# Assignment 3

(Due: Wednesday, March 19 – in class)

1. Estimate the following version of the New Keynesian Phillips Curve

$$\pi_t = E[\pi_{t+1}] + \kappa(y_t - y_t^n).$$

using ordinary least squares by following the steps below.

**For this question, please hand in a joint solution with your computational group.**

<u>Naive Approach</u>

(a) Describe which data (and at what frequency) you are using for inflation, expected inflation and the output gap.

(b) Plot your time series for the price level, inflation and changes in inflation. Which of these series are stationary?

**<u>Bonus:</u>** Test for stationarity of these series.

(c) Report your results for the regression. Why is this approach "naive"?

<u>Rational Expectations Approach</u>

(d) Express your regression equation in terms of changes in inflation $\pi_{t+1} - \pi_t$ as the explained variable. [Hint: Expected changes in inflation are subsumed in an error term.]

(e) Report your results for the regression.

Extended Model – Partially Backward-looking

Consider now the extended version of the Phillips curve given by

$$\pi_t = (1-\nu)E_t[\pi_{t+1}] + \nu\pi_{t-1} + \kappa(y_t - y_t^n).$$

(f) Express again your regression equation in terms of changes in inflation $\pi_{t+1} - \pi_t$ as the explained variable. [Hint: Expected changes in inflation are again subsumed in error terms.]

(g) Report your results for the regression.

2. Consider the New Keynesian model described by the equations

$$\begin{aligned} \pi_t &= \beta E_t[\pi_{t+1}] + \kappa(y_t - y_t^n) \\ y_t - y_t^n &= -\frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - r_t^n) + E_t[y_{t+1} - y_{t+1}^n] \\ i_t &= \rho + \phi_\pi \pi_t + \phi_y(y_t - y_t^n). \end{aligned}$$

Set the policy parameters so that $\phi_\pi = 1.5$ and $\phi_y = 0.125$. Assume that productivity follows an AR(1) process given by

$$a_t = \rho_a a_{t-1} + \epsilon_t.$$

For the parameters of the productivity process $(\rho_a, \sigma_a)$ use the values that you have estimated before.

The natural level of output associated with flexible prices is given by

$$y_t^n = \psi a_t - \xi,$$

where $\psi = \frac{1+\nu}{\sigma\alpha+\nu+(1-\alpha)}$ and $\xi = \frac{\alpha \log \frac{\epsilon}{\alpha(\epsilon-1)}}{\sigma\alpha+\nu+(1-\alpha)}$.

This implies that the natural rate of interest is given by

$$r_t^n = \rho + \sigma\psi E_t[a_{t+1} - a_t]$$

(a) Choose parameters $(\theta, \epsilon, \beta, \rho, \sigma, \nu)$ in the Canadian context and justify their values.

Computational Part

(b) Compute IRFs for $i_t, r_t, r_t^n, \pi_t, y_t, y_t^n$ and $x_t = y_t - y_t^n$ in `DYNARE` for a technology shock. Interpret your results.

(c) Now set $\theta$ close to 1. How do your IRFs change? Interpret your results.

(d) Now set $\epsilon$ close to 1. How do your IRFs change? Interpret your results.

3. Consider a government purchasing goods from firms. The total level of government consumption is given by

$$G_t = \left( \int_0^1 G_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

and is financed by lump-sum taxes $T$. The government chooses its demand of individual goods so as to maximize its consumption for any given level of expenditure (and, hence, lump-sum taxes).

(a) Derive the total demand for an individual good $i$.

(b) Derive an aggregate market clearing condition for goods and log-linearize the condition around a steady state where the shares of government consumption and private consumption are given by $s_g = G/Y$ and $s_c = C/Y$ respectively.

(c) How do government purchases influence the IS equation? [Hint: Use the market clearing condition in the log-linearized Euler equation and express the output gap again as a deviation of the real interest rate from the natural rate of interest, $r_t^n$.]