ECON 815

A Classical Monetary Economy

Winter 2015

Firms

There is no capital.

Taking prices and nominal wages as given, firms solve

$$\max_{N_t} P_t Y_t - W_t N_t$$

subject to

$$Y_t = A N_t^{\alpha}$$

where $\alpha \in (0,1)$.

FOC:

$$\frac{W_t}{P_t} = \alpha A N_t^{\alpha-1}$$

TFP follows an AR(1) process

$$\log A_t = \rho \log A_{t-1} + \epsilon_t$$

where $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$.

Households

$$\max_{C_t, N_t, B_t, M_t/P_t} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\left(\frac{M_t}{P_t}\right)^{1-\nu}}{1-\nu} + \frac{(1-N_t)^{1-\eta}}{1-\eta} \right) \right]$$

subject to

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t - T_t \quad \text{for all } t$$

- T_t are transfers (government, profits)
- $Q_t = \frac{1}{1+i_t}$ is the **nominal** bond price
- M_t is nominal money holdings

Using $A_t = B_t + M_t$ for financial assets, we have

$$P_t C_t + Q_t A_t + (1 - Q_t) P_t \frac{M_t}{P_t} \leq A_{t-1} + W_t N_t - T_t$$

Money Demand Equation

$$
\begin{aligned}
\frac{W_t}{P_t} &= \frac{C_t^\sigma}{(1-N_t)^\eta} \\
1 &= \beta E_t \left[\frac{C_t^\sigma}{C_{t+1}^\sigma} \frac{P_t}{P_{t+1}} (1+i_t) \right] \\
\frac{M_t}{P_t} &= C_t^{\frac{\sigma}{\nu}} \left(\frac{i_t}{1+i_t} \right)^{-\frac{1}{\nu}}
\end{aligned}
$$

The last equation is money demand.

Real balances

- increase with output (income) ...
- ... and decrease with the nominal interest rate.

The IS Equation

For savings, the (expected) real interest rate matters

$$(1 + i_t(s^t))\frac{P(s^t)}{P(s^{t+1})} = (1 + i_t(s^t))\frac{1}{1 + \pi(s^{t+1})} = (1 + r(s^{t+1}))$$

This yields the **Fisher equation**

$$r_t = i_t - E_t[\pi(s^{t+1})].$$

The intertemporal Euler equation is approximately given by

$$0 = \log\beta - \sigma E_t[c_{t+1}] + \sigma c_t - E_t(\pi_{t+1}) + i_t$$

or

$$c_t = E_t[c_{t+1}] - \frac{1}{\sigma}\left(i_t - \bar{r} - E_t[\pi_{t+1}]\right)$$

The consumption-savings decision depends on deviations of the (expected) real interest rate r_t from the steady state interest rate $\bar{r}$.

Why is there money?

Money is costly. Why? Forgive interest rate for bonds, in order to hold money (rate-of-return dominance).

Hence, people would like to economize on holding real balances as much as possible.

In the background, there is some service money provides that bonds cannot.

- Cash-in-Advance constraint
- Search models
- Turnpike and OG models

The Friedman Rule

What would the optimal monetary policy be?

The social marginal costs of producing real balances is zero.

But there is a private marginal cost of holding money which is the lost nominal interest rates.

Hence, one needs to set $i_t = 0$ for all t.

This implies that prices have to decline at the discount rate

$$\frac{P_{t+1}}{P_t} = \beta$$

or – equivalently – that there is deflation according to the rate of time preference

$$1 + \pi = \frac{1}{1 + \bar{r}}.$$

Classical Dichotomy

The real side follows an RBC model without capital.

1) IS equation: $c_t = f(r_t - \bar{r})$

2) Marginal product of labor determines the real wage.

3) Market clearing: $y_t = c_t$

We have two equations that pin down nominal variables (i_t, m_t, π_t).

1) Fisher equation: $E_t[\pi_{t+1}] = i_t - r_t$
2) Money demand equation.

Conclusion: We have two dichotomous blocks for the model economy. Monetary policy either picks the money supply or nominal interest rates to determine nominal variables.

Monetary Policy

Consider an **interest rate rule**, where a central bank chooses i_t in response to shocks and the money supply adjusts passively to satisfy the money demand equation.

Problem: The price level and the inflation rate will be indeterminate.

Why? Consider a policy that sets $i_t = \bar{r}$ for all t.

From the Fisher equation, it must be the case that $E_t[\pi_{t+1}] = \bar{r} - r_t$.

Take any difference equation $\pi_{t+1} = p_{t+1} - p_t = \bar{r} - r_t + \epsilon_{t+1}$ such that $E_t[\epsilon_{t+1}] = 0$.

Conclusion: Expected inflation is pinned down by shocks that influence r_t, but actual inflation is not.

Feedback Rules and the Taylor Principle

Consider now the rule $i_t = \bar{\iota} + \phi_\pi \pi_t$.

For example, $\bar{\iota} = \bar{r} + 2\%$ is a neutral, nominal interest rate consistent with an inflation target.

The Fisher equation now satisfies

$$\pi_t = \frac{1}{\phi_\pi} \left(E_t[\pi_{t+1}] + r_t - \bar{\iota} \right).$$

If $\phi_\pi > 1$ (**Taylor Principle**), we can iterate forward to obtain a unique stationary solution which only depends about future expectations of real interest rates $E_t[r_{t+n}]$.

In what follows, we will always <u>assume</u> this stationary solution so that after a one-time shock we will converge back to the long-run steady state.