do you conclude?

(d) Can you conclude that every one-bedroom apartment costs less than every two-bedroom apartment?

(e) Which is more useful to a potential buyer, the confidence interval in a) or the hypothesis test in c) ?

6. **(10 Marks)** Spam e-mail has become a serious and costly nuisance. An office manager believes that the average amount of time spent by office workers reading and deleting spam exceeds 25 minutes per day. To test this belief, he takes a random sample of 18 workers and measures the amount of time each spends reading and deleting spam. The results are listed here. If the population of times is normally distributed with a standard deviation of 12 minutes, can the manager infer at the 1% significance level that he is correct?

35	48	29	44	17	21	32	28	34
23	13	9	11	30	42	37	43	38

7. **(10 Marks)** University bookstores order books that instructors adopt for their courses. The number of copies ordered matches the projected demand. However, at the end of the semester the bookstore has too many copies on hand and must return them to the publisher. A bookstore has a policy that the proportion of books returned should be kept as small as possible. The average is supposed to be less than 10%. To see whether the policy is working, a random sample of book titles was drawn and the fraction of the total originally ordered that are returned is recorded and listed here.

4 15 11 7 5 9 4 3 5 8

(a) Can we infer at the 10% significance level that the mean proportion of returns is less than 10%?

(b) Repeat a) at the 1% significance level.

(c) Find the P-value for the test.

8. **(10 Marks)** A parking control officer is conducting an analysis of the amount of time left on parking meters. A quick survey of 15 cars that have just left their metered parking spaces produced the following time (in minutes).

22 15 1 14 0 9 17 31 18 26 23 15 33 28 20

(a) Estimate with 95% confidence the mean amount of time left for all the city's meters.

(b) Construct the 95% confidence interval.

(c) What hypothesis can you test with this interval?

(d) Test the hypothesis referred to in part c)

9. (**10 Marks**) You want to see if a redesign of the cover of a mail-order catalog will increase sales. A very large number of customers will receive the original catalog, and a random sample of customers will receive the one with the new cover. For planning purposes, you are willing to assume that the mean sales for the new catalog will be approximately Normal with $\sigma = 50$ dollars and that the mean sales for the original catalog will be $\mu = 25$ dollars. You decide to use a sample size of $n = 900$. You wish to test

$$H_0 : \mu = 25$$
$$H_A : \mu > 25$$

You decide to reject H_0 if $\bar{x} > 26$ and to accept H_0 otherwise.

(a) Find the probability of a Type I error, that is, the probability that your test rejects H_0 when in fact $\mu = 25$ dollars.

(b) Find the probability of a Type II error when $\mu = 28$ dollars. This is the probability that your test accepts H_0 when in fact $\mu = 28$.

(c) Find the probability of a Type II error when $\mu = 30$.

(d) The distribution of sales is not Normal, because many customers buy nothing. Why is nonetheless reasonable in this circumstance to assume that the mean will be approximately Normal?

10. (**10 Marks**) Each month a clothing store conducts an inventory and calculates the losses due to theft. The store would like to reduce these losses and is considering two methods. The first is to hire a security guard and the second is to install cameras. To help decide which method to choose, they hired a security guard for 6 months. During the next 6-month period, the store installed cameras. The monthly losses were recorded and are listed here. The manager decided that since the cameras were cheaper that the guard, he would install the cameras unless there was enough evidence to infer that the guard was better. What should the manager do?

Security Guard	355	284	401	398	477	254
Cameras	486	303	270	386	411	435

Formula Sheet

Statistics Formulas

Notation

- All summations are for $i = 1, \ldots, n$ unless otherwise stated.
- $\sim$ means 'distributed as'

Population Mean

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Sample Mean

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i.$$

Population Variance

$$\begin{aligned}\sigma^2 &= \frac{1}{N}\sum_{i=1}^{N}[x_i - \mu]^2 \\ &= \frac{1}{N}\sum_{i=1}^{N} {x_i}^2 - \mu^2\end{aligned}$$

Sample Variance

$$s^2 = \tfrac{1}{n-1}\textstyle\sum_{i=1}^{n}[x_i - \bar{X}]^2$$

Alternatively

$$s^2 = \frac{1}{n-1}[\sum_{i=1}^{n} x_i^2 - n\bar{X}^2]$$

Grouped Data (with k classes)

$$\bar{X} = \frac{1}{n}\sum_{j=1}^{k} \nu_j f_j \quad \text{where } \nu_j \text{ is the class mark for class } j$$

$$s^2 = \frac{1}{n-1}\sum_{j=1}^{k} f_j(\nu_j - \bar{X})^2$$

Probability Theory

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad \text{(additive law)}$$

$$P(A \cap B) = P(B)P(A \mid B) \quad \text{(multiplicative law)}$$

If the E_i are mutually exclusive and exhaustive events for $i = 1, \ldots, n$, then

$$P(A) = \sum_{i}^{n} P(A \cap E_i) = \sum_{i}^{n} P(E_i)P(A \mid E_i)$$

$$P(E_i \mid A) = \frac{P(E_i)P(A \mid E_i)}{P(A)} \quad \text{(Bayes' Theorem)}$$

Counting Formulae

$$P_R^N = \frac{N!}{(N-R)!}$$

$$C_R^N = \binom{N}{R} = \frac{N!}{(N-R)!R!}$$

Random Variables

Let X be a discrete random variable, then:

$$\begin{aligned} \mu_x &= E[X] = \sum_x xP(X=x) \\ \sigma_x^2 &= V[X] = E[(x-\mu_x)^2] = \sum_x (x-\mu_x)^2 P(X=x) \\ \sigma_x^2 &= E[X^2] - (E[X])^2 \end{aligned}$$

The **covariance** of X and Y is

$$\begin{aligned} Cov[X,Y] &= \sigma_{xy} = E\left[(X-\mu_x)(Y-\mu_y)\right] \\ &= E[XY] - E[X] \times E[Y] \end{aligned}$$

The **correlation coefficient** of X and Y

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

If X and Y are independent random variables and a, b, and c are constants, then:

$$\begin{aligned} E[a+bX+cY] &= a + b\mu_x + c\mu_y \\ V[a+bX+cY] &= b^2\sigma_x^2 + c^2\sigma_y^2 \end{aligned}$$

If X and Y are correlated then

$$V[a + bX + cY] = b^2\sigma_x^2 + c^2\sigma_y^2 + 2bc\sigma_{xy}$$

Coefficient of Variation (CV)

$$\begin{aligned} CV &= \frac{\sigma}{\mu} \times 100\% \text{ for population} \\ CV &= \frac{s}{\bar{X}} \times 100\% \text{ for sample} \end{aligned}$$

Univariate Probability Distributions

Binomial Distribution: For $x = 0, 1, 2, ..., n$ and :

$$Pr[X = x] = \binom{n}{x} \pi^x (1 - \pi)^{n-x}$$

$$E[X] = n\pi$$

$$V[X] = n\pi(1 - \pi)$$

Uniform Distribution: For $a < x < b$:

$$f(x) = \frac{1}{b - a}$$

$$E[X] = \frac{a + b}{2}$$

$$V[X] = \frac{(b - a)^2}{12}$$

Normal Distribution:For $-\infty < x < \infty$:

$$f(x) = (2\pi\sigma^2)^{-1/2} exp[\frac{-(x - \mu)^2}{2\sigma^2}]$$

$$E[X] = \mu$$

$$V[X] = \sigma^2$$

$$\begin{aligned} X &\sim N(\mu_X, \sigma_X^2) \\ Z &= \frac{X - \mu_X}{\sigma_X} \sim N(0,1) \end{aligned}$$

Estimators in General

If $\hat{\theta} \sim N(\theta, V[\hat{\theta}])$, say, then under appropriate conditions:

$$Z = \frac{\hat{\theta} - \theta}{SD[\hat{\theta}]} \sim N(0,1)$$

Confidence Intervals:

(a) $100(1-\alpha)\%$ confidence interval for $\theta : \hat{\theta} \pm Z_{\alpha/2} SD[\hat{\theta}]$, with known variance.

(b) If $V[\hat{\theta}]$ is unknown, then

$$t = \frac{\hat{\theta} - \theta}{SE[\hat{\theta}]} \sim t$$

with appropriate degrees of freedom. $100(1-\alpha)\%$ confidence interval for $\theta : \hat{\theta} \pm t_{\alpha/2} SE[\hat{\theta}]$, where $SE[\hat{\theta}]$ is an estimator of $SD[\hat{\theta}]$.

Estimating Means and Proportions

$$\bar{X} = \frac{1}{n} \sum X_i; \quad s^2 = \frac{1}{n-1} \sum (X_i - \bar{X})^2$$

$$E[\bar{X}] = \mu; \quad V[\bar{X}] = \frac{\sigma^2}{n}$$

$$p = \frac{X}{n}; \quad E[p] = \pi; \quad V[p] = \frac{\pi(1-\pi)}{n}$$

Differences of Means and Proportions for Independent Samples

$$s^2_{pool} = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

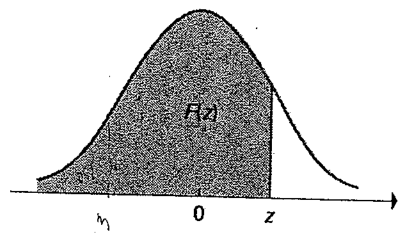
$$E[\bar{X}_1 - \bar{X}_2] = \mu_1 - \mu_2; \quad V[\bar{X}_1 - \bar{X}_2] = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

If variances are assumed (i.e. $\sigma_1^2 = \sigma_2^2$) to be the same we may , estimate

$$s_{\bar{X}_1 - \bar{X}_2} = \sqrt{s^2_{pool}(\frac{1}{n_1} + \frac{1}{n_2})}$$

$$\begin{aligned} p_{pool} &= \frac{X_1 + X_2}{n_1 + n_2}; \\ E[p_1 - p_2] &= \pi_1 - \pi_2; \\ V[f_1 - f_2] &= \frac{\pi_1(1-\pi_1)}{n_1} + \frac{\pi_2(1-\pi_2)}{n_2} \end{aligned}$$

Table 1 Cumulative Distribution Function of the Standard Normal Distribution



z	F(z)	z	F(z)	z	F(z)	z	F(z)	z	F(z)	z	F(z)
.00	.5000										
.01	.5040	.31	.6217	.61	.7291	.91	.8186	1.21	.8869	1.51	.9345
.02	.5080	.32	.6255	.62	.7324	.92	.8212	1.22	.8888	1.52	.9357
.03	.5120	.33	.6293	.63	.7357	.93	.8238	1.23	.8907	1.53	.9370
.04	.5160	.34	.6331	.64	.7389	.94	.8264	1.24	.8925	1.54	.9382
.05	.5199	.35	.6368	.65	.7422	.95	.8289	1.25	.8944	1.55	.9394
.06	.5239	.36	.6406	.66	.7454	.96	.8315	1.26	.8962	1.56	.9406
.07	.5279	.37	.6443	.67	.7486	.97	.8340	1.27	.8980	1.57	.9418
.08	.5319	.38	.6480	.68	.7517	.98	.8365	1.28	.8997	1.58	.9429
.09	.5359	.39	.6517	.69	.7549	.99	.8389	1.29	.9015	1.59	.9441
.10	.5398	.40	.6554	.70	.7580	1.00	.8413	1.30	.9032	1.60	.9452
.11	.5438	.41	.6591	.71	.7611	1.01	.8438	1.31	.9049	1.61	.9463
.12	.5478	.42	.6628	.72	.7642	1.02	.8461	1.32	.9066	1.62	.9474
.13	.5517	.43	.6664	.73	.7673	1.03	.8485	1.33	.9082	1.63	.9484
.14	.5557	.44	.6700	.74	.7704	1.04	.8508	1.34	.9099	1.64	.9495
.15	.5596	.45	.6736	.75	.7734	1.05	.8531	1.35	.9115	1.65	.9505
.16	.5636	.46	.6772	.76	.7764	1.06	.8554	1.36	.9131	1.66	.9515
.17	.5675	.47	.6803	.77	.7794	1.07	.8577	1.37	.9147	1.67	.9525
.18	.5714	.48	.6844	.78	.7823	1.08	.8599	1.38	.9162	1.68	.9535
.19	.5753	.49	.6879	.79	.7852	1.09	.8621	1.39	.9177	1.69	.9545
.20	.5793	.50	.6915	.80	.7881	1.10	.8643	1.40	.9192	1.70	.9554
.21	.5832	.51	.6950	.81	.7910	1.11	.8665	1.41	.9207	1.71	.9564
.22	.5871	.52	.6985	.82	.7939	1.12	.8686	1.42	.9222	1.72	.9573
.23	.5910	.53	.7019	.83	.7967	1.13	.8708	1.43	.9236	1.73	.9582
.24	.5948	.54	.7054	.84	.7995	1.14	.8729	1.44	.9251	1.74	.9591
.25	.5987	.55	.7088	.85	.8023	1.15	.8749	1.45	.9265	1.75	.9599
.26	.6026	.56	.7123	.86	.8051	1.16	.8770	1.46	.9279	1.76	.9608
.27	.6064	.57	.7157	.87	.8078	1.17	.8790	1.47	.9292	1.77	.9616
.28	.6103	.58	.7190	.88	.8106	1.18	.8810	1.48	.9306	1.78	.9625
.29	.6141	.59	.7224	.89	.8133	1.19	.8830	1.49	.9319	1.79	.9633
.30	.6179	.60	.7257	.90	.8159	1.20	.8849	1.50	.9332	1.80	.9641

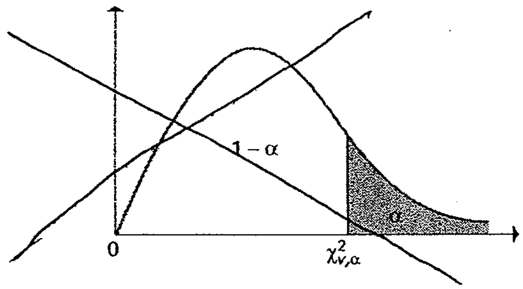
Table 1 Cumulative Distribution Function of the Standard Normal Distribution Continued

z	F(z)	z	F(z)	z	F(z)	z	F(z)	z	F(z)	z	F(z)
1.81	.9649	2.21	.9864	2.61	.9955	3.01	.9987	3.41	.9997	3.81	.9999
1.82	.9656	2.22	.9868	2.62	.9956	3.02	.9987	3.42	.9997	3.82	.9999
1.83	.9664	2.23	.9871	2.63	.9957	3.03	.9988	3.43	.9997	3.83	.9999
1.84	.9671	2.24	.9875	2.64	.9959	3.04	.9988	3.44	.9997	3.84	.9999
1.85	.9678	2.25	.9878	2.65	.9960	3.05	.9989	3.45	.9997	3.85	.9999
1.86	.9686	2.26	.9881	2.66	.9961	3.06	.9989	3.46	.9997	3.86	.9999
1.87	.9693	2.27	.9884	2.67	.9962	3.07	.9989	3.47	.9997	3.87	.9999
1.88	.9699	2.28	.9887	2.68	.9963	3.08	.9990	3.48	.9997	3.88	.9999
1.89	.9706	2.29	.9890	2.69	.9964	3.09	.9990	3.49	.9998	3.89	1.0000
1.90	.9713	2.30	.9893	2.70	.9965	3.10	.9990	3.50	.9998	3.90	1.0000
1.91	.9719	2.31	.9896	2.71	.9966	3.11	.9991	3.51	.9998	3.91	1.0000
1.92	.9726	2.32	.9898	2.72	.9967	3.12	.9991	3.52	.9998	3.92	1.0000
1.93	.9732	2.33	.9901	2.73	.9968	3.13	.9991	3.53	.9998	3.93	1.0000
1.94	.9738	2.34	.9904	2.74	.9969	3.14	.9992	3.54	.9998	3.94	1.0000
1.95	.9744	2.35	.9906	2.75	.9970	3.15	.9992	3.55	.9998	3.95	1.0000
1.96	.9750	2.36	.9909	2.76	.9971	3.16	.9992	3.56	.9998	3.96	1.0000
1.97	.9756	2.37	.9911	2.77	.9972	3.17	.9992	3.57	.9998	3.97	1.0000
1.98	.9761	2.38	.9913	2.78	.9973	3.18	.9993	3.58	.9998	3.98	1.0000
1.99	.9767	2.39	.9916	2.79	.9974	3.19	.9993	3.59	.9998	3.99	1.0000
2.00	.9772	2.40	.9918	2.80	.9974	3.20	.9993	3.60	.9998		
2.01	.9778	2.41	.9920	2.81	.9975	3.21	.9993	3.61	.9998		
2.02	.9783	2.42	.9922	2.82	.9976	3.22	.9994	3.62	.9999		
2.03	.9788	2.43	.9925	2.83	.9977	3.23	.9994	3.63	.9999		
2.04	.9793	2.44	.9927	2.84	.9977	3.24	.9994	3.64	.9999		
2.05	.9798	2.45	.9929	2.85	.9978	3.25	.9994	3.65	.9999		
2.06	.9803	2.46	.9931	2.86	.9979	3.26	.9994	3.66	.9999		
2.07	.9808	2.47	.9932	2.87	.9979	3.27	.9995	3.67	.9999		
2.08	.9812	2.48	.9934	2.88	.9980	3.28	.9995	3.68	.9999		
2.09	.9817	2.49	.9936	2.89	.9981	3.29	.9995	3.69	.9999		
2.10	.9821	2.50	.9938	2.90	.9981	3.30	.9995	3.70	.9999		
2.11	.9826	2.51	.9940	2.91	.9982	3.31	.9995	3.71	.9999		
2.12	.9830	2.52	.9941	2.92	.9982	3.32	.9996	3.72	.9999		
2.13	.9834	2.53	.9943	2.93	.9983	3.33	.9996	3.73	.9999		
2.14	.9838	2.54	.9945	2.94	.9984	3.34	.9996	3.74	.9999		
2.15	.9842	2.55	.9946	2.95	.9984	3.35	.9996	3.75	.9999		
2.16	.9846	2.56	.9948	2.96	.9985	3.36	.9996	3.76	.9999		
2.17	.9850	2.57	.9949	2.97	.9985	3.37	.9996	3.77	.9999		
2.18	.9854	2.58	.9951	2.98	.9986	3.38	.9996	3.78	.9999		
2.19	.9857	2.59	.9952	2.99	.9986	3.39	.9997	3.79	.9999		
2.20	.9861	2.60	.9953	3.00	.9986	3.40	.9997	3.80	.9999		

Table 3 Cumulative Binomial Probabilities Continued

n	x	p .05	.10	.15	.20	.25	.30	.35	.40	.45	.500
	2	.942	.734	.48	.271	.135	.06	.024	.008	.003	.001
	3	.989	.902	.72	.501	.306	.165	.078	.033	.012	.004
	4	.998	.972	.879	.716	.519	.333	.189	.094	.041	.015
	5	1.00	.994	.958	.867	.717	.534	.355	.209	.108	.048
	6	1.00	.999	.988	.949	.861	.722	.549	.374	.226	.119
	7	1.00	1.00	.997	.984	.943	.859	.728	.563	.391	.240
	8	1.00	1.00	.999	.996	.981	.94	.861	.737	.578	.407
	9	1.00	1.00	1.00	.999	.995	.979	.94	.865	.747	.593
	10	1.00	1.00	1.00	1.00	.999	.994	.979	.942	.872	.760
	11	1.00	1.00	1.00	1.00	1.00	.999	.994	.98	.946	.881
	12	1.00	1.00	1.00	1.00	1.00	1.00	.999	.994	.982	.952
	13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.995	.985
	14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.996
	15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999
	16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.000
19	0	.377	.135	.046	.014	.004	.001	.00	.00	.00	.000
	1	.755	.42	.198	.083	.031	.01	.003	.001	.00	.000
	2	.933	.705	.441	.237	.111	.046	.017	.005	.002	.000
	3	.987	.885	.684	.455	.263	.133	.059	.023	.008	.002
	4	.998	.965	.856	.673	.465	.282	.15	.07	.028	.010
	5	1.00	.991	.946	.837	.668	.474	.297	.163	.078	.032
	6	1.00	.998	.984	.932	.825	.666	.481	.308	.173	.084
	7	1.00	1.00	.996	.977	.923	.818	.666	.488	.317	.180
	8	1.00	1.00	.999	.993	.971	.916	.815	.667	.494	.324
	9	1.00	1.00	1.00	.998	.991	.967	.913	.814	.671	.500
	10	1.00	1.00	1.00	1.00	.998	.989	.965	.912	.816	.676
	11	1.00	1.00	1.00	1.00	1.00	.997	.989	.965	.913	.820
	12	1.00	1.00	1.00	1.00	1.00	.999	.997	.988	.966	.916
	13	1.00	1.00	1.00	1.00	1.00	1.00	.999	.997	.989	.968
	14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.997	.990
	15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.998
	16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.000
20	0	.358	.122	.039	.012	.003	.001	.00	.00	.00	.000
	1	.736	.392	.176	.069	.024	.008	.002	.001	.00	.000
	2	.925	.677	.405	.206	.091	.035	.012	.004	.001	.000
	3	.984	.867	.648	.411	.225	.107	.044	.016	.005	.001
	4	.997	.957	.83	.63	.415	.238	.118	.051	.019	.006
	5	1.00	.989	.933	.804	.617	.416	.245	.126	.055	.021
	6	1.00	.998	.978	.913	.786	.608	.417	.25	.13	.058
	7	1.00	1.00	.994	.968	.898	.772	.601	.416	.252	.132
	8	1.00	1.00	.999	.99	.959	.887	.762	.596	.414	.252
	9	1.00	1.00	1.00	.997	.986	.952	.878	.755	.591	.412
	10	1.00	1.00	1.00	.999	.996	.983	.947	.872	.751	.588
	11	1.00	1.00	1.00	1.00	.999	.995	.98	.943	.869	.748
	12	1.00	1.00	1.00	1.00	1.00	.999	.994	.979	.942	.868
	13	1.00	1.00	1.00	1.00	1.00	1.00	.998	.994	.979	.942
	14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.998	.994	.979
	15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.998	.994
	16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999
	17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.000

Table 8 Cutoff Points for the Student's *t* Distribution



For selected probabilities, α, the table shows the values $t_{v,\alpha}$ such that $P(t_v > t_{v,\alpha}) = \alpha$, where t_v is a Student's *t* random variable with *v* degrees of freedom. For example, the probability is .10 that a Student's *t* random variable with 10 degrees of freedom exceeds 1.372.

v	α				
	0.100	0.050	0.025	0.010	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
∞	1.282	1.645	1.960	2.326	2.576