ECON 815

Two-Period Economies

Winter 2015

Basic Model Set-up

- one good for period 1 (good 1) and period 2 (good 2)
- endowment y_1 and y_2
- people (measure 1) have utility over consumption of these goods

$$u(c_1, c_2) = u(c_1) + \beta u(c_2)$$

- competitive markets for trading these goods ex ante

Budget constraint:

$$p_1 c_1 + p_2 c_2 \leq p_1 y_1 + p_2 y_2$$

Equilibrium

Definition: An equilibrium is a set of prices (p_1, p_2) and an allocation (c_1, c_2) such that

(i) people maximize utility subject to the budget constraint taking prices as given

(ii) markets clear, i.e. $c_1 = y_1$ and $c_2 = y_2$.

Solution:

$$\frac{u'(y_1)}{\beta u'(y_2)} = \frac{p_1}{p_2}$$

Intertemporal Choice

Suppose people can borrow or lend at interest rate r.

Three markets – for goods in each period and a market for saving and borrowing.

People have now the budget constraints

$$\begin{aligned} c_1 + s &= y_1 \\ c_2 &= y_2 + (1+r)s \end{aligned}$$

Markets clearing requires $c_1 = y_1$, $c_2 = y_2$ and $s = 0$.

Solution:

$$\frac{u'(y_1)}{\beta u'(y_2)} = (1+r)$$

Equivalence

Note that

$$\frac{p_1}{p_2} = (1 + r)$$

Hence, both ways are equivalent with interest rates being a ratio of intertemporal prices.

Suppose further that $y_1 = y_2$.

We simply get $(1 + r) = 1/\beta$ or – in terms of the discount rate – $r = \theta$.

Sequential vs. Present-Value Budgets

More generally, we have

$$a_{t+1} = (1+r)a_t + y_t - c_t$$

where a_t are total assets owned by a household in period t.

Iterating forward we get

$$a_t = \sum_{i=0}^{\infty} (1+r)^{-i} (c_{t+i} - y_{t+i})$$

where we assume that $\lim_{i \to \infty} (1+r)^{-i} a_{t+i} = 0$.

Using $t = 0$, we thus obtain a net present value budget constraint from sequential constraints.

Euler equation – Examples

1) Quadratic utility $u(c_t) = ac_t - bc_t^2$ and $(1 + r_t) = 1/\beta$

Euler equation is $c_t = c_{t+1}$, so that consumption is constant.

If consumption is uncertain tomorrow, we have

$$c_{t+1} = c_t + \epsilon_{t+1}$$

for some ϵ_{t+1} with $E_t[\epsilon_{t+1}] = 0$.

This is the exact case of *certainty equivalence*.

We can neglect (higher moments of) uncertainty.

2) CRRA utility $u(c_t) = c_t^{1-\alpha}/(1-\alpha)$ with $\alpha > 0$

From the Euler equation, we have that $c_t^{-\alpha} = \beta(1 + r_{t+1})c_{t+1}^{-\alpha}$.

Hence:

$$\frac{\ln c_{t+1} - \ln c_t}{\ln \beta + r_{t+1}} = \frac{1}{\alpha}$$

The elasticity of intertemporal substitution is given by the inverse of the coefficient of relative risk aversion.

Moreover for the variance in consumption growth we approximately have

$$V(\ln c_{t+1} - \ln c_t) \simeq \left(\frac{1}{\alpha}\right)^2 V(r_{t+1})$$

The data imply that α needs to be fairly large.

Adding Production and Investment

Firms:

- have technology and are owned by people
- borrow goods x from people in period 1
- convert these goods into capital $x = k$
- pay back goods plus interest $(1 + r)x$ in period 2

Production:

$$f(k) + (1 - \delta)k$$

Assumptions:

- $f(0) = 0$, $f'(0) = \infty$ and $f'(\infty) = 0$
- $f' > 0$ and $f'' < 0$

Firms maximize profits:

$$\max_k \Pi = \max_k f(k) + k(1-\delta) - (1+r)k$$

Since people get the profits, the udget constraints are

$$\begin{aligned} c_1 &\leq y_1 - x \\ c_2 &\leq y_2 + (1+r)x + \Pi \end{aligned}$$

Definition: And equilibrium is an interest r and an allocation (x, k, c_1, c_2) such that

(i) people maximize utility taking the interest rate and profits as given

(ii) firms maximize profits taking the interest rate as given

(iii) markets clear, i.e. $c_1 = y_1 - x$, $c_2 = y_2$ and $x = k$.

Solution

For profit maximization to have a solution that corresponds to an equilibrium, we need to have that

$$f'(k) + (1 - \delta) = (1 + r)$$

or

$$f'(k) = r + \delta$$

Solution:

$$\text{MRT} = f'(k) + (1 - \delta) = (1 + r) = \frac{p_1}{p_2} = \frac{u'(c_1)}{\beta u'(c_2)} = \text{IMRS}$$

Since $c_1 = y_1 - k$ and $c_2 = y_2 + f(k) + (1 - \delta)$, we have a nonlinear equation in k.

Example

- $u(c) = \ln c$
- $f(k) = k^\alpha$ and $\delta = 0$
- $y_1 = y$ and $y_2 = 0$

Let's look at a **social planner** who simply picks an allocation that maximizes utility for people, while respecting all (technological) constraints.

$$\max_k \ln(y - k) + \beta \ln(k^\alpha)$$

Solution:

$$\begin{aligned} c_1 &= \frac{1}{1+\alpha\beta} y \\ k &= \frac{\alpha\beta}{1+\alpha\beta} y \\ c_2 &= \left(\frac{\alpha\beta}{1+\alpha\beta} y \right)^\alpha \end{aligned}$$

Key insight:

Solution depends on parameters (α, β, y) and the model structure.

Let's use our first-order condition:

$$\text{MRT} = \alpha k^{\alpha-1} = \frac{c_2}{\beta c_1} = \text{IMRS}$$

Using the market clearing conditions, we obtain exactly the same result.

This is just a consequence of the two fundamental theorems of welfare economics.