Economics 815 **Winter 2015**

Macroeconomic Theory **Thorsten Koeppl**

Assignment 1

(Due: Thursday, January 29 – in class)

1. Consider the following economy. The economy lasts for two periods. A representative household has an endowment of a single good equal to $y = 1$ in the first period, but no endowment in the second period. Preferences over consumption are given by

$$u(c_1, c_2) = \frac{c_1^{1-\gamma}}{1-\gamma} + \beta \frac{c_2^{1-\gamma}}{1-\gamma}$$

where $\gamma > 0$ and $\beta \in (0, 1)$. The household can invest into capital in period 1 to obtain a deterministic gross return of r in period 2.

The capital is being used by a representative firm to produce output in the second period according to the production function

$$f(k) = k^{\alpha}$$

where $\alpha \in (0, 1)$. Once capital has been used, it fully depreciates. The firm is owned by the household who receives all the firm's profits after it has payed for the rental of capital.

(a) Set up the household's decision problem with a sequence of budget constraints.

(b) Set up the firm's decision problem.

(c) Define a competitive equilibrium for this economy.

(d) Derive a single equation that describes the competitive equilibrium in terms of capital k invested in the first period and the parameters of the model (α, β, γ).

<u>Computational part</u>

(e) Set $\alpha = 0.3$, $\beta = 0.9$ and $\gamma = 2$. Solve numerically for the equilibrium capital stock k, consumption c_1 and c_2 and the equilibrium interest rate r.

(f) Produce graphs for the equilibrium capital stock, the interest rate and consumption as a function of the parameter $\gamma \in (0, 10)$. [WARNING: $\gamma = 1$ is a special case. Which one?]

(g) Provide economic intuition for the resulting graph.

2. Consider the following economy with uncertainty. Households have preferences that are given by

$$u(c_1, c_2) = E_0 \left[\frac{c_1^{1-\gamma}}{1-\gamma} + \beta \frac{c_2^{1-\gamma}}{1-\gamma} \right]$$

where $\gamma > 0$ and $\beta \in (0, 1)$.

In each period, the household has a stochastic endowment $\underline{y}$ or $\bar{y}$. Uncertainty is described by the following Markov transition matrix

$$\Pi = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$$

where the first (second) row gives the probabilities to go from $\underline{y}$ ($\bar{y}$) today to a new state tomorrow.

(a) Assume that the state today is given by $\underline{y}$. What is the probability distribution across states tomorrow?

(b) Find the long-run, stationary distribution across the two states $\underline{y}$ and $\bar{y}$.

Set $\beta = 0.9$ and $\gamma = 2$ and assume that $\underline{y} = 1 < 2 = \bar{y}$. Consider now an asset – a risk-free bond – that is available in period 1 and delivers a payoff of 1 in units of the consumption good.

(c) Calculate the equilibrium price for the asset for $y_1 = \underline{y}$ and $y_1 = \bar{y}$.

(d) Calculate the unconditional mean price for the bond in period 0 assuming that we start out from the long-run stationary distribution you have found in part (b).

Consider now an asset that is risky – equity – as it delivers $1 - \epsilon + x/0.9$ when $y_2 = \underline{y}$ and $1 - \epsilon - x/0.1$ when $y_2 = \bar{y}$ in units of the consumption good. Assume that both ϵ and x are sufficiently small, so that the asset yields a positive payoff in both states.

(e) Calculate the price for this asset in period 1 assuming that $y_1 = \underline{y}$.

(f) For this case, find the range of values for x such that the price for the risky asset is larger than the price of the risk-free asset. Interpret your answer by using the covariance decomposition for asset pricing.

Computational part

Consider now that the payoff for equity is given precisely by the endowment process.

(g) Plot the return difference between the two assets as a function of $\gamma \in (0, 10)$ for $y_1 = \underline{y}$, $y_1 = \bar{y}$ and for the unconditional mean.

3. Consider the following two-period economy. There are two representative households which both have an endowment of $y = 1$ in both periods and preferences given by

$$u(c_1, c_2) = \ln c_1 + \beta_i \ln c_2$$

where $\beta_1 < \beta_2$ so that household 1 discounts the future more than household 2. These households can save or borrow across periods at interest rate $1 + r$.

Find the competitive equilibrium for this economy and interpret your result. [Hint: Derive the intertemporal budget constraints from the sequential ones and use it to find the equilibrium.]