# **ECON 815**

# Optimal Monetary Policy

Winter 2015

# Long-run Wedge (SS)

Efficient allocation:

$$C^{\sigma}N^{\eta} = \alpha A N^{\alpha - 1}$$

Monopolistic competition:

$$C^{\sigma}N^{\eta} = \frac{W}{P} = \frac{\epsilon - 1}{\epsilon} \alpha A N^{\alpha - 1}$$

so that labour is paid less than its marginal product.

There is a wedge in the labour supply condition similar to a tax on labour income.

A subsidy for labour (financed by a lump-sum tax) can remove this inefficiency and is given by

$$\frac{\epsilon - 1}{\epsilon} (1 + \tau) = 1.$$

## Short-run Wedges

Real shocks lead to variation in the desired average mark-up.

$$\mu_t = \frac{P_t}{W_t} \alpha A_t N_t^{\alpha - 1} \neq \mu$$

Differences in prices lead to differences in demand across goods.

$$C_t(i) \neq C_t(j)$$

## Key Idea:

We need a time-varying subsidy that stabilizes the desired mark-up.

Monetary policy can then counteract fluctuations due to sticky prices.

### Full Stabilization of Prices

**Assumption:** There is a  $\tau_t > 0$  so that  $y_t^n = y_t^*$  for all t.

All real shocks are "undone" by varying the subsidy on labour supply.

Starting from a SS with P, monetary policy aims to stabilize the average mark-up  $\mu_t$  perfectly.

Why? Firms have then no reason to change prices when allowed to do so  $P_t^* = P_{t-1} = P$ . Sticky prices do not matter then anymore.

#### How does MP do this?

- 1. Choose nominal interest rate so that  $i = r_t^n$ .
- 2. This implies that output gap is zero in equilibrium forever.
- Phillips curve gives zero inflation given optimal price setting by firms.

# Problem I – Multiple Solutions

For  $i_t = r_t^n$ , we have

$$\left[\begin{array}{c} x_t \\ \pi_t \end{array}\right] = \left(\begin{array}{cc} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{array}\right) \left[\begin{array}{c} E_t[x_{t+1}] \\ E_t[\pi_{t+1}] \end{array}\right]$$

Unique (stationary) solution requires that number of EV of the matrix are both less than 1.

But only one is, so that there are multiple (stable) solutions.

Sunspots are possible, implying that monetary policy cannot control prices.

# Digression – Blanchard & Kahn (1980)

Stable rational expectations equilibria

$$E_t[y_{t+1}] = Ay_t + B\epsilon_{t+1}$$

- ▶ y is a vector of state (backward looking) and control (forward looking) variables
- $\triangleright$   $\epsilon$  is shocks

Need to look at eigenvalues of A that are greater than 1.

Case 1 – Uniqueness and Saddle-Path Stability: number of EV greater than 1 equal to number of control variables

Case 2 – Multiplicity: not enough EV larger than 1

Case 3 – No stable solution: too many EV larger than 1

## Problem II – $r^n$ is not observable

Consider welfare losses given by

$$W = \frac{1}{2}E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{\kappa}{\lambda} x_t^2 + \frac{\epsilon}{\lambda} \pi_t^2 \right) \right]$$

or in form of an average per-period loss function

$$L = \frac{1}{2} \left[ \frac{\kappa}{\lambda} Var[x_t] + \frac{\epsilon}{\lambda} Var[\pi_t] \right]$$

where  $\kappa/\lambda = \sigma + \frac{\eta + (1-\alpha)}{\alpha}$  and losses are to be understood as % losses from steady state.

Observe that the frictions  $(\epsilon, \theta)$  only influence the inflation variability term.

Consider now the rule

$$i_{t} = \rho + \phi_{\pi} \pi_{t} + \phi_{y} (\log Y_{t} - \log Y_{SS}^{*})$$

$$= \rho + \phi_{\pi} \pi_{t} + \phi_{y} (\log Y_{t} - \log Y_{t}^{n} + \log Y_{t}^{n} - \log Y_{SS}^{*})$$

$$= \rho + \phi_{\pi} \pi_{t} + \phi_{y} x_{t} + v_{t}$$

where  $Y_{SS}^*$  is the efficient steady state.

Only actual output measures matter relative to the long-run efficient steady state.

This is the same specification as we had with monetary policy shocks.

Set  $\phi_y \simeq 0$  to minimize variability and only react to inflation.

This looks like an inflation targeting regime.

Perfectly stabilizes inflation in a unique, stable equilibrium.

### General Problem

We have some loss function for the central bank.

$$\min_{x_t, \pi_t} E_0 \left[ \sum_{t=0}^{\infty} \beta^t f(\pi_t, x_t) \right]$$

We have a Phillips curve as a constraint.

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa x_t + u_t$$

Key variables are model dependent:

- ightharpoonup f is some function related to welfare (approximation)
- $\triangleright$   $x_t$  is some measure of the output gap
- $\triangleright \kappa$  is a parameter
- $\triangleright$   $u_t$  captures shocks relative to the relevant output gap

We get nominal interest rates for implementing optimal policy from the IS equation.

# Example I

### Assumption:

- steady state is efficient
- there are short-run deviations (why?  $\tau$  cannot react)

Relevant output gap:  $x_t = y_t - y_t^*$ 

Loss function:  $f = \pi_t^2 + \omega x_t^2$ , with  $\omega$  possibly set to  $\frac{\kappa}{\epsilon}$ 

Shocks:  $u_t = \kappa(y_t^* - y_t^n)$ 

# Example II

## Assumption:

- steady state is inefficient
- why? cannot set  $\tau > 0$

Short-run output gap:  $x_t = y_t - y_t^*$ 

Long-run output gap:  $x = y_{SS}^n - y_{SS}^* < 0$ 

Loss function:  $f = \frac{1}{2} \left( \pi_t^2 + \omega (x_t - x)^2 \right) - \frac{\lambda}{\epsilon^2} (x_t - x)$ 

The last term captures the welfare costs of long-run distortions.

Shocks:  $u_t = \kappa ((y_t^* - y_{SS}^*) - (y_t^n - y_{SS}^n))$