Economics 250
Functions of Random Variables

The expressions we need for functions of RVs are in the formula sheet. But this note shows where those expressions come from.

1. Suppose x is a discrete RV with mean μ_x and standard deviation σ_x. Then a second RV is related to x like this: $y = a + bx$.

The expectation of y:

$$\begin{aligned} E(y) = E(a+bx) &= \sum (a+bx)P(x) \\ &= \sum aP(x) + \sum bxP(x) \\ &= a + b\sum xP(x) = a + b\mu_x, \end{aligned}$$

simply using our summation rules and the fact that probabilities sum to 1. This way we can find the mean of y without having to create a long column of all the y values.

Now the variance:

$$\begin{aligned} \sigma_y^2 = \sum (y-\mu_y)^2 P(y) &= \sum (a+bx-a-b\mu_x)^2 P(x) \\ &= \sum (bx - b\mu_x)^2 P(x) \\ &= b^2 \sum (x-\mu_x)^2 P(x) \\ &= b^2\sigma_x^2. \end{aligned}$$

Thus $\sigma_y = b\sigma_x$. (We found this same relationship for sample statistics at the start of the course.)

2. Now suppose x and y are RVs and c and d are numbers. A third RV is given by

$$w = cx + dy,$$

so that it is a combination of the original two RVs.

Notice that when we see the E symbol that is simply a shorthand for 'weight observations by probabilities and then add them up.' Thus the mean:

$$\mu_w \equiv E(w) = E(cx+dy) = E(cx) + E(dy) = cE(x) + dE(y) \equiv c\mu_x + d\mu_y.$$

This simply uses the properties of the summation operator to split up the sum and factor out the constants.

The variance:

$$\begin{aligned} \sigma_w^2 &= E(cx + dy - c\mu_x - d\mu_y)^2 \\ &= E[c(x-\mu_x) + d(y-\mu_y)]^2 \\ &= E[c^2(x-\mu_x)^2 + d^2(y-\mu_y)^2 + 2cd(x-\mu_x)(y-\mu_y)] \\ &= c^2\sigma_x^2 + d^2\sigma_y^2 + 2cd\sigma_{xy}. \end{aligned}$$