ECON 815

The Lucas-Phelps Model

Winter 2015

Key Idea

Firms only observe the price for their good, but not the price level.

- price could be high b/c people like the good
- price could be high b/c because the price level is high

In the first case, the firm should increase its output, but in the second case it should not.

With incomplete information, the firm faces a signal extraction problem. It needs to determine the reason for why its price is high.

Upward sloping aggregate supply curve

Based on this information problem, increases in the money supply have (short-term) positive effects on equilibrium output.

Household's Supply Decision

Household can produce good i according to

$$Y_i = L_i$$

Taking its price as given, the household solves

$$\max_{L_i} E_i[C_i - \frac{1}{\gamma} L_i^{\gamma}]$$

$$\text{subject to}$$

$$PC_i = P_i Y_i$$

$$Y_i = L_i$$

where $\gamma < 1$.

FOC:

$$L_i = \left(E_i \left[\frac{P_i}{P} \right] \right)^{\frac{1}{\gamma - 1}}$$

Aggregate Demand

Define aggregate demand as $\ln Y = \int_i \ln Y_i^d$ and the price level as $\ln P = \int_i \ln P_i$.

Quantity theory equation

$$Y = \frac{MV}{P}$$

where M is the money supply and V is a (velocity) shock.

Total demand is split by the household among goods according to

$$Y_i^d = e^{z_i} \left(\frac{P_i}{P} \right)^{-\eta} Y$$

where z_i is a zero mean preference shock and η is the price-elasticity of demand.

Given P_i, demand for good i could be high because the price level (P) is high or because the preference for the good (z_i) is high.

Benchmark – Full Information Equilibrium

Suppose households observe both the price level P and the price P_i.

Equilibrium requires that supply equals demand

$$L_i = Y_i = Y_i^d$$

so that prices are given by

$$P_i = \left(e^{z_i} Y\right)^{\frac{\gamma-1}{1+\eta(\gamma-1)}} P$$

Using the log and integrating over goods, this implies that

$$\ln Y = 0 \text{ or } Y = 1.$$

Conclusion:

Money is neutral ($P = MV$), the aggregate supply curve is vertical and shocks to V influence the price level, but not equilibrium output.

Imperfect Information

Suppose households only observe p_i.

Otherwise:

- $z_i \sim \mathcal{N}(0, \sigma_z)$
- $v \sim \mathcal{N}(0, \sigma_v)$

Households only learn the true values of z_i and p after they have made their production decisions.

Hence, their supply decision depends on the <u>expected</u> relative price

$$\ln L_i = l_i = \frac{1}{\gamma - 1} E_i[p_i - p | p_i]$$

where we assume rational expectations, apply certainty equivalence and use small case letters for the log of variables.

Signal Extraction Problem

Step 1: Prices p_i need to such that the market for good i clears

$$\frac{1}{\gamma - 1}(p_i - E_i[p|p_i]) = z_i - \eta(p_i - p) + m + v - p$$

Step 2: Since p and p_i will be normal, $E[p|p_i]$ will be linear in p_i. Hence, p_i is a linear function of p and some noise

$$p_i = a + bp + \epsilon_i$$

where $\epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon)$

Step 3: Integrating shows that p_i must be a noisy signal of the (random) price level p

$$p_i = p + \epsilon_i$$

and $p_i|p \sim \mathcal{N}(p, \sigma_\epsilon)$

Step 4: Apply now Bayes' Law to obtain

$$E[p|p_i] = \frac{\sigma_\epsilon}{\sigma_p + \sigma_\epsilon} E[p] + \frac{\sigma_p}{\sigma_p + \sigma_\epsilon} p_i$$

Aggregate Supply

Individual output is given by

$$l_i = \alpha(p_i - E[p])$$

In equilibrium, aggregate demand equals aggregate supply

$$m + v - p = y = \int_i y_i^d = \int_i l_i = \alpha(p - E[p]).$$

Taking expectation on both sides shows $E[p] = m$.

Conclusion:

$$p = \frac{1}{1+\alpha} v + m$$
$$y = \frac{\alpha}{1+\alpha} v$$

Unexpected changes in the money supply will have real effects.

NK Models and Microfoundations

Standard New Keynesian Models assume sticky prices.

Firms are updating their prices only with a delay.

This can be justified by

- costs associated with price changes (menu costs)
- costly information acquisition (inattentiveness)
- costly information processing (rational inattention)

In what follows, we *assume* that firms update their prices only periodically and in a random fashion ("Calvo-pricing").