

# ECON 815

## The RBC Model – Using DYNARE

Winter 2014

## RBC Model

We can write our RBC model as

$$\begin{aligned} \theta(1 - n_t)^{-\eta} &= (1 - \alpha) \frac{y_t}{n_t} c_t^{-\gamma} \\ c_t^{-\gamma} &= \beta E_t \left[ c_{t+1}^{-\gamma} \left( \alpha \frac{y_{t+1}}{k_{t+1}} + (1 - \delta) \right) \right] \\ c_t + k_{t+1} &= y_t + (1 - \delta)k_t \\ y_t &= z_t k_{t+1}^\alpha n_t^{1-\alpha} \\ \log z_t &= \rho \log z_{t-1} + \epsilon_t \end{aligned}$$

We have four endogenous variables we want to look at:

- ▶ output  $y$
- ▶ labour supply  $n$
- ▶ consumption  $c$
- ▶ capital  $k$

## Model in DYNARE – Logs

$$\begin{aligned}
 (1 - \exp(n))(-\eta) &= (1 - \alpha) * \exp(-\gamma * c + y - n); \\
 \exp(-\gamma * c) &= \beta * \exp(-\gamma * c(+1)) * (\alpha * \exp(y(+1) - k) + (1 - \\
 &\quad \exp(c) + \exp(k) = \exp(y) + (1 - \delta) * \exp(k(-1))); \\
 \exp(y) &= \exp(z + \alpha * k(-1) + (1 - \alpha) * n); \\
 z &= \rho * z(-1) + e;
 \end{aligned}$$

### Important issues:

- ▶ decision (jump) variables have current index
- ▶ state variables in period  $t$  have index  $-1$ :
  - ▶  $k(-1)$  is capital just before period  $t$
  - ▶  $z(-1)$  is productivity just before shock in period  $t$
- ▶ model will treat all variables in logs –  $z$  is in logs already,  $c$  is  $\log c$  now

## Policy Functions

The solution of the linearized model is in percentage deviations from steady state.

The interpretation of the coefficients is according to

$$\mathbf{k} = \bar{\mathbf{k}} + \mathbf{a}_1(\mathbf{k}(-1) - \bar{\mathbf{k}}) + \mathbf{a}_2(\mathbf{z}(-1) - \bar{\mathbf{z}}) + \mathbf{a}_3\mathbf{e}. \quad (1)$$

Hence, the solution distinguishes between the effects of the current shock and the lagged term from the AR(1) shock process.

Beyond this, DYNARE delivers

- ▶ impulse response functions
- ▶ autocorrelations
- ▶ second order moments

## IRFs

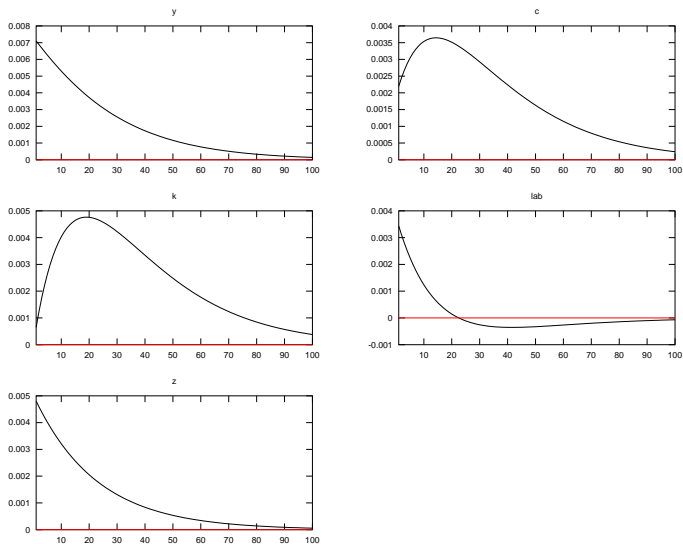


Figure: Baseline model – Canadian Data

# An Almost Linear Model

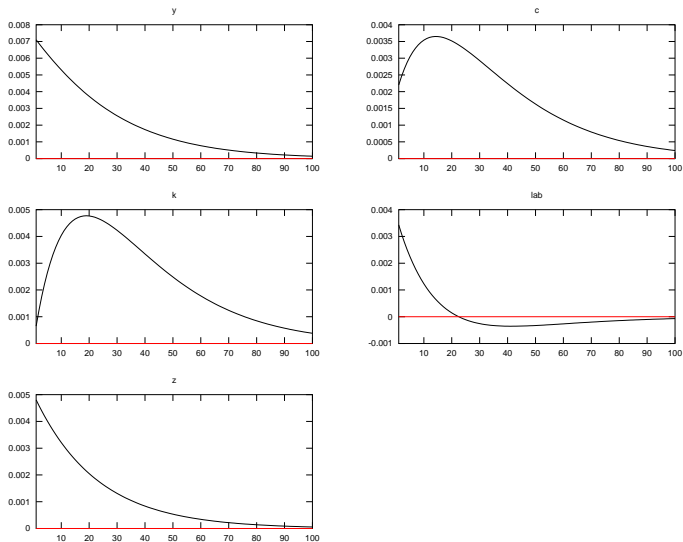


Figure: Baseline model – 2nd order approximation

## Correlations

### MATRIX OF CORRELATIONS

Variables	y	c	k	lab	z
y	1.0000	0.9220	0.8357	0.6758	0.9950
c	0.9220	1.0000	0.9832	0.3377	0.8788
k	0.8357	0.9832	1.0000	0.1599	0.7767
lab	0.6758	0.3377	0.1599	1.0000	0.7460
z	0.9950	0.8788	0.7767	0.7460	1.0000

### COEFFICIENTS OF AUTOCORRELATION

Order	1	2	3	4	5
y	0.9659	0.9327	0.9005	0.8691	0.8387
c	0.9943	0.9866	0.9772	0.9661	0.9536
k	0.9986	0.9948	0.9888	0.9808	0.9710
lab	0.8999	0.8075	0.7224	0.6439	0.5716
z	0.9562	0.9143	0.8743	0.8360	0.7994

## Match with Data

Golden ratios?

- ▶  $c/y = 0.79$  and  $k/y = 8.26$  from calibration

Covariance with output?

- ▶ labor – 0.67 vs. 0.7 (data); consumption – 0.92 vs. 0.53 (data)

Relative standard deviations?

- ▶ labor/output – 28% vs. 97% (data)
- ▶ consumption/output – 78% vs. 72% (data)
- ▶ also usually compare SD of output from model to the one in data



# Critique – No Amplification

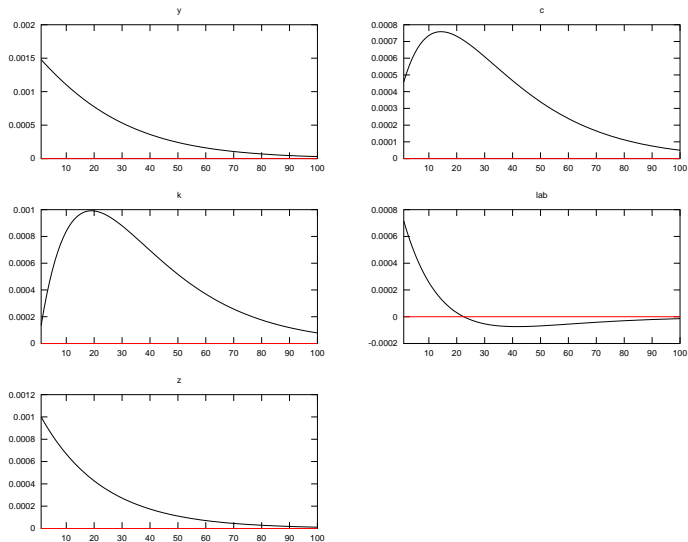


Figure: Low shocks – Canadian Data

## Critique – No Propagation

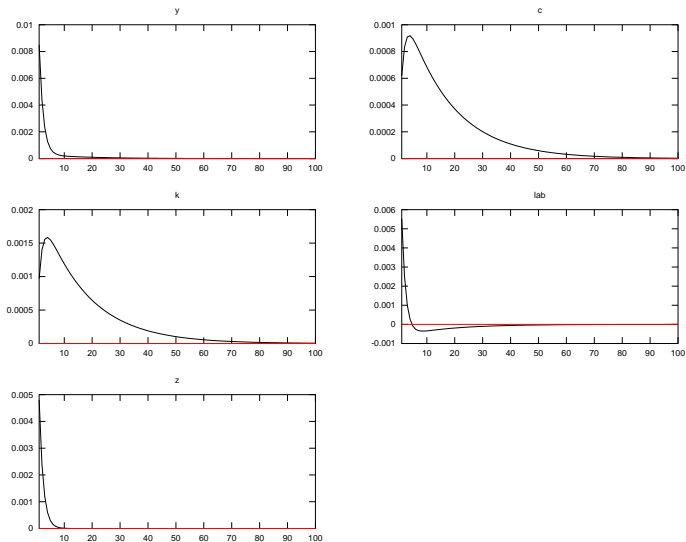


Figure: Baseline model – Canadian Data

# Critique – Labor Supply Elasticity

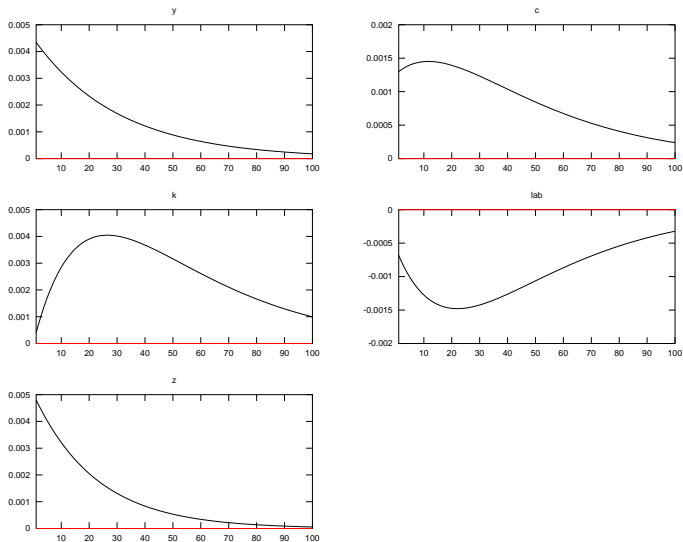


Figure: Model for  $\gamma = \eta = 5$

Increasing the parameter for the intertemporal elasticity of substitution

- ▶ makes labour supply response *countercyclical*
- ▶ improves labour volatility relative to output volatility (60% for the example)
- ▶ improves consumption volatility relative to output volatility (56% for the example)
- ▶ this is highlighted even more whenever the spread  $\eta - \gamma$  increases

Hence, the model fits best for the linear labour model – interpreted as changes in employment.

## Summary

### Ken Rogoff:

“The real business cycle results..., are certainly productive. It has been said that a brilliant theory is one which at first seems ridiculous and later seems obvious. There are many that feel that (RBC) research has passed the first test. But they should recognize the definite possibility that it may someday pass the second test as well.”

### My assessment:

It shows the power of the DSGE approach. But it would be foolish to think that business cycles are entirely driven by highly persistent technology shocks and the large – and efficient – reaction of labor input to such shocks.

The RBC model is particularly “vulnerable” to changes in the intertemporal elasticity of substitution. But one can argue that this is precisely what we are most interested in (employment responses)!