18

Derivative Securities
Jarrow + Turnbull.

CREDIT RISK

18.0 INTRODUCTION

The pricing and hedging models we describe in the previous chapters are based on the four fundamental assumptions we first discussed in Chapter 2.

Assumption A1. No market frictions

Assumption A2. No counterparty risk

Assumption A3. Competitive markets

Assumption A4. No arbitrage opportunities

Of these four assumptions, the second one—no counterparty risk—is perhaps the least defensible. In practice, bankruptcy and the nonexecution of contracts are of major concern in all business transactions, including the trading of financial instruments, especially over-the-counter contracts. When entities borrow and lend, Assumption A2 implies that there is only one rate—the default-free rate. However, in practice, there are many rates for borrowing. These rates differ based on the perceived ability (or likelihood) of the borrower to repay the loan.

For exchange-traded derivative securities, such as options and futures, there is the risk that the writer of the security will be unable to meet the obligations of the contract. As a consequence, exchanges have set up clearing corporations to reduce concern about this counterparty risk (see Chapter 1). A clearing corporation is the counterparty to all exchange-traded derivatives. In the United States the triple-A-credit rated Options Clearing Corporation helps reduce counterparty risk in the execution of exchange-traded option contracts. Many investment banks have also set up triple-A-credit rated derivative product companies in response to concern about creditworthiness in the over-the-counter market.

These observations suggest that if we want to understand corporate bond markets and over-the-counter contracts, we need to understand the pricing and hedging of derivatives in the presence of counterparty risk, that is, the relaxation of Assumption A2.

The pricing and hedging of financial securities under the relaxation of Assumption A2 is an exciting and new area of research in financial economics. Here we present the modeling paradigm formulated by Jarrow and Turnbull (1995). This paradigm can be shown to include all other modeling approaches as special cases, hence its suitability for presentation. The material is no more difficult than any of the other areas studied in this text. However, the notation is slightly more complicated in order to keep track of bankruptcy and its likelihood of occurrence.

This model considers pricing and hedging of derivatives under two forms of credit risk. The first form we examine is the pricing of derivatives written on assets subject to default risk. An example is the pricing of derivatives written on corporate bonds, where default may occur on the part of the corporation. The second form we examine is the pricing of derivatives where the writer of the derivative might default. For example, consider an over-the-counter option written on a Treasury bond. There is no default risk arising from the underlying asset—the Treasury bond. However, default risk arises from the fact that the writer of the option may not be able to honor the obligation if the option is exercised. This form of risk is referred to as **counterparty risk**. We will describe a simple approach to the pricing and hedging of both forms of credit risk. We will give a number of examples, including the pricing of credit-risky annuities, floating rate debt, swaps, and derivatives.

In the over-the-counter market counterparty risk is a major concern to financial institutions and regulatory bodies. One of the main concerns of bank regulators is to evaluate management's ability to measure and control risk and to determine when risk exposures become excessive relative to the institution's capital position.[1] The traditional approach of specifying minimum or maximum balance sheet ratios such as equity to total assets is inappropriate for two reasons. First, derivative securities such as swaps and options that are now a significant source of risk do not appear on the balance sheet. Second, the static nature of balance sheet ratios is woefully inadequate in assessing the risk of derivative securities. The last part of this chapter describes some recent regulatory developments in this regard.

18.1 PRICING CREDIT-RISKY BONDS

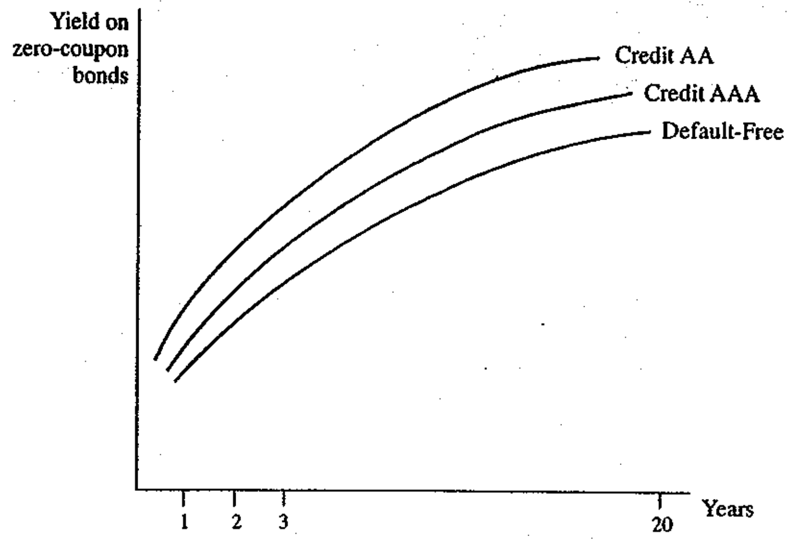
In this section we study the pricing of bonds (loans) subject to default risk, called **credit-risky bonds**. Our approach to pricing credit-risky bonds allocates firms to particular risk classes—AAA, AA, and so on—based on their current creditworthiness.[2] A typical set of credit-risky term structures is shown in Figure 18.1.

Illustrated in this figure are the yields on zero-coupon bonds issued by firms with different credit risk. The default-free curve is derived using Treasury bills, notes, and bonds, as described in Chapter 13. A corporation in credit class AAA is assumed to have the least credit risk among corporate firms. Its yields are higher than those of the Treasury's due to default risk. Firms of lower credit than AAA, such as those in credit class AA, trade at a lower price and thus at a higher yield than AAA, and so forth.

[1]See Part 1 of the Federal Reserve Trading Manual (1994).

[2]An extended form of this structure is for each firm to belong to its own separate credit class. In this case, the analysis applies firm by firm.

FIGURE 18.1 *Term Structures*



Given these term structures of zero-coupon bond prices for the different credit classes, pricing coupon bonds issued by the different credit classes is a straightforward exercise. For example, consider a coupon bond issued by a firm in credit class ABC with coupons c_j paid at times T_j for $j = 1, \ldots, n$ and a face value F paid at the time T_n.

Let $v(0, T)$ represent the date-0 value of a promised dollar at date T issued by the ABC credit class firm. Then, the coupon bond's value at date 0, denoted $v_c(0)$, is given by

$$v_c(0) = \sum_{j=1}^{n} c_j v(0, T_j) + F v(0, T_n),$$

which is an arbitrage-free pricing relation. Its proof is identical to the proof of Expression (13.4) in Chapter 13. The zero-coupon term structures for the different credit classes are the only inputs necessary for pricing credit-risky coupon bonds.

In fact, we have already used this result previously in Chapter 14 when valuing plain vanilla interest rate swaps. In that case, the two different term structures were the default-free and the LIBOR curves. We used those curves to determine the present value of the swap's cash flows.

Next, suppose we want to price a derivative written on a zero-coupon bond issued by a firm with credit rating ABC. We must price this derivative in such a way that it is

(1) consistent with the absence of arbitrage, (2) consistent with the relevant initial term structures of interest rates, and (3) consistent with a positive probability of default. This is a two-step procedure. First, we need to construct a lattice of one-period interest rates to model the default-free term structure. This construction is described in Chapter 15. Second, we need to perform a similar, but more complex, construction for the zero-coupon bonds of the firm belonging to the risk class ABC. We now turn to these tasks.

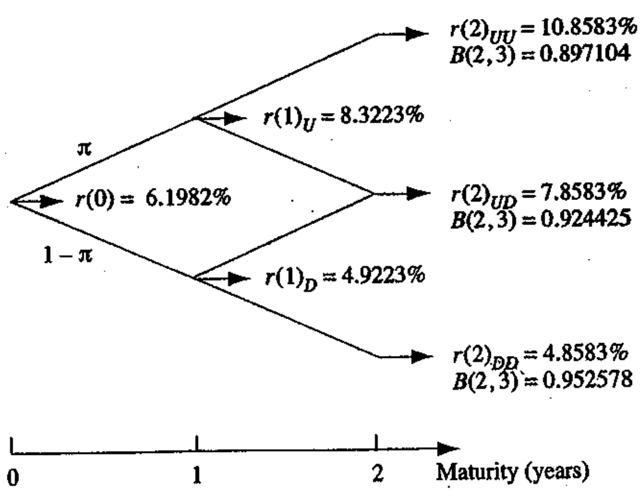
Lattice of Default-Free Interest Rates

Here we recall the construction of the lattice for default-free interest rates studied in Chapter 15. The term structure for default-free zero-coupon bonds is assumed to be identical to that given in Table 15.1 of Chapter 15, and the lattice for short-term interest rates shown in Figure 15.3 is reproduced here as Figure 18.2.

We have selected the lattice approximating normally distributed interest rates. Alternatively, we could have used the Black, Derman, and Toy (1990) lognormal model given by the lattice in Figure 15.6.

We note that all of the insights concerning the pricing of default-free interest rate derivatives shown in Chapter 15 apply based on these figures. In the next section we extend this model to handle credit-risky bonds.

FIGURE 18.2 *Default-Free Spot Interest Rates*



π is the martingale probability of an up-state ($\pi = 0.5$).
$1 - \pi$ is the martingale probability of a down-state ($1 - \pi = 0.5$).

Risky Debt

Let us examine the pricing of credit-risky debt. We want to value a zero-coupon bond for a firm belonging to the credit class ABC.

Let $v(t, T)$ denote the value at date t of a zero-coupon bond issued by the firm. The debt matures at date T, and the bondholders are *promised* the face value of the bond at maturity. Let the face value be \$1. Because the firm's debt is risky, there is a positive probability that the firm might default over the life of the bond. If default occurs, the bondholders will receive less than the promised amount.

It is instructive to view the pricing of credit-risky bonds in terms of a foreign currency analogy. Imagine a hypothetical currency, called *ABCs*. In terms of this currency, we can view the debt issued by the firm as default-free. Indeed, at maturity the bondholder is issued the face value of debt in *ABCs*. But this hypothetical currency is useless to the bondholder; we need to define an exchange rate that converts this hypothetical currency to dollars. After all, the bondholders are interested in the dollar value of their *ABCs*.

There are two cases to consider. If default has not occurred before or at date t, then the exchange rate is unity. If default did occur, it is assumed that we get some fraction δ of a dollar for each *ABC*. This is the same as receiving the fraction δ of the face amount of the debt in the event of default. The fraction δ is also called the **payoff ratio** or **recovery rate**.

Defining e_t as the date-t exchange rate per *ABCs*, we have

$$e_t = \begin{cases} 1 \text{ with probability } 1 - \lambda(t)\,\Delta & \text{if} \quad \text{no default} \\ \delta \text{ with probability } \lambda(t)\,\Delta & \text{if} \quad \text{default,} \end{cases} \tag{18.1}$$

where $0 \leq \delta < 1$ and $\lambda(t)\,\Delta$ is the martingale probability of default conditional upon no default at or before date $t - \Delta$.

We are interested in the martingale probabilities of default because we want to develop pricing formulas that are arbitrage-free.[3] This approach is analogous to that taken in Chapter 15 for default-free derivatives and draws upon the work in Chapter 6.

If default has occurred at or before date $t - \Delta$, it is assumed that the bond remains in default and the payoff ratio remains constant at δ dollars:

$$e_t \equiv \delta. \tag{18.2}$$

The conditional martingale probabilities of default can be estimated using the observed term structures of interest rates. We will discuss how to do this in the following text.

To simplify the analysis, we are going to assume that the default process is independent of the level of the default-free interest rate. This implies that interest rates be-

[3]The existence and uniqueness of these martingale probabilities of default is discussed in Jarrow and Turnbull (1995).

ing either "high" or "low" have no effect on the probability of default. This is a useful first approximation and its relaxation is discussed in Jarrow and Turnbull (1995).

Credit-Risky Debt

This section shows how to determine the martingale probabilities of default using the observed term structures of zero-coupon bond prices. We use an example to illustrate the procedure.

EXAMPLE

Martingale Probabilities of Default

This example illustrates the procedure for determining the martingale probabilities of default. In Table 18.1, we are given two sets of prices for zero-coupon bonds of maturities one, two, and three years. The first is for default-free bonds and the second is for bonds belonging to credit class ABC.

The default-free bonds at each maturity are seen to be more valuable than the equivalent maturity bond issued by the firm in credit class ABC. This difference reflects the likelihood of default and the recovery rate. We want to estimate the martingale probabilities of default implicit in these bond prices.

Before we make our estimate, however, we must first specify the payoff ratio δ in the event of default. This value comes from our credit risk analysts who estimate that, given the nature of the debt, we expect to receive $0.32 on the dollar in the event of default.[4]

Consider first the one-year ABC zero-coupon bond. For simplicity, we take the interval in the lattice to be one year. At maturity, the credit-risky bond's value is

$$v(1,1) = \begin{cases} 1 \text{ with probability } 1 - \lambda(0)\,\Delta & \text{if no default} \\ \delta \text{ with probability } \lambda(0)\,\Delta & \text{if default,} \end{cases} \tag{18.3}$$

where $\Delta = 1$ and $\delta = 0.32$.

TABLE 18.1 *Prices of Zero-Coupon Bonds*

MATURITY (YEARS) T	DEFAULT-FREE $B(0,T)$	CREDIT CLASS ABC $v(0,T)$
1	0.9399	0.9361
2	0.8798	0.8703
3	0.8137	0.7980

[4] For different types of bonds, average recovery rates are given in Moody's Special Report (1992).

The default process is shown in Figure 18.3. Given that default has not occurred at date $t = 0$, the conditional (martingale) probability that default occurs at $t = 1$ is denoted by $\lambda(0) \times \Delta$, where Δ is the time interval. In this example, $\Delta = 1$. The conditional (martingale) probability that default does not occur is $1 - \lambda(0)\,\Delta$.

We can use the term structures of interest rates for default-free and credit class ABC bonds to infer the value of $\lambda(0)$. We know from Chapter 6 that normalized prices are a martingale under the martingale probabilities:[5]

$$\frac{v(0,1)}{A(0)} = \tilde{E}\left[\frac{v(1,1)}{A(1)}\right], \tag{18.4}$$

where $A(t)$ denotes the value of the money market account at date t.

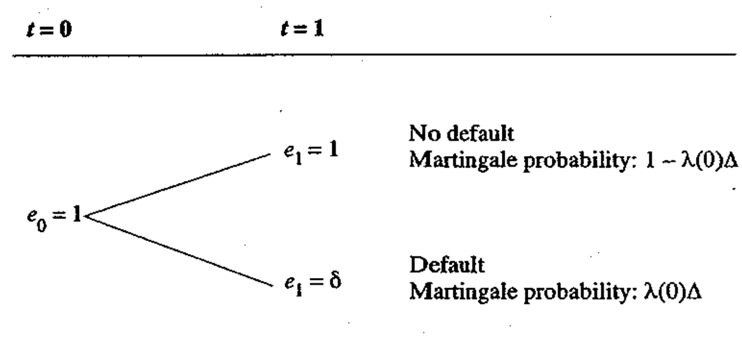
We use Expression (18.4) to solve for the value of $\lambda(0)$. First, by construction we have that $A(0) = 1$, and from Figure 18.2 we see that

$$A(1) = \exp[r(0)] = \exp(0.06198) = 1.0639.$$

Substituting Expression (18.3) into (18.4), given no default at time 0, we have

$$v(0,1) = \{1 \times [1 - \lambda(0)] + \delta \times \lambda(0)\}/1.0639.$$

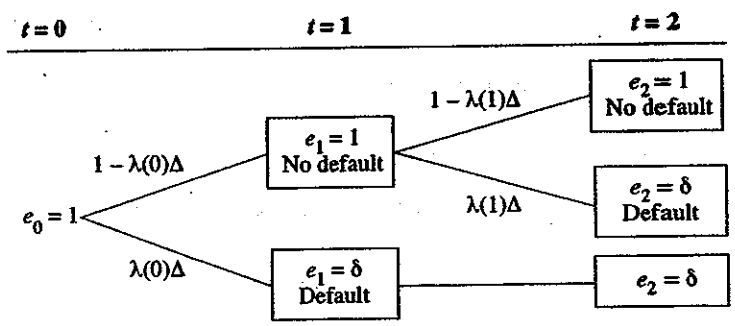
FIGURE 18.3 *One-Period Credit-Risky Debt Default Process*



$\lambda(0)\Delta$ is the martingale probability that default occurs at date $t = 1$, conditional upon the fact that default has not occurred at date $t = 0$. In this example $\Delta = 1$.

[5] The reader is referred to Jarrow and Turnbull (1995) for technical details.

FIGURE 18.4 *Two-Period Credit-Risky Debt Default Process*



$\lambda(1)\Delta$ is the martingale probability that default occurs at date $t = 2$, conditional upon the fact that default has not occurred at date $t = 1$.

From Table 18.1, we know that $v(0,1) = 0.9361$ and $\delta = 0.32$. Therefore,

$$0.9361 = \frac{1}{1.0639}\{[1 - \lambda(0)] + 0.32 \times \lambda(0)\}. \tag{18.5}$$

Solving for the martingale probability of default gives

$$(1 - 0.32) \times \lambda(0) = 1 - (0.9361 \times 1.0639)$$

or

$$\lambda(0) = 0.0059.$$

The pricing of the two-period zero-coupon bond is slightly more complicated because, at the end of the first period, both interest rates and the default status of the firm are uncertain.

The default process is shown in Figure 18.4. If default has occurred at date $t = 1$, the bond is assumed to remain in default. If default has not occurred at date $t = 1$, then one period later, at date $t = 2$, either default occurs or it does not. The martingale probability of default occurring at date $t = 2$, conditional upon the fact that default has not occurred at date $t = 1$, is $\lambda(1)\,\Delta$. The conditional (martingale) probability that default does not occur at date $t = 2$ is $1 - \lambda(1)\,\Delta$.

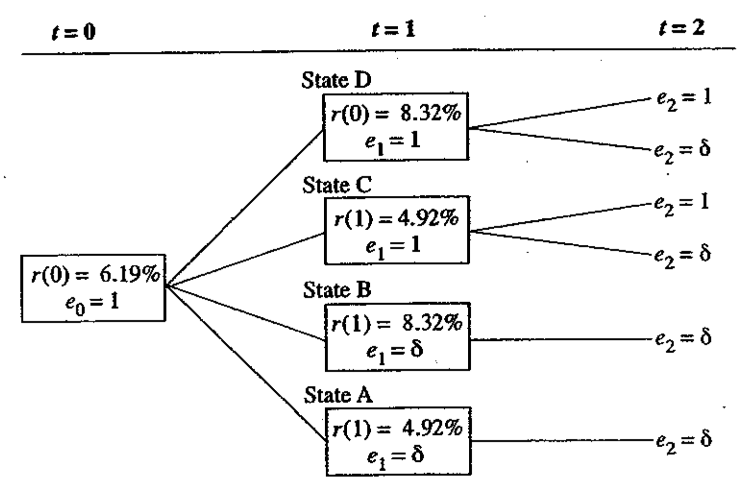
Combining Figure 18.4 with Figure 18.2 gives all the possible states, as shown in Figure 18.5. There are four possible states labeled A, B, C, and D, corresponding to all combinations of interest rates going up or down combined with default or no default. The martingale probabilities of each state occurring at date 1 are given as the multiplicative product of the separate probabilities of interest rates going up or down times the probabilities of default or no default. The multiplicative product is the implication of the assumption that the interest rate process and the bankruptcy process are statistically independent.

We use an analogous argument to determine the conditional martingale probability of default $\lambda(1)$ implicit in the bond prices.

Let us start at state A, at date $t = 1$. The value of a default-free bond that matures at $t = 2$ is

$$B(1,2) = \exp(-0.0492).$$

FIGURE 18.5 *Two-Period Credit-Risky Debt: Determining the Implied Probabilities*



State	Martingale probability of occurrence
A	$(1-\pi)\lambda(0)$
B	$\pi\lambda(0)$
C	$(1-\pi)[1-\lambda(0)]$
D	$\pi[1-\lambda(0)]$

For simplicity we set $\Delta = 1$ where the time interval is one year.

In state A, default has occurred at date $t = 1$, so the payoff to the risky bond at date $t = 2$ is

$$v(2,2) = \delta.$$

The value in state A at date $t = 1$, given that default has occurred, is

$$\frac{v(1,2)}{A(1)} = E^{\pi}\left[\frac{v(2,2)}{A(2)}\right].$$

Now from Figure 18.5, in state A we have that the value of the money market is

$$A(2) = A(1)\exp(0.0492).$$

Substitution yields

$$\begin{aligned} v_A(1,2) &= \frac{1}{\exp(0.0492)}\delta \\ &= B(1,2)_d\delta. \end{aligned}$$

The subscript d refers to the down-state for the default-free spot rate of interest.

A similar argument applies if state B occurs. In state B the firm has defaulted, so

$$\begin{aligned} v_B(1,2) &= \frac{1}{\exp(0.0832)}\delta \\ &= B(1,2)_u\delta, \end{aligned}$$

where

$$B(1,2)_u = \exp(-0.0832).$$

The subscript u refers to the up-state for the default-free spot rate of interest.

If state C occurs, the argument is more interesting. In state C, default has not occurred, so that one period later, at maturity, one of two possible states can occur:

$$v(2,2) = \begin{cases} 1 \text{ with probability } 1 - \lambda(1) & \text{if} \quad \text{no default} \\ \delta \text{ with probability } \lambda(1) & \text{if} \quad \text{default.} \end{cases}$$

Therefore, in state C the value of the bond is

$$v_C(1,2) = \frac{1}{\exp(0.0492)}\{[1-\lambda(1)] + \delta\lambda(1)\}$$
$$= B(1,2)_d\{[1-\lambda(1)] + \delta\lambda(1)\}.$$

In state D, a similar argument applies because the firm has not yet defaulted:

$$v_D(1,2) = \frac{1}{\exp(0.0832)}\{[1-\lambda(1)] + \delta\lambda(1)\}$$
$$= B(1,2)_u\{[1-\lambda(1)] + \delta\lambda(1)\}.$$

Today we know the value of the risky bond $v(0, 2) = 0.8703$ from Table 18.1. Now

$$\frac{v(0,2)}{A(0)} = E^{\pi}\left[\frac{v(1,2)}{A(1)}\right].$$

Referring to Figure 18.5, there are four possible states. Therefore,

$$v(0,2) = \frac{1}{\exp(0.0619)}\{(1-\pi)\lambda(0)v_A(1,2) + \pi\lambda(0)v_B(1,2)$$
$$+ (1-\pi)[1-\lambda(0)]v_C(1,2) + \pi[1-\lambda(0)]v_D(1,2)\}.$$

Substituting the previously computed values of the bond in the four different states gives

$$v(0,2) = \frac{1}{\exp(0.0619)}[(1-\pi)\exp(-0.0492) + \pi\exp(-0.0832)]\lambda(0)\delta$$
$$+ \frac{1}{\exp(0.0619)}[(1-\pi)\exp(-0.0492) + \pi\exp(-0.0832)]$$
$$\times [1-\lambda(0)]\{[1-\lambda(1)] + \lambda(1)\delta\}.$$

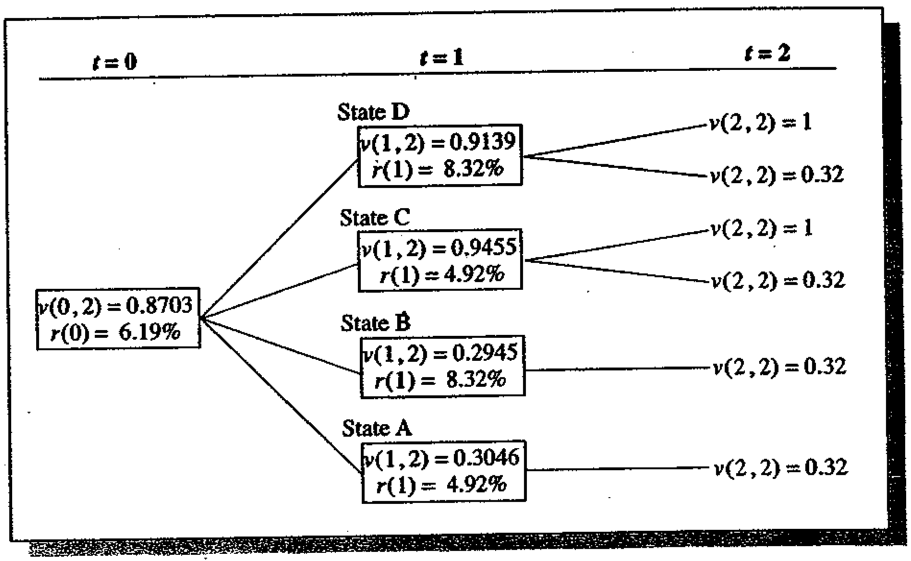
The above calculation can be simplified by considering the pricing of a two-period default-free zero-coupon bond. Consider the value of the default-free bond at $t = 1$:

$$B(1,2) = \begin{cases} \exp(-0.0832) & \text{with martingale probability } \pi \\ \exp(-0.0492) & \text{with martingale probability } 1-\pi. \end{cases}$$

The value of the default-free bond today at $t = 0$ is

$$B(0,2) = \frac{1}{\exp(0.0619)}[(1-\pi)\exp(-0.0492) + \pi\exp(-0.0832)].$$

Figure 18.6 *Two-Period Credit-Risky Debt: Summary of Results*



Therefore, substituting into the expression for $v(0,2)$ yields

$$v(0,2) = B(0,2)\{\lambda(0)\delta + [1 - \lambda(0)]([1 - \lambda(1)] + \lambda(1)\delta)\}. \tag{18.6}$$

This is the equation we use to solve for $\lambda(1)$.

From Table 18.1 we have $B(0,2) = 0.8798$, $v(0,2) = 0.8703$, and $\delta = 0.32$, and we have previously computed $\lambda(0) = 0.0059$. Substituting these values into Expression (18.6) and solving for $\lambda(1)$ gives

$$\lambda(1) = 0.0100.$$

The martingale probability of default at date 1 as implied by the bond prices, $\lambda(1)$, is seen to be almost twice that of default at date 0, $\lambda(0)$.

Given that $\lambda(0) = 0.0059$ and $\lambda(1) = 0.0100$, the pricing of the two-period credit-risky debt is summarized in Figure 18.6.

We leave it as an exercise for the reader, using the prices of the three-year bonds, to prove that

$$\lambda(2) = 0.0127. \quad \blacksquare$$

In summary, this example shows how to infer the martingale probabilities of default using the term structures for default-free bonds and credit-risky bonds.

Formalization

Here we formalize the preceding example. This involves little more than replacing numbers with symbols. However, this formalization provides additional insights into the pricing of credit-risky derivatives.

First we consider Expression (18.5). The value of a one-period, zero-coupon, default-free bond is given by

$$B(0,1) = \frac{1}{\exp(0.0619)}.$$

Thus, given no default at time 0, we can rewrite Expression (18.5) in the form

$$v(0,1) = B(0,1)\{[1 - \lambda(0)] + \delta\lambda(0)\}.$$

The default process is defined by Expression (18.3):

$$e_1 = \begin{cases} 1 \text{ with probability } 1 - \lambda(0)\,\Delta & \text{if} \quad \text{no default} \\ \delta \text{ with probability } \lambda(0)\,\Delta & \text{if} \quad \text{default,} \end{cases}$$

conditional upon no default at $t = 0$.

The expected value, conditional upon no default at $t = 0$, is

$$E_0^{\pi}[e_1 \mid \text{no default}] = [1 - \lambda(0)\,\Delta] + \delta[\lambda(0)\,\Delta].$$

In this section, for clarity of the exposition, we need to indicate via a subscript the date at which the expectation is computed. For this expectation, it is time 0, hence the zero subscript.

We can rewrite Expression (18.3) one last time as

$$v(0,1) = B(0,1)E_0^{\pi}[e_1 \mid \text{no default}]. \tag{18.7}$$

This is the generalized version of the risky zero-coupon bond's valuation formula. It states that the risky zero-coupon bond's date-0 price is the discounted expected payoff to the risky bond at date 1. The expectation is computed under the martingale probabilities.

A similar expression holds for the two-period credit-risky bond, for which (referring to Figure 18.5), if we are in either state A or state B, default has occurred and

$$e_2 = \delta.$$

Therefore, conditional on default having occurred at date $t = 1$,

$$E_1^{\pi}[e_2 \mid \text{default}] = \delta.$$

If we are in state C or state D, default has not occurred, so

$$e_2 = \begin{cases} 1 \text{ with probability } 1 - \lambda(1)\,\Delta & \text{if} \quad \text{no default} \\ \delta \text{ with probability } \lambda(1)\,\Delta & \text{if} \quad \text{default} \end{cases}$$

and the conditional expected value is

$$E_1^{\pi}[e_2 \mid \text{no default}] = [1 - \lambda(1)\,\Delta] + \delta[\lambda(1)\,\Delta].$$

Today, at $t = 0$, default has not occurred, so

$$e_1 = \begin{cases} 1 \text{ with probability } 1 - \lambda(0)\,\Delta & \text{if} \quad \text{no default} \\ \delta \text{ with probability } \lambda(0)\,\Delta & \text{if} \quad \text{default.} \end{cases}$$

Therefore, when viewed from date 0,

$$\begin{aligned} E_0^{\pi}[e_2 \mid \text{no default}] = {} & [1 - \lambda(0)\,\Delta]\,E_1^{\pi}[e_2 \mid \text{no default}] \\ & + [\lambda(0)\,\Delta]E_1^{\pi}[e_2 \mid \text{default}]. \end{aligned} \tag{18.8}$$

Using these general expressions, we can rewrite Expression (18.6) in the form

$$\begin{aligned} v(0,2) &= B(0,2)\{\lambda(0)\,\Delta E_1^{\pi}[e_2 \mid \text{default}] + [1 - \lambda(0)\,\Delta]E_1^{\pi}[e_2 \mid \text{no default}]\} \\ &= B(0,2)E_0^{\pi}[e_2 \mid \text{no default}]. \end{aligned} \tag{18.9}$$

This expression states that the date-0 price of the two-year, credit-risky, zero-coupon bond is its discounted expected payoff at date 2 under the martingale probabilities.

Expressions (18.7) and (18.9) are special cases of the general formulas

$$v(0,T) = B(0,T)E_0^{\pi}[e_T \mid \text{no default}] \quad \text{if} \quad \text{no default} \tag{18.10a}$$

and

$$v(0,T) = B(0,T)\delta \quad \text{if} \quad \text{default,} \tag{18.10b}$$

both for a risky zero-coupon bond of maturity T.

Expression (18.10a) gives the risky zero-coupon bond's value if the firm is not in default, and Expression (18.10b) gives the risky zero-coupon bond's value if the firm is in default. The value not in default, Expression (18.10a), is greater than the value in default; Expression (18.10b), because $E_0^{\pi}[e_T \mid \text{no default}] > \delta$. This follows because if default has not yet occurred, there is some positive probability that the firm will pay its promised face value of a dollar at the bond's maturity. An example helps to clarify these calculations.

EXAMPLE **Computations of Discounted Expected Payoffs (Continued)**

This example illustrates the computation of the discounted expected payoffs using Expressions (18.7) and (18.10a,b). Referring to the numbers in the previous example, we have

$$\lambda(0) = 0.0059,$$
$$\lambda(1) = 0.0100,$$

and

$$\lambda(2) = 0.0127.$$

To determine the value of a three-year, zero-coupon, credit-risky bond, we must first compute the expected value of the exchange rate at $t = 3$, conditional upon no default at $t = 2$:

$$\begin{aligned} E_2^\pi[e_3 \mid \text{no default}] &= [1 - \lambda(2)] + \delta\lambda(2) \\ &= (1 - 0.0127) + 0.32(0.0127) \\ &= 0.9914. \end{aligned}$$

If default has occurred at $t = 2$, then

$$E_2^\pi[e_3 \mid \text{default}] = \delta.$$

At $t = 1$, the expected value of the exchange rate at $t = 3$, conditional upon no default at $t = 1$, is given by

$$\begin{aligned} E_1^\pi[e_3 \mid \text{no default}] &= [1 - \lambda(1)]E_2^\pi[e_3 \mid \text{no default}] + \lambda(1)E_2^\pi[e_3 \mid \text{default}] \\ &= 0.9846. \end{aligned}$$

If default has occurred at $t = 1$, then

$$E_1^\pi[e_3 \mid \text{default}] = \delta.$$

At $t = 0$, the expected value of the exchange rate at $t = 3$, conditional upon no default at $t = 0$, is given by

$$\begin{aligned} E_0^\pi[e_3 \mid \text{no default}] &= [1 - \lambda(0)]E_1^\pi[e_3 \mid \text{no default}] + \lambda(0)E_1^\pi[e_3 \mid \text{default}] \\ &= 0.9807. \end{aligned}$$

Therefore, using Expressions (18.10a,b), the price of the three-year risky zero-coupon bond is

$$\begin{aligned} v(0,3) &= 0.8137 \times 0.9807 \\ &= 0.7980, \end{aligned}$$

which agrees with Table 18.1. ■

Interpretation of Expression (18.10a,b)

We pause to discuss the interpretation of Expression (18.10a,b), which is an important and intuitive result. It is important both because it provides a practical way of computing the martingale probabilities of default using market data and because it can be used for pricing derivatives on credit-risky cash flows. It is intuitive because the second term in Expression (18.10a), $E_0^{\pi}[e_T \mid \text{no default}]$, is the date-0 expected value of the promised payoff at date T. We can alternatively rewrite expression (18.10a) using the form

$$E_0^{\pi}[e_T \mid \text{no default}] = v(0,T)/B(0,T), \tag{18.11}$$

and the right side can be interpreted as a credit spread. This credit spread will prove useful in subsequent applications.

Value of the Claim in the Event of Default

We have assumed that if default occurs, bondholders will receive the fraction δ per promised dollar at the maturity of the bond. The present value of this payment is $\delta B(t,T)$, assuming that default has occurred by date t.

An alternative assumption is to assume that the present value of the claim to bondholders is proportional to the value of the bond just prior to default. This assumption is associated with the work of Duffie and Singleton (1997). It is a simple exercise to incorporate this assumption into our previous analysis. This proportionality assumption and others can be found in Jarrow and Turnbull (1999).

18.2 PRICING OPTIONS ON CREDIT-RISKY BONDS

We demonstrate in this section how to price options on credit-risky bonds. Consider an option written on the debt issued by *ABC* Company. We illustrate the analysis with a simple example, which we will then formalize.

EXAMPLE **Put Option**

This example illustrates the pricing of a put option on *ABC* debt. Consider a put option with a maturity of one year. At maturity, the option allows you to sell a one-year zero-coupon bond issued by the *ABC* firm for a strike price of 0.93.

To price this option, we follow the same risk-neutral valuation procedure that we have used throughout this text. For simplicity of exposition, we maintain our assumption that the length of the lattice interval is one year so that we can use all the results summarized in Figure 18.6. In practice, we would use intervals of shorter length.

We start by considering the value of the option at its maturity date. At the maturity of the option, there are four possible values for the underlying risky zero-coupon bond in states A, B, C, and D. The option values are shown in Figure 18.7. Pricing the put option at date 0 using the risk-neutral valuation procedure gives

$$\frac{p(0)}{A(0)} = E^{\pi}\left[\frac{p(1)}{A(1)}\right], \tag{18.12}$$

where $p(t)$ is the value of the put option at date $t = 0, 1$.

We next discuss how to evaluate Expression (18.12). First, the values of the money market account at $t = 0$ and $t = 1$ are

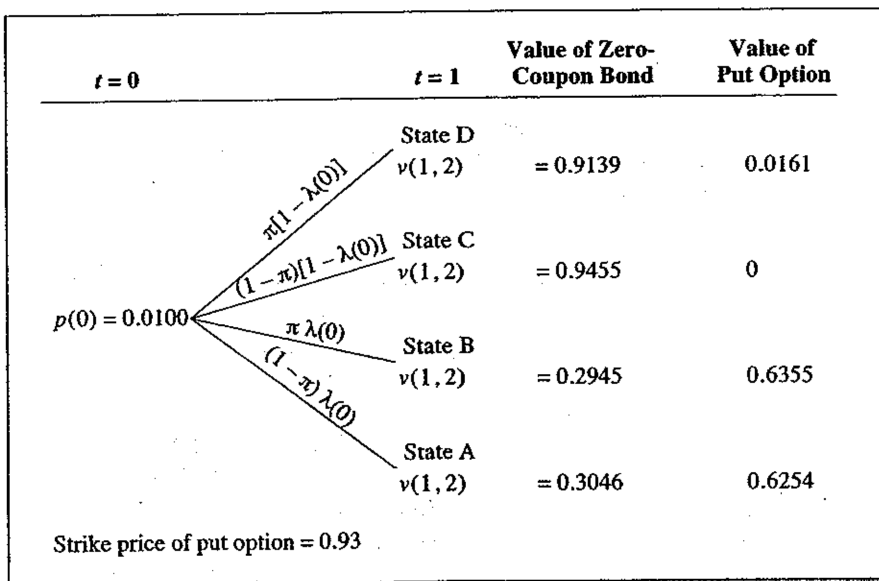
$$A(0) \equiv 1$$

and

$$A(1) = \exp(0.06198) = 1.0639.$$

Next, in states A and B at date 1, default has occurred on the underlying *ABC* zero-coupon bond. The value of the put option varies over these two states because the value of the underlying *ABC* zero-coupon bond varies due to the interest rate risk.

FIGURE 18.7 *Pricing a Put Option Written on Credit-Risky Debt*



In states C and D at $t = 1$, the underlying ABC bond is not in default.
The date-0 value of the option based on Expression (18.12) is

$$p(0) = \frac{1}{1.0639}\{[1 - \lambda(0)][\pi 0.0161 + (1 - \pi)0] + \lambda(0)[\pi 0.6355 + (1 - \pi)0.6254]\}. \qquad (18.13)$$

Given that $\pi = 0.5$ and $\lambda(0) = 0.0059$, then

$$p(0) = \frac{1}{1.0639}\{[1 - \lambda(0)]0.0805 + \lambda(0)0.6292\} = 0.0110. \quad \blacksquare$$

Formalization

Here we formalize the preceding analysis. This involves little more than replacing the numerical values in the previous example with symbols.

We can write Expression (18.12) in the form

$$\begin{aligned} p(0) &= [1 - \lambda(0)][\pi p(1, U; \text{no default}) + (1 - \pi)p(1, D; \text{no default})]/A(1) \\ &\quad + \lambda(0)[\pi p(1, U; \text{default}) + (1 - \pi)p(1, D; \text{default})]/A(1) \\ &= [1 - \lambda(0)][\text{Value of option, given default did not occur}] \\ &\quad + \lambda(0)[\text{Value of option, given default occurs}]. \end{aligned} \qquad (18.14)$$

The preceding expression readily generalizes, which is important because it implies that we can always separate the problem of pricing an option on credit-risky debt into two simpler problems. First, we can price an option assuming that default does not occur over the life of the option. Second, we can price an option given that default does occur. Then, we take an expectation across these two values.

These two more simple problems can be solved using the methods described in Chapter 15. Indeed, if the debt never defaults, the debt is default-free and the techniques of Chapter 15 apply. If the debt defaults, there is no more default risk. Thus, after default, the debt is default-free again and the techniques of Chapter 15 apply. In the latter case, however, the payoff is $\delta < 1$ and not 1.

Hedging

Here we study the hedging of options on risky debt. To hedge and/or to construct a synthetic option, we employ the same procedures introduced in Chapter 5 for the binomial pricing model. The only difference is that instead of only two outcomes after each node in the lattice, now there are four. The modifications necessary to handle this difference are straightforward and best illustrated using the previous example.

EXAMPLE

Put Option (Continued)

This example illustrates constructing a synthetic option using risky debt and default-free debt.

To hedge the put option's values given in Figure 18.7, we need four assets because there are four possible outcomes given by states A, B, C, and D. We choose the two *ABC* zero-coupon risky bonds maturing at dates 1 and 2, and the two default-free zero-coupon bonds maturing at dates 1 and 2.

The initial cost of constructing this portfolio is

$$V(0) = n_1 v(0,1) + n_2 v(0,2) + n_3 B(0,1) + n_4 B(0,2),$$

where

n_1 equals the number of one-year *ABC* zero-coupon bonds,
n_2 equals the number of two-year *ABC* zero-coupon bonds,
n_3 equals the number of one-year default-free zero-coupon bonds, and
n_4 equals the number of two-year default-free zero-coupon bonds.

The prices of these bonds are contained in Table 18.1, so

$$V(0) = n_1(0.9361) + n_2(0.8703) + n_3(0.9399) + n_4(0.8798).$$

The holdings n_1, n_2, n_3, n_4 must be determined so that this portfolio's value at date 1 matches the put option's values across all possible states at date 1. That is,

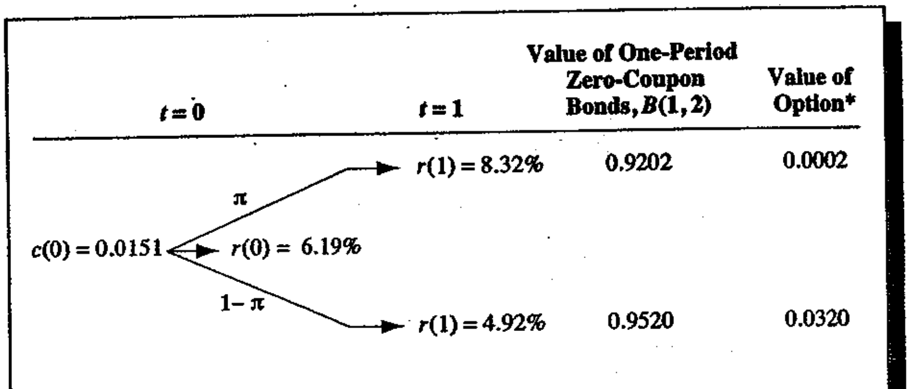
$$\begin{aligned}
&\text{(state D)} \quad n_1 1 + n_2(0.9139) + n_3 1 + n_4(0.9201) = 0.0161 \\
&\text{(state C)} \quad n_1 1 + n_2(0.9455) + n_3 1 + n_4(0.9520) = 0 \\
&\text{(state B)} \quad n_1(0.32) + n_2(0.2945) + n_3 1 + n_4(0.9201) = 0.6355 \\
&\text{(state A)} \quad n_1(0.32) + n_2(0.3046) + n_3 1 + n_4(0.9520) = 0.6254.
\end{aligned}$$

We obtained the values for the prices in these equations as follows. The date-1 values for $v(1,1)$ are obtained from the definition of their payoff at maturity, with $\delta = 0.32$ in default. The date-1 values for $v(1,2)$ are obtained from Figure 18.7. The date-1 values of $B(1,1)$ are always 1. Finally, the date-1 values for $B(1,2)$ are available in Figure 18.8.

The solutions to these equations are

$$\begin{aligned}
n_1 &= -0.6567 \\
n_2 &= -0.2791 \\
n_3 &= 1.1379 \\
n_4 &= -0.2283.
\end{aligned}$$

Figure 18.8 *Pricing a Default-Free Treasury Bill Call Option*



*Strike price is 0.92, where $\pi = 0.5$ represents the martingale probability of an up-jump in spot interest rates.

These holdings of the four bonds give the synthetic put option. The initial cost of the synthetic put is

$$V(0) = -0.6567(0.9361) - 0.2791(0.8703) + 1.1379(0.9399) \\ - 0.2283(0.8798) \\ = 0.0110,$$

which equals the value of the put option obtained from Expression (18.13), as is to be expected. ■

18.3 PRICING VULNERABLE DERIVATIVES

We examine here the pricing of vulnerable derivatives. **Vulnerable derivatives** are derivative securities subject to the additional risk that the writer of the derivative might default. For example, consider an over-the-counter (OTC) option written by a financial institution on a Treasury bill. There is no default risk associated with the underlying asset—the Treasury bill. However, the writer of the option, a financial institution, may default, so there is the risk that if the option is exercised, the writer may be unable to fulfill the obligation to make the required payment. The previous methodology can handle this situation. A simple example is used to illustrate the procedure.

EXAMPLE

Valuing a Vulnerable Call Option

This example illustrates the procedure for valuing a vulnerable call option. Initially, let's assume there is no risk of the writer defaulting.

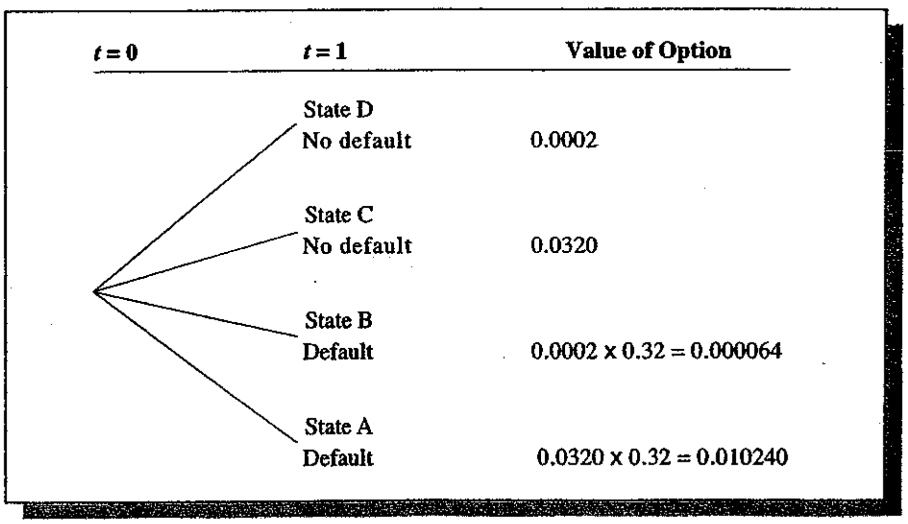
Consider a call option written on a Treasury bill. The maturity is one year, and at expiration the option holder can purchase a one-year Treasury bill at a strike price of 0.92. The option is valued using the information summarized in Figure 18.8. The lattice of interest rates comes from Figure 18.5.

The date-0 value of the call option is

$$\begin{aligned} c(0) &= \frac{1}{A(1)}[\pi \times 0.0002 + (1-\pi) \times 0.0320] \\ &= \frac{1}{1.0639}[0.5 \times 0.0002 + 0.5 \times 0.0320] \\ &= 0.0151. \end{aligned}$$

Now assume that the financial institution writing the option belongs to the ABC credit class. Consequently, we can use the results summarized in Figure 18.6 for the financial institution. When the option matures, there are four possible states, A, B, C, and D, depending on whether interest rates go up or down and whether the writer defaults. The four states are shown in Figure 18.9, which is similar in nature to Figure 18.7.

FIGURE 18.9 *Pricing a Vulnerable Call Option*



In states A and B, the writer defaults. By assumption, claimholders receive as the payoff 32 percent of the value of their option in these states. In states C and D, the writer does not default, and the entire payment is received.

The value of the vulnerable option today is the discounted expected value of these payments, that is,

$$c_V(0) = \frac{1}{1.0639}\{[1-\lambda(0)][\pi 0.0002 + (1-\pi)0.0320] + \lambda(0)[\pi 0.0002 + (1-\pi)0.0320] \times 0.32\}. \tag{18.15}$$

Given that $\pi = 0.5$ and $\lambda(0) = 0.0059$,

$$\begin{aligned} c_V(0) &= [1-\lambda(0)]0.0151 + \lambda(0)0.0048 \\ &= 0.0150. \end{aligned}$$

The difference in these option prices paid to a default-free writer versus a writer in credit class ABC is small—only 0.0001—which is to be expected given that the martingale probability of default is small. ■

Formalization

Now we formalize the analysis in the previous example, which again essentially involves replacing numbers with symbols.

Let $c(1)$ represent the date $t = 1$ value of the option in the absence of the writer defaulting, and let $c_V(1)$ represent the value of a vulnerable option.

At maturity, the payoff to the option is

$$c_V(1) = \begin{cases} c(1) & \text{if} \quad \text{no default} \\ \delta c(1) & \text{if} \quad \text{default,} \end{cases}$$

where δ represents the fractional payoff the holder receives if default occurs.

The date-0 value of the option in the absence of default is

$$c(0) = E^{\pi}\left[\frac{c(1)}{A(1)}\right].$$

Using Expression (18.15), the value of the vulnerable option is

$$\begin{aligned} c_V(0) &= [1-\lambda(0)]c(0) + \lambda(0)\delta c(0) \\ &= E^{\pi}(e_1)c(0) \end{aligned} \tag{18.16}$$

because $E^{\pi}(e_1) = [1-\lambda(0)] + \lambda(0)\delta$.

This expression says that the value of a vulnerable option is the value of an option written by a default-free entity times the expected payoff from the option writer at the maturity date of the option.

This result has an important implication. Given that there is a positive probability of default, the expected payoff is

$$E^{\pi}(e_1) < 1,$$

which implies that a vulnerable option must always be worth less than a non-vulnerable option, that is,

$$c_V(0) < c(0). \tag{18.17}$$

Expression (18.16) holds for any European option that matures at date T:

$$c_V(0) = E^{\pi}(e_T)c(0). \tag{18.18a}$$

Using Expression (18.11), Expression (18.18a) can be rewritten in the form

$$c_V(0) = [v(0,T)/B(0,T)]c(0). \tag{18.18b}$$

This form of the expression is useful in practice because it involves pricing a vulnerable option in terms of a credit risk spread for the writer—the term inside the square bracket on the right side—and the price of a non-vulnerable option.

EXAMPLE

Pricing a Vulnerable Option

This example illustrates the use of Expression (18.18b).

Consider a firm that wants to buy a five-year interest rate cap on the six-month default-free interest rate. Three institutions offer to sell the firm a cap. The institutions, however, have different credit ratings. Institution A belongs to credit class A, Institution B belongs to credit class B, and Institution C belongs to credit class C. Credit class A has a lower risk of default than credit class B, and credit class B has a lower risk of default than credit class C.

The term structure details are given in Table 18.2, Part A, for default-free interest rates and the three credit classes. The value of the caplets, assuming no counterparty risk, is calculated using the program Bonds/Caps/Normal. The prices of the caplets are given in Table 18.2, Part B.

To incorporate the effects of counterparty risk, we use Expression (18.18b). Consider the last caplet. The value in the absence of counterparty risk is \$15,620.

For institution A belonging to credit class A, using the figures from the last row of Table 18.2, Part A,

$$\begin{aligned} v_A(0,4.5)/B(0,4.5) &= 0.753875/0.775249 \\ &= 0.9724. \end{aligned}$$

TABLE 18.2 *Pricing a Vulnerable Cap*

PART A
TERM STRUCTURE DATA

MATURITY (YEARS)	DEFAULT-FREE × 100	CREDIT CLASS A × 100	CREDIT CLASS B × 100	CREDIT CLASS C × 100
0.5	97.4892	97.2260	97.1870	96.1244
1.0	94.9635	94.4443	94.3320	92.3261
1.5	92.4320	91.6633	91.4518	88.6109
2.0	89.9028	88.8909	88.5610	84.9835
2.5	87.3835	86.1342	85.6727	81.6727
3.0	84.8806	83.3999	82.7984	78.0064
3.5	82.3998	80.6939	79.9480	74.9480
4.0	79.9464	78.0215	77.1301	71.4161
4.5	77.5249	75.3875	74.3521	68.2698

PART B
PRICING THE CAPLETS

MATURITY (YEARS)	VALUE OF CAPLET*	CREDIT CLASS A	CREDIT CLASS B	CREDIT CLASS C
0.5	80	79.78	79.75	78.88
1.0	1,050	1,044.26	1,043.02	1,020.84
1.5	2,890	2,865.97	2,859.35	2,770.53
2.0	5,120	5,062.37	5,043.58	4,839.84
2.5	7,450	7,343.49	7,304.14	6,963.12
3.0	9,730	9,560.26	9,491.31	8,942.00
3.5	11,880	11,634.05	11,526.51	10,805.64
4.0	13,840	13,506.77	13,352.45	12,363.27
4.5	15,620	15,189.35	14,980.73	13,755.25
Total	67,680	66,286.30	65,680.84	61,539.37
Difference		1,393.70	1,999.16	6,140.63
		2.06%	2.95%	9.07%

*Volatility	1.0 percent
Volatility Reduction Factor	0.1
Cap Rate	7.00 percent
Principal	$10 million

Therefore, using Expression (18.18b), the value of the caplet is

$$
\begin{aligned}
&\$15,620 \times 0.9724 \\
&= \$15,189.35,
\end{aligned}
$$

as shown in Table 18.2, Part B.

The values of the other caplets are calculated in a similar way. The credit risk of Institution A lowers the value of the cap by approximately 2 percent, for Institution B, 2.95 percent, and for Institution C, 9.07 percent. ■

Hedging

Let us examine hedging vulnerable options. To hedge and/or construct a synthetic vulnerable option we employ the same procedure introduced in Chapter 5 for the binomial pricing model. The only difference is that instead of there being two outcomes after each node in the tree, there are now four. The modifications necessary to handle this difference are straightforward and best illustrated using the preceding example.

EXAMPLE **Hedging a Vulnerable Option**

Consider the vulnerable call option example at the start of this section. To hedge the call option's values given in Figure 18.9 we need four assets, because there are four possible outcomes given in states A, B, C, and D. We choose the two risky zero-coupon bonds maturing at dates 1 and 2, and the two default-free zero-coupon bonds maturing at dates 1 and 2.

The initial cost of constructing this portfolio is

$$V(0) = n_1 v(0,1) + n_2 v(0,2) + n_3 B(0,1) + n_4 B(0,2),$$

where

n_1 equals the number of one-year ABC zero-coupon bonds,
n_2 equals the number of two-year ABC zero-coupon bonds,
n_3 equals the number of one-year default-free zero-coupon bonds, and
n_4 equals the number of two-year default-free zero-coupon bonds.

The prices of these bonds are contained in Table 18.1, so

$$V(0) = n_1 (0.9361) + n_2(0.8703) + n_3(0.9399) + n_4(0.8798).$$

The holdings n_1, n_2, n_3, n_4 must be determined so that this portfolio's date-1 value matches the vulnerable call's date-1 values across all possible states, that is,

$$\begin{aligned}
&\text{(state D)} \quad n_1 1 + n_2(0.9139) + n_3(1) + n_4(0.9201) = 0.0002\\
&\text{(state C)} \quad n_1 1 + n_2(0.9455) + n_3(1) + n_4(0.9520) = 0.0320\\
&\text{(state B)} \quad n_1(0.32) + n_2(0.2945) + n_3(1) + n_4(0.9201) = 0.000064\\
&\text{(state A)} \quad n_1(0.32) + n_2(0.3046) + n_3(1) + n_4(0.9520) = 0.010240.
\end{aligned}$$

We obtained the prices in these equations as follows. The date-1 values for $v(1,1)$ are obtained from the definition of their payoff at maturity, with $\delta = 0.32$ in default. The date-1 values for $v(1,1)$ are obtained from Figure 18.7. The date-1 values of $B(1,1)$ are always 1. Finally, the date-1 values for $B(1,2)$ are available in Figure 18.8.

The solutions to these equations are

$$n_1 = -0.9160$$
$$n_2 = 1.0058$$
$$n_3 = -0.0036$$
$$n_4 = 0.0006.$$

These holdings give the synthetic vulnerable call option portfolio.

The initial cost of the synthetic vulnerable call is

$$\begin{aligned} V(0) &= -0.9160(0.9361) + 1.0058(0.8703) - 0.0036(0.9399) \\ &\quad + 0.0006(0.8798) \\ &= 0.0150, \end{aligned}$$

which agrees with the value given earlier by Expression (18.15), as it should. ■

Risk Management

The pricing of counterparty risk is important for risk management. From this pricing one can compute the reserve that should be set aside to account for the dollar value of counterparty risk. The preceding methodology provides a simple, robust approach to measure this reserve.

18.4 VALUATION OF A SWAP

Here we study the valuation of a swap where the counterparties have different credit risk.

Consider a simple interest rate swap for which we are receiving fixed payments $\overline{R}$ from a firm in a given credit class, say, the ABC class. In return, we are making floating rate payments. Payments are due at dates $t_1, t_2, \ldots, t_N$, where the subscript N denotes the total number of payments.

To price this swap, we must price each side of the swap separately. First we value the fixed payments, and then we value the floating payments.

Fixed Payment Side

The present value of an annuity of fixed payments reflects the credit risk of default from a firm in class ABC. The present value of the payments is

$$\begin{aligned} &v(0,t_1)\overline{R} + v(0,t_2)\overline{R} + \ldots + v(0,t_N)\overline{R} \\ &\equiv v_{\overline{R}}(0). \end{aligned} \tag{18.19}$$

Each fixed payment of $\overline{R}$ is discounted via the rate appropriate for the firm in class ABC promising the payment.

TABLE 18.3 *Value of Floating Rate Payments*

DATE	VALUE AT $t = 0$
t_1	$1 - B(0,t_1)$
t_2	$B(0,t_1) - B(0,t_2)$
.	.
.	.
.	.
t_N	$B(0,t_{N-1}) - B(0,t_N)$
SUM	$1 - B(0,t_N)$

Floating Payment Side

For reasons of simplicity, we assume that we are default-free and that we make floating rate payments using the Treasury rate as the reference interest rate. In practice, the LIBOR interest rate is used.

The value of the floating rate payments is calculated using the results from Chapter 14 and is summarized in Table 18.3.

From Table 18.3, the total value of the floating rate payments is

$$V_F(0) = 1 - B(0,t_N). \tag{18.20}$$

The value of the swap is the value of the fixed payments less the floating payments, that is,

$$\begin{aligned} V_S(0) &\equiv v_{\bar{R}}(0) - V_F(0) \\ &= v_{\bar{R}}(0) - [1 - B(0,t_N)]. \end{aligned} \tag{18.21}$$

When the swap is initiated, the swap rate $\bar{R}$ is set such that the value of the swap is zero. An example will clarify these expressions.

EXAMPLE

Calculating the Swap Rate

This example illustrates the valuation of a swap using Expression (18.21). The swap rate is calculated by equating the present value of the floating rate payments with the present value of the fixed rate payments.

Using the term structure data in Table 18.4, we want to determine the swap rate for a two-year swap.

The valuation of the floating rate payments is given by

$$\begin{aligned} V_F(0) &= [1 - B(0,t_N)]100 \\ &= 100 - 90.7214 \\ &= 9.2786. \end{aligned}$$

TABLE 18.4 *Determination of Swap Rate*

MATURITY (YEARS) T	DEFAULT-FREE $B(0, T) \times 100$	CREDIT CLASS ABC $v(0, T) \times 100$
0.5	97.7272	97.2508
1.0	95.4164	94.4826
1.5	93.0780	91.7064
2.0	90.7214	88.9320

assuming a notional principal of $100.

The valuation of the fixed rate payments using Table 18.4 is given by

$$(\overline{R}/2) \times (97.2508 + 94.4826 + 91.7064 + 88.9320) = \overline{R} \times 186.19,$$

where $\overline{R}$ is the annual swap rate.

The swap rate is defined to be that rate $\overline{R}$ that satisfies

$$9.2786 = \overline{R} \times 186.19,$$

implying

$$\overline{R} = 4.98\%. \quad \blacksquare$$

Note that a number of implicit assumptions are used in determining the swap rate. The swap rate assumes that if default occurs on the fixed side, payments continue on the floating rate side. This is because the floating payment side is valued at the default-free rate, which assumes that the payments occur with certainty.

On the other hand, in default, the fixed rate side pays δ for each promised payment over the life of the swap. It might be argued that it makes sense for default to occur by the counterparty only if the present value of the fixed payments is greater than the present value of the floating rate payments. If the swap is viewed in isolation, this argument makes sense. However, default can occur for many different reasons. If we have many contracts with this counterparty, default on one contract may trigger default on all the contracts, which is the case if "netting" is allowed. The issue of netting is discussed later in this chapter.

18.5 CREDIT DEFAULT SWAPS

Here we show how to use the martingale approach to price a **simple credit default swap**. We start by describing a credit default swap and then show how to price it.

Let us consider a one-year credit default swap. The basic structure is shown in Figure 18.10. In this credit default swap, the bank is buying default protection from the counterparty. The counterparty's default exposure is to two companies. These companies are specified in the contract. These companies are usually referred to as reference credits. After the first default by one of the reference credits, the counterparty's exposure to subsequent defaults is terminated. In the event of a default by one of the two reference credits, the counterparty pays a fixed amount to the bank. In return for this default insurance, the bank pays a premium to the counterparty when the swap is initiated. We will show how to determine this premium.

To illustrate how to price this form of credit default swap, a simple numerical example will be considered. The tenor of the swap is assumed to be one year. For simplicity, we divide the one year into two six-month intervals. In practice, shorter intervals would be used.

The counterparty is assumed to belong to credit class A and the two reference credits belong to credit class C, which is a lower credit class than A. We summarize the martingale default probabilities in Table 18.5.

Conditional on no defaults by the two reference credits at date $t-1$, payment by the counterparty at date t is described by one of four mutually exclusive and exhaustive events:

1. first credit defaults, second credit does not default,
2. first credit does not default, second credit defaults,
3. first credit defaults, second credit defaults, or
4. first credit does not default, second credit does not default.

If one of the first three events occur, the counterparty makes a fixed payment, F, to the bank. If event four happens, no payment occurs.

The probability that the first (second) credit does not default at date t, conditional upon no default at date $t-1$, is $[1-\lambda_c(t-1)\Delta]$, where $\lambda_c(t-1)$ is the (martingale) conditional probability of default occurring at date t for a firm in credit class

FIGURE 18.10 *A Simple Credit Default Swap*

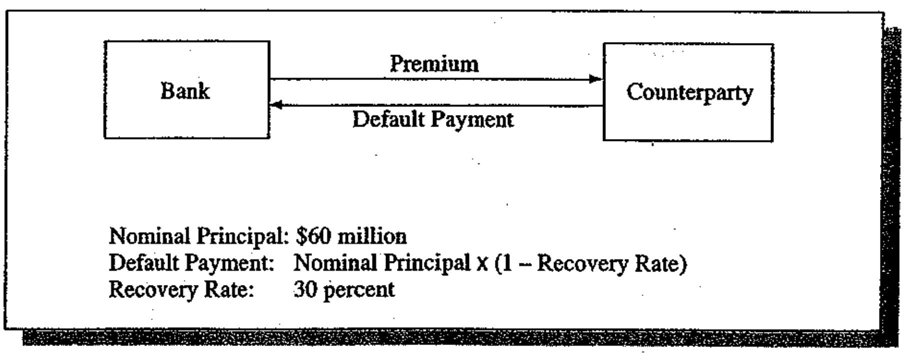


Table 18.5 *Term Structure Data*

		Credit Class A*		Credit Class C	
Maturity (Years)	Default-Free* × 100	Price × 100	Martingale Probabilities of Default	Price × 100	Martingale Probabilities of Default
0.5	97.4892	97.2260	0.01	96.1244	0.0412
1.00	94.9635	94.4443	0.0103	92.3261	0.0413
Recovery Rates		0.46		0.32	

*This data comes from Table 18.2.

C, conditional upon no default at date $t-1$ and where Δ is the length of the interval between dates $t-1$ and t.

Assuming independence between the event of default for the first credit and the second credit, the conditional probability of event four occurring is $[1-\lambda_c(t-1)\Delta]^2$. Recall that both reference credits are in credit class C. This independence assumption is imposed for simplicity. Basically, it states that the defaults in the two companies would be caused by idiosyncratic events, that is, events that are not related. This assumption may be easily relaxed.

To summarize the payment by the counterparty to the bank, it will prove useful if we define the following indicator function. Conditional upon no default at date $t-1$, we define

$$\mathbf{1}(t) \equiv \begin{cases} 0 & \text{with probability } [1-\lambda_c(t-1)\Delta]^2 \\ 1 & \text{with probability } 1-[1-\lambda_c(t-1)\Delta]^2. \end{cases} \tag{18.22}$$

If $\mathbf{1}(t)=0$ at date t, it implies event four has occurred and no payment is made by the counterparty to the bank; if $\mathbf{1}(t)=1$ at date t, it implies that either event one, two, or three has occurred and the counterparty makes a payment, F, to the bank.

If a default has occurred at or prior to date $t-1$, we define

$$\mathbf{1}(t) \equiv 0, \tag{18.23}$$

implying that the counterparty's exposure is terminated.

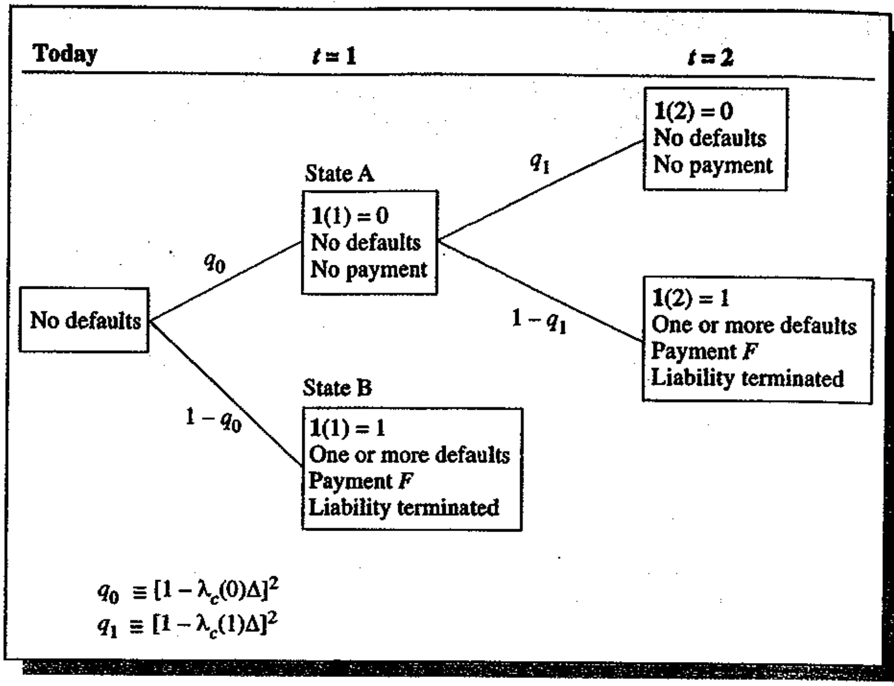
In this example, the credit swap has a maturity of one year. The default payment process over the two intervals is shown in Figure 18.11.

Referring to Figure 18.11, if no defaults have occurred at date $t=1$, state A, the value of the swap is

$$V_A(1) \equiv B(1,2)[0 \times q_1 + F \times (1-q_1)], \tag{18.24}$$

where $q_1 \equiv [1-\lambda_c(1)\Delta]^2$.

FIGURE 18.11 *Default Payment Process*



If one or more defaults occur at date $t = 1$, state B, then

$$V_B(1) \equiv F. \tag{18.25}$$

Today the value of the swap is

$$V(0) = B(0,2)q_0[0 \times q_1 + F \times (1 - q_1)] + B(0,1)(1 - q_0)F, \tag{18.26}$$

where $q_0 \equiv [1 - \lambda_c(0)\Delta]^2$.

Using the data in Table 18.5 gives

$$q_0 = [1 - 0.0412 \times 0.5]^2 = 0.95922$$
$$q_1 = [1 - 0.0413 \times 0.5]^2 = 0.95913,$$

so that

$$\begin{aligned} V(0) &= 0.9496 \times 0.95922[0 + F(1 - 0.95913)] + 0.9749 \times (1 - 0.95922)F \\ &= F(0.0770). \end{aligned}$$

This calculation determines the value of the swap in terms of the default payment F. Note that the analysis assumes that the counterparty does not default. This assumption can be relaxed using the analysis given in Section 18.3. Our discussion of credit default swaps is now complete.

18.6 REGULATION

We now discuss the current regulatory issues involving derivative securities. The risks arising from a portfolio of derivative securities include (1) market or price risk, (2) liquidity risk, (3) credit risk, (4) clearing or settlement risk, (5) operations and systems risk, and (6) legal risk.[6] We discuss each of these risks in turn.

1. **Market or Price Risk** is the risk from unanticipated changes in the value of a financial instrument or portfolio of financial instruments. This includes the risk arising from model misspecification.
2. **Liquidity Risk** is the risk of being unable to close out open positions within a reasonable time and in sufficient quantities at a reasonable price.
3. **Credit Risk** is the risk that a counterparty defaults on its obligations.
4. **Clearing or Settlement Risk** is the risk that technical difficulties interrupt delivery or settlement despite the counterparty's ability or willingness to perform. This type of risk also occurs when a counterparty defaults.
5. **Operations and Systems Risk** is the risk of human error; fraud; inadequate internal controls; or that the systems will fail to fully record, monitor, and account for transactions or positions.
6. **Legal Risk** is the risk that an action by a court or regulatory body will invalidate a contract.[7]

These risks are not unique to derivatives, being common to many financial activities.

Regulators of the financial system act to ensure that the management of financial institutions act prudently by not assuming excessive risk and by having sufficient capital reserves. The Bank of International Settlement (B.I.S.)[8] in July 1988 issued a report containing proposals describing the minimum capital reserve requirements for banks.[9]

[6]This list combines and generalizes the factors identified in the Federal Reserve Board Trading Manual (1994) and the General Accounting Office Report (1994).

[7]The British House of Lords ruled on January 24, 1991 that the swap contracts entered into by the London Borough of Hammersmith and Fulham were unlawful. Because this rule affected all UK local authorities, it was estimated that international banks would lose around £600 million in the estimated closeout value of their swaps. See Shirreff (1991).

[8]The B.I.S. was established in 1930 in Basle, Switzerland, by Western European banks.

[9]Bank for International Settlements, "Proposals for International Convergence of Capital Adequacy Standards," July 1988. The report was issued by the Basle Committee on Banking Regulations and Supervisory Practices with the endorsement of the central bank governors of the Group of Ten countries (Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, the United Kingdom, and the United States of America).

The report states certain minimum capital standards for on- and off-balance-sheet items and assigns different capital requirements for different classes of assets. In July 1993, the Group of Thirty[10] issued a report[11] to improve the supervisory and capital requirements of the "off-balance-sheet" risks related to derivative activity for dealers and end users.[12] The United States General Accounting Office[13] in May 1994 issued a report on financial derivatives and the actions needed to protect the financial system.[14] The B.I.S. 1988 Accord was extended by the 1996 Amendment that became mandatory in 1998. We start by discussing the 1988 Accord and then the 1996 Amendment.[15]

Bank of International Settlement (B.I.S.) 1988 Accord

The B.I.S. reserve requirements start by defining two forms of capital. Tier One Capital is defined as common stockholders' equity, non-cumulative perpetual preferred stock, and minority equity interests in consolidated subsidiaries, less goodwill.[16] Tier Two Capital is defined as cumulative perpetual preferred shares, subordinated debt with an original average maturity of at least five years, and other forms of long-term capital.

Given these definitions, there are two forms of capital requirements. The first states that

$$\frac{\text{Tier One Capital}}{\text{Risk-Weighted Amount}} > 0.08$$

TABLE 18.6 *Risks Weights for On-Balance-Sheet Asset Category*

RISK WEIGHTS (%)	ASSET CATEGORY
0	Cash, gold bullion, claims on OECD Treasury bonds
20	Claims on OECD banks and OECD public sector entities
50	Uninsured residential mortgages
100	Corporate bonds, equity, real estate

[10] The Group of Thirty is an international financial policy organization whose members include representatives of central banks, international banks, and securities firms.

[11] "Derivatives: Practices and Principles," a report prepared by the Global Derivatives Study Group, the Group of Thirty, Washington, D.C., July 1993.

[12] End users typically enter into derivative transactions for many reasons, such as to reduce financing costs and/or alter their risk exposure. Dealers or intermediaries cater to the needs of end users by making markets in over-the-counter derivatives.

[13] The General Accounting Office is a branch of the U.S. Government.

[14] General Accounting Office, *Financial Derivatives*, May 1994, Washington, D.C.

[15] For a detailed description see Crouhy, Galai, and Mark (1998a).

[16] Tier One Capital includes only permanent shareholder equity (issued and fully paid ordinary shares/common stock and perpetual non-cumulative preference shares) and disclosed reserves (share premiums, retained profits, general reserves, and legal reserves).

TABLE 18.7 *Add-on Factors**

MATURITY	INTEREST RATE	EXCHANGE RATE AND GOLD	EQUITY	PRECIOUS METALS EXCEPT GOLD	OTHER COMMODITIES
One year or less	0	1.0	6.0	7.0	10.00
Over one year to five years	0.5	5.0	8.0	7.0	12.0
Over five years	1.5	7.5	10.0	8.0	15.0

*Expressed as percentages.

and the second states that

$$\frac{\text{Tier Two Capital}}{\text{Risk-Weighted Amount}} > 0.08.$$

It is required that at least 50 percent of capital must be Tier One Capital. The risk-weighted amount for on-the-balance-sheet items is defined as the sum of principal × risk weight for each asset. The B.I.S. defines the risk capital weights (see Table 18.6 for a partial list). For example, the risk weight for commercial loans is 1.00, 0.5 for residential mortgages, and 0.0 for Treasury bills.

For off-balance-sheet items, the determination is more complicated. We start by defining a number of terms.

Current exposure

The B.I.S. first calculates the current mark-to-market value of a deal and then defines the **current exposure** as follows:

$$\text{Current Exposure} \equiv \text{Max}\{\text{Mark-to-Market Value}, 0\}.$$

Add-on amount

The **add-on amount** is computed by multiplying the notional amount of the asset by the B.I.S.-required add-on factor, as shown in Table 18.7. Note that there are five categories—interest rate, exchange rate and gold, equity, precious metals except gold, and other commodities. The interest rate category includes single-currency interest rate swaps, forward rate agreements, interest rate futures, and interest rate options (purchased). The exchange rate and gold category includes foreign currency swaps, forward rate agreements, futures, and options (purchased).

In 1995, the initial B.I.S. Accord was modified to allow banks to reduce their credit equivalent when bilateral agreements are in place.[17]

[17]Details are given in Crouhy, Galai, and Mark (1998b).

Credit equivalent amount

The **credit equivalent amount** equals the sum of the current exposure and the add-on amount, that is,

Credit Equivalent Amount = Current Exposure + Add-on Amount.

Risk-weighted amount

The **risk-weighted amount** is the product of the credit equivalent amount and the **counterparty credit weight**, that is,

Risk-Weighted Amount ≡ Credit Equivalent Amount
× Counterparty Credit Weight.

Table 18.8 provides a partial list of the counterparty credit weights.

The risk-weighted amount is defined to equal the sum of the risk-weighted amount for the on-the-balance-sheet assets plus the risk-weighted amount for the off-balance-sheet items.

The 1998 Accord completely ignores capital adequacy for marketable securities in the trading book.[18] This issue is addressed in the 1996 Accord.

B.I.S. 1996 Amendment

In the 1996 Amendment, a bank must hold capital to cover the market risk associated with debt and equity positions in the trading book and foreign exchange and commodity positions in both the trading and banking books. Market risk is measured for on- and off-balance-sheet traded instruments. On-balance-sheet instruments are subject only to market risk capital charges. Off-balance-sheet instruments are subject to market risk and credit capital charges.

TABLE 18.8 *Risk Weights for Off-Balance-Sheet Credit Equivalents by Type of Counterparty*

RISK WEIGHTS (%)	TYPE OF COUNTERPARTY
0	OECD governments
20	OECD banks and public sector entities
50	Corporate counterparties

[18]The trading book means the bank's proprietary positions in financial instruments, whether on- or off-balance-sheet, that are intentionally held for short-term trading. All trading book positions must be marked-to-market or marked-to-model every day.

A bank's overall capital requirements will be the sum of the following:

1. The credit risk capital charges described by the 1988 Accord. They apply to all the positions in the trading and banking books, over-the-counter derivatives, and off-balance sheet commitments *but* exclude debt and equity traded securities in the trading book and *all* positions in commodities and foreign exchange.
2. Market risk capital charges for the debt and equity positions in the trading book and foreign exchange and commodity positions in the trading and banking books.

Market risk is defined to cover general market risk and specific risk. General market risk refers to the risk arising from general market movements, or systematic risk. Examples of general market risk are changes in the S&P 500 index, commodity prices, foreign exchange rates, and interest rates. Specific risk refers to unanticipated changes in the market value of individual positions due to factors other than general market movements. Examples of specific risk are unanticipated changes in credit quality and liquidity and events unique to the individual asset.

The Amendment also gives banks the choice, subject to certain conditions, to develop their own internal models to assess market risks or to follow a standardized approach. This means that sophisticated institutions that already have an independent risk management division in place will have the choice between using their own internal value-at-risk (VAR) model under the internal models approach or following the standardized approach. The new capital requirement related to market rates should be offset by the fact that it will no longer be necessary to cover credit risk using the 1998 Accord for debt and equity traded securities in the trading book and all positions in commodities and foreign exchange. The internal models approach will generate greater savings in capital than the standardized models approach. However, the costs of developing an internal VAR model and maintaining it to the required standard to gain regulatory approval are not insubstantial.[19]

The Amendment introduces a third tier of capital to meet market risk requirements. Tier Three Capital consists of short-term subordinated debt with an original maturity of at least two years. It must be unsecured and fully paid up. It is subject to lock-in clauses that prevent the issuer from repaying the debt before maturity. Tier Three Capital cannot support capital requirements arising from the banking book.

Limitations of the 1988 Accord and 1996 Amendment

The Amendment does not alter the main rules applying to credit risk. There are a number of significant weaknesses to these rules.

[19] Crouhy, Galai, and Mark (1998b) estimate these savings to be in the order of 20 to 50 percent.

Limited differentiation[20]

The 1988 Accord provides a limited differentiation of credit risk into broad categories; see Tables 18.6 and 18.8. Why does an exposure to, say, Turkey—an OECD country with a B1 Moody's credit rating—receive no charge, while exposure to a corporate credit risk with a AAA rating receives a charge? Why are all corporate credit risks treated equally, regardless of credit rating?

Static measures of default risk

The Accord uses an 8 percent-of-capital requirement as protection for corporate credit risk, regardless of the actual credit risk. If a bank must hold 8 percent of capital against corporate loans, it has an incentive to lend to high-yield issuers.

Maturity of credit risk exposure

Capital credit charges are set at the same level regardless of the maturity of the credit exposure. The current rules make no distinction between current and future credit exposures.

Simplified counterparty risk calculation

Look at Table 18.7, which gives the add-on charges for the credit equivalent amount. It treats all equity investments as having the same risk. A similar comment applies to the other categories.

Integration of credit and market risk

The current approach categorizes credit risk under three headings: (1) banking book credit risk, (2) trading book specific risk, and (3) counterparty risk. Yet with the models we have described in this chapter, market and credit risk cannot be separated.

18.7 WHAT CAN GO WRONG?

There are many reasons why losses can occur. We list a number of examples; see Table 18.9.

Wrong model

The Bank of Tokyo-Mitsubishi suffered an $80 million loss in 1997 due to swaptions that traders calibrated to market prices of at-the-money swaptions using a Black, Derman, and Toy model. The model did not correctly price out-of-the-money swaptions and was incapable of pricing Bermuda swaptions.

Operational risk

The BankBoston Corporation in 1998 discovered about $73 million in irregular loans secured by fraudulent or nonexistent collateral to friends of a bank's employee.[21]

[20]See the International Swaps and Derivatives Association (1988) document for a more detailed discussion.
[21]New York Times, March 19, 1998.

TABLE 18.9 *Recent Corporate Losses*

COMPANY	LOSS (PRETAX, IN MILLIONS)	TYPE OF TRANSACTION
Metallgesellschaft (Germany) (1993)	$1,340	Oil Derivatives
Procter & Gamble (U.S.) (1994)	$102	Leveraged Currency Swaps
Air Products & Chemicals (U.S.) (1993)	$60	Leveraged Interest Rate and Currency Swaps
Paine Webber (U.S.) (1994)	$180	Mortgage-Based Structured Notes

The Sumitomo Corporation recorded a loss of $2.6 billion arising from unauthorized trades by one of its commodity traders over an eleven-year period.[22] The company was also fined $150 million for allowing one of its traders to corner the copper market.

Rogue traders are not unique to financial institutions. Philip Services Corporation, a metal company in Ontario, Canada, revealed a loss of approximately US$120 million due to rogue trading in copper.[23]

Recipes for Risk[24]

Arrogance and incentives

All too frequently trouble arises when little thought is given to a particular policy and the incentives generated by the policy. To create risk, just blend arrogance and incentives together. What do we mean by arrogance? An example is picking an accounting policy for a product without adequately questioning the appropriateness of the policy's underlying assumptions. Incentives usually provide rewards based on positive accounting performance over short periods. If the adopted accounting policy creates profits that have low short-term correlation with economic profits, then the firm is at risk.

Example 1

Consider a seven-year zero-coupon bond with a yield to maturity of 6 percent, trading at $665 per $1,000 face value. Suppose accrual accounting is used, and the accrual is calculated on a straight-line basis as (1000 − 665)/7 = $48 instead of on a

[22]New York Times, May 12, 1998.

[23]Globe and Mail, Toronto, March 6, 1998.

[24]We are making extensive use of a talk given by Lee Wakeman, "The Changing Role of the Financial Engineer" (1996). We thank him for many helpful discussions.

compound basis of $40 over the first year. If bonuses are based on accounting profits, there is an incentive for traders to buy and hold.

Example 2

It is customary for trading desks at many banks to be funded by their treasury departments at rates close to LIBOR. Suppose the borrowing rate is 5 percent. Using the data given in Example 1, this creates a carrying charge of $\$665 \times 0.05 = \$33$ per year per $1,000 face value. If the zero-coupon bond is accrued on a compound basis, it will accrue $40 for a net holding profit of $7 per $1,000 face value.

This type of accounting policy creates an incentive for high-yield and emerging-market debt desks to maintain larger than necessary inventories. Such inventories are justified with the response that they "are necessary to maintain an inventory in order to respond quickly to customer buy orders." RAROC systems if properly designed can reduce these incentives.[25]

Credulity

Senior managers normally participate in short-term, profit-based bonus schemes as described above. Large trading profits lead to large bonuses, creating an incentive for these managers to believe the traders reporting the profits and to dismiss questions raised by risk managers.

In the fairy tale "Peter Pan," Peter asks the children in the audience to shout "I believe, I believe" in order to revive the poisoned Tinkerbell. After several repetitions at increasing volume of this "I believe" mantra, the fading light that represents a sick Tinkerbell glows bright again.

No matter how enthusiastically managers invoke this mantra, some profits insist on disappearing.

Tinkerbell 1: Kidder Peabody

Kidder Peabody dismissed the head trader of its government bond trading desk for allegedly falsely reporting over $300 million in trading profits. The "trading profits" were generated by a flaw in Kidder Peabody's accounting system.[26]

The importance of having an independent middle office and senior management that is concerned about risk exposure arises in this case. Apparently the risk manager on the trading desk notified senior management about the difficulty of making such profits in one of the world's most efficient capital markets. His concerns were ignored.

Tinkerbell 2: Barings Brothers and Co. Limited[27]

Barings Bank was the oldest merchant bank in the City of London at the time of its collapse. It was founded in 1762 and was privately controlled. On February 25, 1995,

[25] For an introduction to RAROC see Crouhy, Turnbull, and Wakeman (1999).

[26] See the New York Times, July 23, 1998.

[27] We draw upon the Report of the Board of Banking Supervision Inquiry into the Circumstances of the Collapse of Barings. We refer to this as the Bank of England Report. It is an excellent report and should be read by everyone interested in risk management.

Peter Baring informed the Bank of England that Barings Bank had been the victim of a massive fraud. At the end of December, 1994 the cumulative concealed losses were 208 million pounds; the loss by February 27, 1995 was 827 million pounds.

Barings Futures Singapore (BFS) was an indirect subsidiary of Barings Bank. This office executed trades in three kinds of financial futures: the Nikkei 225 contract, the ten-year Japanese government bond (JGB) contract, and the three-month Euroyen contract. It also traded options on these financial futures. These contracts could be traded on the Osaka Securities Exchange (OSE), the Tokyo Stock Exchange, the Tokyo Futures Exchange, and the Singapore International Monetary Exchange (SIMEX). The original function of the BFS office was to execute trades for its clients. In mid-1993, BFS began trading for its own account, arbitraging any differences between the SIMEX contracts and the equivalent contracts on the Japanese markets. These trades were viewed as being non-directional. If they were long in the Nikkei contracts traded on SIMEX, they would be short in Nikkei contracts traded in Japan. Until the collapse, Barings' management in London believed the trading conducted by BFS to be essentially risk-free and very profitable.

Nick Leeson moved to the BFS office in March, 1992. Account 88888, opened in July, 1992, was recorded at BFS as a client's account. Details of all client accounts were sent to London in four different reports: the trade file, the price file, the margin file, and the file containing details of positions. In July, 1992, Leeson ordered that for account 88888 only the margin file was to be sent to London. Contrary to stated policy, Leeson took directional positions. Profits were recorded in a different account and losses in account 88888. This account was also used to record trading positions. To help meet margin calls, Leeson also sold options. Again, this was contrary to stated policy. In early 1993 he was appointed as general manager. He was in charge of the front and back offices at BFS.

Barings' senior management were given a number of warning signals but chose to ignore them.

1. An initial audit report in August, 1994 identified the lack of segregation between the front and back offices in BFS's operations.
2. The high profitability of BFS's trading activities relative to the low risk as perceived and authorized by Barings' management in London was a clear signal that something was amiss.

   In one week when Leeson reported profits of US$10 million, senior management commented:[28] "Wow! That is impressive. . . . You know, if he makes US$10 million doing arbitrage in a week, what is that? About a half a billion a year. That is pretty good doing arbitrage. That guy is a turbo arbitrageur!"
3. Management in London were sent daily reports from BFS. At no time did they realize that while they received the margin report for account 88888, they did not receive the trade, price, and position reports associated with the account. This discrepancy was not noticed or appreciated.

[28]See p. 50, 3.63, Bank of England Report.

Why did Leeson risk sending this report to London? The margin account provided the rationale for the cash needed by Leeson to meet the Osaka and SIMEX margin calls.

4. In February, 1993, SIMEX sent a letter to BFS with respect to a transaction for account 88888. On January 11, 1995, reference was again made to account 88888 in a letter sent by SIMEX to BFS. On January 27, 1995, SIMEX sent a letter to BFS asking for assurance that it could fund margin calls arising from Leeson's trading.

Despite all these warning signals, senior management, whose bonuses depended on the level of profits, were true believers in their turbo arbitrageur. He was providing Barings Bank with most of its profits. We leave the last word to the Bank of England Report with respect to the profits reported by Leeson and management's willingness to accept them at face value:[29] "As the Exchanges were open and competitive markets, this suggests a lack of understanding of the nature of the business."

18.8 SUMMARY

This chapter takes as exogenous both the term structure of zero-coupon corporate bonds for firms within a given risk class and the term structure of zero-coupon default-free bonds. Using standard arguments, we show how to extract the conditional martingale probabilities of default. Given these probabilities, we show how to price options on credit-risky bonds and how to price vulnerable options. This analysis is then used to examine the pricing of swaps and the credit exposure in swaps. Lastly, we briefly discuss regulatory issues and capital requirements.

We make the simplifying assumption that the martingale default probabilities are independent of the martingale probabilities for the default-free spot interest rates, an assumption that can be relaxed and generalized in numerous ways. Jarrow, Lando, and Turnbull (1997) let the default probabilities for firm *ABC* depend on a current credit rating given by an external agency, like Standard & Poor's, Inc. or Moody's. This creates a Markov chain in credit ratings, in which historical default frequency data can be utilized. Lando (1994) allows the default probability to be dependent upon the level of spot interest rates. This last modification appears promising in the area of Eurodollar contracts.

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[29]See p. 50, 3.66, Bank of England Report.

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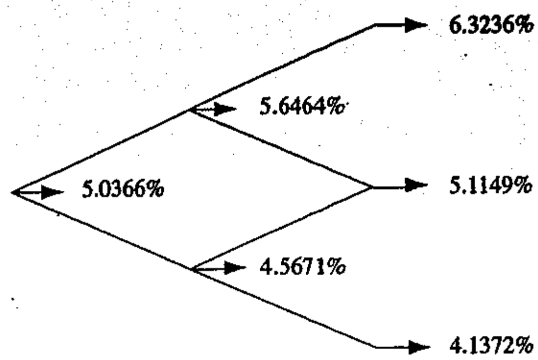
QUESTIONS

Question 1

In the accompanying figure, you are given a lattice of spot interest rates for the default-free term structure. The interval between the nodes is 0.5 year. You are also given the conditional martingale probabilitites of default for a firm in a particular credit class. If default occurs, the recovery rate is 48 cents per dollar.



Conditional martingale probabilities of default:

$\lambda(0) = 0.007$ for the first interval,
$\lambda(1) = 0.008$ for the second interval,
$\lambda(2) = 0.010$ for the third interval.

a) Determine the prices of zero-coupon default-free bonds with maturities 0.5, 1.0, and 1.5 years.
b) Determine the prices of zero-coupon credit-risky bonds with maturities 0.5, 1.0, and 1.5 years.

Question 2

Given the information in Question 1, determine the value of a six-month put option written on a zero-coupon credit-risky bond. At the expiry of the option, the maturity of the underlying credit-risky bond is six months. The strike price of the option is $98. The face value of the zero-coupon bond is $100.

Question 3

Given the information in Question 1, determine the value of a six-month put option written on a default-free zero-coupon bond. At the expiry of the option, the maturity of the underlying default-free zero-coupon bond is six months. The strike price of the option is $98. The face value of the zero-coupon bond is $100. The writer of the option is a firm that belongs to the same credit class as that described in Question 1.

Question 4

Given the following information, please answer the following questions.

a) A three-year bond with a face value of $100 pays a coupon of $2 $^5/_8$ every six months. If the bond is default-free, what is its value?

b) If the bond is issued by a firm belonging to credit class A, what is the value of the bond?

c) If the bond is issued by a firm belonging to credit class B, what is the value of the bond?

MATURITY (YEARS) T	DEFAULT-FREE $B(0,T)$	CREDIT CLASS A $v(0,T)$	CREDIT CLASS B $v(0,T)$
0.5	0.9772	0.9751	0.9748
1.0	0.9540	0.9501	0.9492
1.5	0.9303	0.9242	0.9228
2.0	0.9065	0.8980	0.8962
2.5	0.8825	0.8713	0.8692
3.0	0.8585	0.8445	0.8419
Recovery Rate		0.45	0.38

Question 5

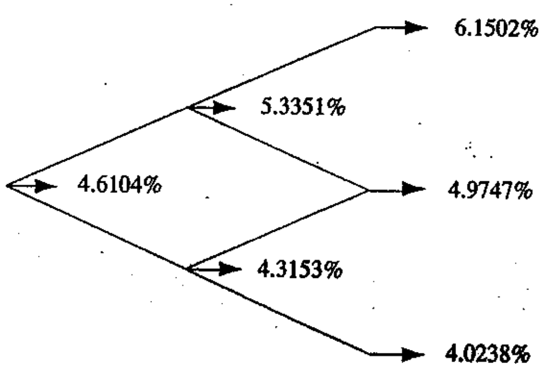
Use the information in Question 4 to answer the following questions.

a) Determine the conditional martingale probabilities of default for credit class A.

b) Determine the conditional martingale probabilities of default for credit class B.

Question 6

Use the information in Question 4 to answer the following. A call option with a maturity of six months is written on a zero-coupon bond by a firm belonging to credit class A. At the expiry of the option, the maturity of the zero-coupon bond is one year. The strike price of the option is 95, face value being $100. The lattice of spot interest rates for the default-free process is given in the accompanying figure.



Interval length: $\Delta = 0.5$ years.

Determine the value of the option.