Money, Markets and Dynamic Credit*

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Abstract

This paper presents an integrated theory of money and dynamic credit. I study financial intermediation when both the intermediary and individuals have private information. I show that money is essential in solving the two-sided incentive problems under the dynamic credit arrangement. First, requiring settlement with money can induce market trades that generate information-revealing prices to discipline the intermediary. Second, it is optimal for the intermediary to issue money that can record its own history of being used in settlements, and to require settlements be made with only money that has been returned to the intermediary every settlement period. This arrangement effectively reduces individuals’ incentives to deviate and allows intermediation to achieve efficient allocations.

Key words: private money; dynamic credit
JEL classifications: E4, G2

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1 Introduction

In this paper, I present an integrated theory of money and dynamic credit. I address the following questions: what is the optimal arrangement of financial intermediation when both individuals and intermediaries (such as banks) have private information? Is money essential when dynamic credit arrangements are available? These are fundamental questions concerning any modern economy where both money and long-term credits are used extensively. Whether intermediation is designed efficiently is critical for the performance of the financial system as a major source of lending.

To answer these questions, I develop a dynamic model where money and credit coexist. In the model, individuals receive random endowment shocks over time. At the beginning of time, individuals compete in offering the optimal contract so as to become an intermediary. As a result, an intermediary arises endogenously and uses long-term credit contracts to achieve dynamic risk sharing. An individual’s entitlement to future utilities depends on his reported histories of endowments. There is a two-sided incentive problem under private information. On one hand, individual endowments are private information. The contract must be incentive compatible for individuals to reveal their true endowments. On the other hand, the intermediary has private information about the random aggregate state because he can filter out the idiosyncratic shocks by aggregating the individuals’ truthful reports.

I establish that money is essential in solving the incentive problems un-
nder the dynamic credit arrangement, when there is private information and aggregate uncertainty. Requiring nominal settlements can induce market trades, which generate information-revealing prices to discipline the intermediary. Such market trades cannot take place if settlements are made with real goods. Therefore, money and markets play an important role in resolving the intermediary’s incentive problem. Nevertheless, money’s function as a store of value poses another problem in contract design because it augments individual incentives to deviate from settlements. Since individuals can use money to purchase their preferred goods, it becomes more tempting to hide money from the intermediary than hiding and consuming their own endowments.

If the intermediary is not allowed to issue private money, the problem of designing the optimal contract involves evaluating whether payments should be made in real goods or fiat money. The trade-off between real payments and nominal payments is essentially a trade-off between (i) incurring the cost of the intermediary’s incentive compatibility while maintaining less costly individual incentive compatibility, and (ii) fully resolving the intermediary’s incentive problem while incurring a higher cost of individual incentive compatibility. If the severity of the augmented individual incentive problem outweighs that of the intermediary’s incentive problem, it is optimal to require settlements made in real goods. Otherwise, it is optimal to require settlements in fiat money. In this case, fiat money is essential in that it helps resolve the intermediary’s incentive problem and save the overall cost
of incentive compatibility.

If the intermediary is allowed to use private money, the optimal dynamic credit arrangement entails: (i) the intermediary uses record-keeping technology to issue private money. In particular, such money carries a record of its own history of being used in settlements, i.e., whenever it is paid to and from the intermediary; (ii) payments are required to be made with only money that has a “full record” (i.e., has been paid to the intermediary every settlement period.) In the equilibrium, previously hidden money has no value because it will not be accepted as a means of settlement, or a medium of exchange. Essentially, such money stores value only if it has been used properly. This special form of money effectively reduces individuals’ incentives to deviate. In sum, money and dynamic credit complement each other in achieving efficient allocations. Credit explores dynamic incentives to attain optimal risk sharing and money helps alleviate the incentive problems both on the intermediary’s side and on the individual side.

The model of this paper is built upon Atkeson and Lucas (1992), with aggregate risk and an unobserved individual income shock instead of an unobserved individual taste shock. Hence my work is related to the literature on efficient distributions in a dynamic setting, e.g. Thomas and Worrall (1990), Lucas (1992) and Phelan (1995), etc. My work is also complementary to the literature that examines the functioning of money and financial intermediation, e.g., Shi (1996), Aiyagari and Williamson (2000), Williamson (2004), He, Huang and Wright (2005, 2008), Berentsen, Camera and Waller

Shi (1996) and Sun (2007a) also demonstrate that debt repayments in money instead of goods help improve welfare. Shi studies coexistence of money and credit in a search model with divisible commodities. He shows that the competition of money and credit eliminates the inefficient monetary equilibrium where money has a weak purchasing power, when debt is repaid with only money. Sun addresses the problem of monitoring banks with undiversifiable risks and shows that there is no need to monitor a bank if it requires loans to be repaid partly with money. A market arises at the repayment stage and generates information-revealing prices that perfectly discipline the bank. This result is strengthened in the current paper, which features an enduring relationship between the intermediary and individuals. In contrast to the static contract studied in Sun (2007a), here I show that even the more sophisticated contract form, dynamic contracts, can use the help of markets to deal with the intermediary’s incentive problem. In a simple static framework, Khan and Roberds (2007) show that settlement of debt through inside money can be beneficial to exchange by slackening certain incentive constraints. The current paper conveys a similar message in a dynamic environment and it goes further into examining the optimal form of money in making settlements.

Aiyagari and Williamson (2000) and Williamson (2004) study money and credit where individuals have limited participation to financial markets. In
these models, money is held to satisfy the cash-in-advance type of constraints. Berentsen, Camera and Waller (2007) examine money and credit in a search model where money is essential to overcome lack of double coincidence of wants problems. In contrast, the current paper accentuates the role of money to induce market trades that generate information-revealing prices. Although money functions as a medium of exchange, it is not used to overcome trading frictions but information frictions.

He, Huang and Wright (2005, 2008) study money and banking in a search environment. Bank liabilities are identified as a safer instrument than cash while cash is less expensive to hold. In equilibrium, agents may find it optimal to hold a mix of both. Head and Qiu (2008) study an economy where exchange may be conducted using either inside money (bank deposits) or outside money (fiat currency). They show that it improves welfare to have the central bank conducts monetary policy in two aspects: managing short-run interest rate to control the issuance of inside money and controlling the supply of outside money to set the trend inflation rate. The current paper also examines alternative forms of money in improving efficiency. The optimal instrument for inducing truthful revelation is private money that can record its own history of being used to make repayments. Andolfatto and Nosal (2009) construct a model with spatial separation, limited communication friction and limited information friction. They explain why money creation is typically associated with banking. Similarly, the current paper shows that money and intermediation are intertwined in achieving efficient
allocations when there is private information and aggregate uncertainty.

The remainder of the paper is organized as follows. Section 2 describes the environment of the model. Section 3 studies constrained efficient allocations when there is no aggregate uncertainty. Section 4 examines financial intermediation where money is not used to make repayment. Section 5 explores intermediation where fiat money is required as repayment and Section 6 studies intermediation with private money. Section 7 concludes the paper.

2 The Environment

Time is discrete and has an infinite horizon, \( t = 0, 1, \ldots, \infty \). Each period \( t \) consists of two sub-periods, indexed by \( \tau = 1, 2 \). There are two villages indexed by \( i = 1, 2 \). Each village is populated by a continuum of agents who have unit mass, live forever and discount across time \( t \) with factor \( \beta \in (0, 1) \). Individuals from the two villages can visit each other freely at any point in time.

In each period \( t \), agents in village \( i \) receive endowments of type \( i \) goods at \( \tau = i \). For an individual of village 2, the endowment is deterministic at \( \bar{y} \) for all \( t \). The endowment of an individual from village 1 is given by \( y_t = s_t \phi_t \), where \( s_t \) and \( \phi_t \) are both random variables and \( E(y) = \bar{y} \). In particular, \( s_t \) is an aggregate shock, which is common to all agents in village 1. It is i.i.d. across time according to distribution \( \xi \) with support \( S = \{s^1, \ldots, s^I\} \) where \( s^1 > \cdots > s^I = 0 \). The variable \( \phi_t \) is an idiosyncratic shock. It
is i.i.d. over time and drawn in such a way that the law of large numbers applies across type-1 agents, according to probability distribution \( \mu \) with support \( \Phi = \{ \phi^1, \cdots, \phi^J \} \) where \( \phi^1 > \cdots > \phi^J = 0 \). The realization of \( y_t \) is private information and is not verifiable. Individuals do not observe \( s_t \) or \( \phi_t \) specifically by observing their own endowments. All individuals know \( \xi \) and \( \mu \). All goods are perishable across sub-periods. All markets, if any, are perfectly competitive.

Individual preferences are given by the following:

\[
U_1 = E \sum_{t=0}^{\infty} \beta^t u(\varepsilon a_1^t + a_2^t)
\]
\[
U_2 = E \sum_{t=0}^{\infty} \beta^t (b_1^t + b_2^t),
\]

where \( u(x) = -\exp(-\gamma x) \), or constant relative risk aversion. The variable \( a_i^t \) denotes a type-1 agent’s consumption of date-\( t \) type \( i = 1, 2 \) goods and similarly \( b_i^t \) for type-2 agents. Moreover, \( 0 < \varepsilon < \frac{\delta}{\sigma E(\phi)} \) and \( \varepsilon \gamma \leq 1 \). Note that type-1 and type-2 goods are perfect substitutes for agents in village 2. However, agents in village 1 strictly prefer type-2 goods to type-1 goods if given the same amount.
3 Efficient Allocation under No Aggregate Uncertainty

I assume no aggregate uncertainty in this section. When there is no private information, it is efficient for village 1 and village 2 to swap endowments in the following way: \( \bar{y} \) units of type-2 goods for all type-1 goods every period. As a result, agents in village 1 are strictly better off because (i) they strictly prefer type-2 goods to type-1 goods given the same amount and (ii) they are risk averse and prefer a constant life-time consumption stream \( \bar{y} \). In the meanwhile, agents in village 2 are just as well off with or without the swap in that they are risk neutral and that \( E (y) = \bar{y} \).

When type-1 endowments are of private information, the aforementioned swap will not be incentive compatible. Efficiency requires incentive compatible allocations across agents of both villages. The allocation of resources is arranged by a financial intermediary. At the beginning of \( t = 0 \), all agents compete in offering dynamic contracts to become a financial intermediary. If there is no aggregate uncertainty, the intermediary can be either a type-1 or a type-2 agent. However, as will be discussed in the next section, aggregate uncertainty causes incentive problems on the intermediary’s side. In this case, competition to become the intermediary renders that the intermediary be the type of agent who has the least incentives to misrepresent information about the aggregate type-1 endowments. In any period \( t \), a type-2 agent’s marginal utility from consumption of type-1 goods is always equal to one.
In contrast, a type-1 agent’s marginal utility from consumption of type-1 goods in any given period is \( \varepsilon \gamma \exp [\gamma (\varepsilon a_1 + a_2)] \), which is no greater than one given that \( \varepsilon \gamma \leq 1 \). Thus it is less costly to make a contract incentive compatible for a type-1 agent as the intermediary than for a type-2 agent. It follows that an intermediary must be a type-1 agent when there is aggregate uncertainty. Without loss of generality, I only consider a type-1 agent being the intermediary for the rest of this paper, with and without aggregate uncertainty.

One financial intermediary arises through competition. If more than one agent has offered the optimal contract, then one is chosen randomly to be the intermediary.\(^1\) Both the intermediary and agents can credibly enter into enduring and binding contracts. The intermediary has access to a record-keeping technology, which allows only the intermediary to keep track of individual financial histories. In particular, a financial history refers to an individual’s history of receiving funding from and making payments to the intermediary.

Once established, the intermediary writes contracts with both type-1 and type-2 agents in the economy. The contract specifies that every period, (i) at \( \tau = 1 \) the intermediary collects and allocates type-1 goods equally among agents in village 2; (ii) at \( \tau = 2 \) the intermediary collects and allocates type-2 goods across agents in village 1. Part (i) of the contract can be

\(^1\)One can also think of this economy as having intermediaries each with a positive measure of contracted agents. We can treat the economy as if there were one representative intermediary.
considered as the *settlement* stage between the intermediary and the type-1 agents. Provided that the contract is incentive compatible, the intermediary transfers type-1 goods with an expected amount of \( E(y) = \bar{y} \) units to village 2, in exchange for \( \bar{y} \) units of type-2 goods. Given their preferences, agents in village 2 are no worse off accepting the intermediary’s contract. Assume that all of them do so.

The intermediary must design a contract that is incentive compatible for type-1 agents. Given the CARA utility of type-1 agents, this unobserved endowment shock economy is equivalent to an unobserved taste shock economy where the taste shock is multiplicative as in Atkeson and Lucas (1992).\(^2\) I formulate the intermediary’s problem along the lines of Atkeson and Lucas (1992), which is referred to as AL in the rest of this paper. The intermediary of this model is similar to the social planner in AL. The intermediary promises an expected, discounted life-time utility to each of its contracted type-1 agents. The goal of the intermediary is to achieve optimal risk sharing by minimizing the cost of achieving the promised utilities. This cost-minimizing approach is a dual problem to welfare maximization as is discussed in Green (1987). The total allocations of type-2 goods across type-1 agents cannot exceed \( \bar{y} \) in any period. Furthermore, the allocations must be incentive compatible. In particular, the contract exploits an agent’s intertemporal incentives: If a type-1 agent reports a low endowment level, he will be provided consumption of type-2 goods in a risk-sharing way, but with

\(^2\)The equivalence will be demonstrated in the proof of Proposition 1.
a lower promise of the expected discounted future utility, i.e., the continuation value. Otherwise, the agent will be given consumption of type-2 goods and rewarded by a higher promise of the continuation value for reporting a high endowment level. In sum, when deciding on whether to report the true endowment or not, an agent faces the trade-off between current consumption and future continuation value.

The intermediary’s problem of allocating type-2 goods across type-1 agents can be formulated in the following recursive manner. In each period $t$, a type-1 agent is identified with a number $w_t \in D \subseteq R$, which denotes the agent’s entitlement to the continuation value from period $t$ on. Let $\psi_t \in Q$ be a distribution of utilities $w_t$ across all type-1 agents. At $\tau = 2$ of all $t$, the intermediary assigns consumption of $c_t$ units of type-2 goods to a type-1 agent with current entitlement $w_t$ and who has reported the current idiosyncratic shock as $\phi_t$. Let the intermediary’s choice function $h_t$ be the agent’s utility from consuming $c_t$, that is, $h_t(w_t, \phi_t) = -\exp(-\gamma c_t)$. The intermediary also chooses a function $k_t$ that stipulates this agent’s expected utility entitlement from period $t + 1$ on, that is, $w_{t+1} = k_t(w_t, \phi_t)$. According to his $w$-value and his reported current endowment shock, a type-1 agent receives a quantity of type-2 goods in the current period and an expected utility from the period on. Hence the intermediary is faced with a problem of the same form every period, except that the utility distribution $\psi_t$ is updated every period due to the reassignment of expected utilities. Without loss of generality, I assume that all type-1 agents are promised with the same initial entitlement $w_0$. Let
\( \psi_0 \) denote the initial distribution of utility entitlements. The competition for the intermediary position drives the intermediary’s expected profit down to zero. The intermediary earns the same initial utility entitlement, \( w_0 \), as every other type-1 agent.

Let the function \( \varphi^*(\psi_t) \) be the minimum cost of attaining a utility distribution \( \psi_t \). Hence \( \varphi : Q \to \mathbb{R}_+ \cup \{+\infty\} \). Let \( \Psi \) denote the set of all functions \( \varphi : Q \to \mathbb{R}_+ \cup \{+\infty\} \).

**Definition 1** A contract \( \sigma = \{h_t, k_t\}_{t=0}^{\infty} \) attains a given utility distribution \( \psi_0 \) with resources \( \bar{y} \) only if the resource cost satisfies that for all \( t \geq 0 \),

\[
-\frac{1}{\gamma} \int_{D \times \Phi} \log[-h_t(w_t, \phi_t)] d\mu d\psi_t \leq \bar{y}.
\]

To put the above definition in a recursive fashion, a contract \( \sigma \) attains a given utility distribution \( \psi_0 \) with resources \( \bar{y} \) only if both the total consumption of type-2 goods allocated in \( t \),

\[
-\gamma^{-1} \int_{D \times \Phi} \log[-h_t(w_t, \phi_t)] d\mu d\psi_t,
\]

is less than or equal to \( \bar{y} \), and the continuation of the contract \( \sigma \) from period \( t + 1 \) on also attains the utility distribution \( \psi_{t+1} \) with resources \( \bar{y} \). I suppress all time indexes and use a subscript “+1” to identify next-period values. In every period, given the current distribution of promised expected utilities \( \psi_t \),
the intermediary solves the following recursive contract design problem:

\[ T\varphi(\psi) = \inf_{h,k \in B} \max \left\{ -\gamma^{-1} \int_{D \times \Phi} \log[-h(w, \phi)] d\mu d\psi, \varphi(\psi_{+1}) \right\} \]  \hspace{1cm} (1)

where the operator \( T : \Psi \to \Psi \) and \( B \) is the set of functions \( h, k : D \times \Phi \to D \times D \) such that \( h(\cdot, \phi) \) and \( k(\cdot, \phi) \) are Borel measurable and such that

\[ \int_{\Phi} [h(w, \phi) + \beta k(w, \phi)] d\mu = w, \quad \forall \ w \in D \]  \hspace{1cm} (2)

\[ h(w, \phi) + \beta k(w, \phi) \geq h(w, \tilde{\phi}) \exp[-\gamma \varepsilon s(\phi - \tilde{\phi})] + \beta k(w, \tilde{\phi}), \]  \hspace{1cm} (3)

\[ \forall \ s \in S, \phi, \tilde{\phi} \in \Phi; \tilde{\phi} \neq \phi. \]

According to the objective (1), for any given \( h \) and \( k \), the intermediary finds the maximum between the cost of delivering the current utility \( h \) and the cost of delivering future utility \( k \). Then the intermediary chooses \( h \) and \( k \) to minimize the aforementioned maximal cost. The fixed point of (1), \( \varphi^*(\psi) \), is the minimum cost of attaining a utility distribution \( \psi \).

Constraint (2) is the promise-keeping constraint. That is, the contract \( \sigma \) must deliver the expected utility \( w_t \) to a type-1 agent promised with \( w_t \). Constraint (3) is the incentive compatibility constraint, which states that the contract \( \sigma \) induces the type-1 agent to report truthfully. The left-hand side of (3) is the utility gain from reporting the true idiosyncratic shock \( \phi \) and the right-hand side is the utility from misrepresenting the endowment shock with \( \tilde{\phi} \). In the latter case, the agent will be assigned a current utility \( h(w, \tilde{\phi}) \).
and a future utility \( k(w, \tilde{\phi}) \) according to \( \tilde{\phi} \), and he will consume the hidden endowment, \( s(\phi - \tilde{\phi}) \) units of type-2 goods. Since the constraint (3) must be satisfied for any realization of \( s \), it suffices to have the constraint hold for the highest possible realization \( s^1 \) (note that the CARA preference gives negative utilities). Hence (3) reduces to:

\[
h(w, \phi) + \beta k(w, \phi) \geq h(w, \tilde{\phi}) \exp[-\gamma \varepsilon s^1(\phi - \tilde{\phi})] + \beta k(w, \tilde{\phi}), \quad \forall \phi, \tilde{\phi} \in \Phi; \tilde{\phi} \neq \phi. \tag{4}
\]

We have the following proposition for the intermediary’s contract design problem:

**Proposition 2** (i) The contract design problem given by (1) has a fixed point \( \varphi^* \); (ii) the optimal contract \( \sigma^* = \{ h_t^*, k_t^* \}_{t=0}^\infty \) is unique; (iii) for all \( t \), the type-1 agents’ total consumption of type-2 goods is

\[
-\gamma^{-1} \int_{D \times \Phi} \log [-h_t(w_t, \phi_t)] d\mu d\psi_t = \bar{y};
\]

and (iv) the incentive compatibility constraints bind only for those comparing an individual’s utility from reporting the truth with the utility from reporting the next lower idiosyncratic shock level. That is, the following equations hold at the optimum for all \( t \):

\[
h_t(w_t, \phi_t) + \beta k_t(w_t, \phi_t) = h_t(w_t, \tilde{\phi}_t) \exp[-\gamma \varepsilon s^1(\phi_t - \tilde{\phi}_t)] + \beta k_t(w_t, \tilde{\phi}_t), \tag{5}
\]
where $\phi_t = \phi^j$ and $\tilde{\phi}_t = \phi^{j+1}$ for all $j = 1, \cdots, J$.

**Proof.** I first show that the contract design problem (1) maps exactly into the one studied by AL. Then the proposition follows naturally as is proven by AL. In AL, agents face idiosyncratic, multiplicative taste shocks $\theta_t \in \Theta$ distributed according to $\eta$. A typical agent’s preference is given by $E \sum_{t=0}^{\infty} (1-\beta)^t V(c_t) \theta_t$, where $V$ is the CARA utility function. There is a constant endowment of goods over time. The planner has two choice functions $f_t(w_t, \theta_t)$ and $g_t(w_t, \theta_t)$, which assign the current utility and the expected utility entitlement to an agent with promised utility $w_t \in D$ and current shock $\theta_t$. Again all time indexes are suppressed and a subscript “+1” is used to denote the next period values. Given the CARA preference, the planner’s problem in AL is given by the following:

$$T \varphi(\psi) = \inf_{f, g \in B} \max \left\{ -\gamma^{-1} \int_{D \times \Theta} \log[-f(w, \theta)] d\eta d\psi, \quad \varphi(\psi_{+1}) \right\}$$

where the operator $T : \Psi \rightarrow \Psi$ and the optimization is subject to the following constraints

$$\int_{\Theta} [(1-\beta) f(w, \theta) \theta + \beta g(w, \theta)] d\eta = w, \quad \forall w \in D$$

$$(1-\beta) f(w, \theta) \theta + \beta g(w, \theta) \geq (1-\beta) f(w, \tilde{\theta}) \theta + \beta g(w, \tilde{\theta}), \quad \forall \theta, \tilde{\theta} \in \Theta; \quad \tilde{\theta} \neq \theta.$$

Now let $(1-\beta) \theta \equiv \exp(-\gamma \varepsilon s^1 \phi)$, $f(w, \theta) \equiv h(w, \phi) \exp(\gamma \varepsilon s^1 \phi)$ and $g(w, \theta) \equiv$

\[\text{See Atkeson and Lucas (1992), p.p. 433, for the recursive formulation of the planner's problem. Some notations are slightly different here.}\]
$k(w, \phi)$. Then it is obvious that the above planner’s problem is exactly the same as the intermediary’s problem given by (1). Part (i) follows Lemma 4.1 in AL. Part (ii) follows Lemma 5.1 in AL for the CARA case (see the Appendix in AL). Part (iii) follows Lemma 5.2 and Lemma 5.3 in AL for the CARA case (see the Appendix in AL). These proofs show that the infimum cost of resources, $\varphi^*(\psi_1)$, is constant over time. It follows trivially that this infimum cost is equal to $\bar{y}$. For part (iv), see Lemma 6.1 in AL, for the case of two idiosyncratic shock values and see Thomas and Worrall (1990), Lemma 4, for $N$ shock values.

It is not obvious whether the constrained efficient allocation characterized by $\sigma^*$ is implementable when there is aggregate uncertainty. In what follows, I will use $\sigma^*$ as a benchmark to evaluate the outcomes of financial intermediation under aggregate uncertainty. From this point on, the aggregate state $s$ is unobservable.

4 Efficient Allocation under Aggregate Uncertainty

With aggregate uncertainty, the intermediary cannot implement the efficient contract $\sigma^*$ with resources of $\bar{y}$ units of type-2 goods (not without involving any monetary instruments). The reason is that the unobservable aggregate risk causes incentive problems on the intermediary’s side, in addition to in-
individuals’ incentive problems. By making the contracts incentive compatible, the intermediary induces individuals to contribute all their endowments. However, since the aggregate state in village 1 is unobservable to the public, the intermediary has the incentives to misrepresent the aggregate output of type-1 goods to type-2 agents. By doing so, the intermediary gets to keep the hidden type-1 goods for his own consumption.

It follows that any contract offered by the intermediary must address his own incentive problem. The contract can explore dynamic incentives of the intermediary, which allows the intermediary to have profits based on his reported history of aggregate shocks in village 1. The contract specifies that every period, (i) after collecting type-1 goods at $\tau = 1$, the intermediary announces the aggregate state $s_t$ and divides the aggregate type-1 endowment equally among type-2 agents; (ii) then at $\tau = 2$ the intermediary allocates type-2 goods among type-1 agents and the intermediary himself, according to individual reported histories of idiosyncratic shocks and according to the intermediary’s own reported history of aggregate shocks. The goal of the intermediary is to minimize the resource cost of achieving the individual promised utilities and the intermediary’s utility entitlement, while maintaining incentive compatibility for both the individuals and the intermediary himself.

Once again we formulate the intermediary’s problem in a recursive manner. Similar to the type-1 individuals, every period the intermediary is also identified with a number $v_t \in D \subseteq R$, which denotes the intermediary’s en-
itlement to continuation value from period $t$ on. Every period, based on $v_t$ and his report of $s_t$, the intermediary is entitled to consumption of $c_t$ units of type-2 goods and an expected utility entitlement, $v_{t+1}$, from period $t+1$ on. Let $f_t(v_t, s_t) = -\exp(-\gamma c_t)$ and $v_{t+1} = g_t(v_t, s_t)$. The intermediary is induced to report truthfully by the trade-off between current consumption and future utility entitlements. Moreover, the intermediary uses $h_t(w_t, \phi_t)$ and $k_t(w_t, \phi_t)$ to assign individual current consumption and future utilities, respectively.

As previously, assume that all type-1 agents are promised with the same initial entitlement $w_0$ and $\psi_0$ is the initial distribution of utility entitlements. Let $v_0$ be intermediary’s initial entitlement. At $t = 0$, the equilibrium for financial intermediation satisfies: (i) $v_0 \geq w_0$ and (ii) $v_0$ is the minimum initial entitlement for the intermediary with which the incentive compatible contract attains $\psi_0$ with resources $\bar{y}$. Condition (i) is the participation constraint for the intermediary. In particular, a type-1 individual must be no worse off being the intermediary than otherwise. Condition (ii) is the outcome of competition. In particular, competition among type-1 individuals drives the intermediary’s expected, discounted life-time utility down to the lowest level required to attain $\psi_0$ using the incentive compatible contract. Thus the equilibrium minimum initial entitlement for the intermediary is given by $v_0(\psi_0)$.

Let the function $\varphi_{nm}^*(\psi_t, v_t)$ be the minimum cost of attaining a utility distribution $\psi_t$ and the intermediary’s entitlement $v_t$. Then $\varphi_{nm}^*(\psi_t, v_t) =$
\( \varphi^*(\psi_t) + \varphi_{in}^*(v_t) \), where \( \varphi^*(\psi_t) \) is the minimum cost of attaining the distribution \( \psi_t \) and \( \varphi_{in}^*(v_t) \) is the minimum cost of attaining utility \( v_t \) for the intermediary. Hence \( \varphi_{in} : D \to \mathbb{R}_+ \cup \{+\infty\} \). Let \( \Psi_{in} \) denote the set of all functions \( \varphi_{in} : D \to \mathbb{R}_+ \cup \{+\infty\} \).

**Definition 3** A contract \( \sigma = \{f_t, g_t, h_t, k_t\}_{t=0}^\infty \) attains a given utility distribution \( \psi_0 \) and a given utility \( v_0 \) with resources \( \bar{y} \) only if the resource cost satisfies that for all \( t \geq 0 \),

\[
-\gamma^{-1} \log[-f(v_t, s_t)] - \frac{1}{\gamma} \int_{D \times \Phi} \log[-h_t(w_t, \phi_t)]d\mu \psi_t \leq \bar{y}.
\]

To put the above definition in a recursive fashion, a contract \( \sigma \) attains \( \psi_0 \) and \( v_0 \) with resources \( \bar{y} \) only if in any aggregate state \( s_t \), both the total consumption of type-2 goods allocated in \( t \),

\[
-\gamma^{-1} \log[-f(v_t, s_t)] - \gamma^{-1} \int_{D \times \Phi} \log[-h_t(w_t, \phi_t)]d\mu \psi_t,
\]

is less than or equal to \( \bar{y} \), and the continuation of the contract \( \sigma \) from period \( t + 1 \) on also attains the utility distribution \( \psi_{t+1} \) and the utility \( v_{t+1} \) with resources \( \bar{y} \). Note that the allocations to the intermediary and those to the individuals are separable. Then the intermediary’s recursive contract design problem essentially has two components: (i) the component of allocating resources across type-1 individuals, which is the same problem given by (1)
subject to conditions (2) and (3); (ii) the component of assigning incentive compatible allocations for the intermediary given $v_t$ and $s_t$. The solutions to the first component have been characterized in the previous section. In particular, it has a fixed point $\varphi^*(\psi_0)$. Let us consider the second component of the intermediary’s problem in a recursive fashion: A contract $\sigma$ attains utility $v_0$ for the intermediary with resources $\bar{y} - \varphi^*(\psi_0)$ only if in any aggregate state $s_t$, the intermediary’s consumption of type-2 goods in $t$, $-\gamma^{-1} \log [-f(v_t, s_t)]$, is less than or equal to $\bar{y} - \varphi^*(\psi_0)$, and the continuation of the contract $\sigma$ from period $t + 1$ on also attains the utility $v_{t+1}$ with resources $\bar{y} - \varphi^*(\psi_0)$.

Suppress all time indexes and use a subscript “+1” to identify next-period values. Given $v_t$, the intermediary’s second component problem is given by:

$$T\varphi_{in}(v) = \inf_{f, g \in A} \max_{s \in S} \left\{ \max \left\{ -\gamma^{-1} \log [-f(v, s)], \varphi_{in}[g(v, s)] \right\} \right\}, \quad (6)$$

where the operator $T : \Psi_{in} \to \Psi_{in}$ and $A$ is the set of functions $f, g : D \times S \to D \times D$ such that $f(\cdot, s)$ and $g(\cdot, s)$ are Borel measurable and such that

$$\int_S [f(v, s) + \beta g(v, s)] d\xi = v, \quad \forall v \in D \quad (7)$$

$$f(v, s) + \beta g(v, s) \geq f(v, \tilde{s}) \exp[-\gamma \epsilon E(\phi)(s - \tilde{s})] + \beta g(v, \tilde{s}), \quad \forall s, \tilde{s} \in S; \tilde{s} \neq s. \quad (8)$$

According to the objective (6), for any given $f, g$ and $s$, the intermediary
finds the maximum between the cost of delivering the current utility \( f \) and that of delivering future utility \( g \). Then the intermediary finds the maximal cost across all possible state \( s \). Finally, the intermediary chooses \( f \) and \( g \) to minimize the maximal cost required to deliver both current utility and future utility in all possible states. The fixed point of (6), \( \varphi^*_\text{in}(v) \), is the minimum cost of attaining a utility \( v \) for the intermediary.

Constraint (7) is the promise-keeping constraint. The contract \( \sigma \) must deliver the expected utility \( v \) to the intermediary. Constraint (8) is the incentive compatibility constraint, which states that the contract \( \sigma \) induces the intermediary to report aggregate information truthfully. The left-hand side of (8) is the utility gain from reporting the true shock \( s \) and the right-hand side is the utility from misrepresenting the shock with \( \bar{s} \). In the latter case, the intermediary is entitled to a current utility \( f(v, \bar{s}) \) and a future utility \( g(v, \bar{s}) \) according to his report \( \bar{s} \). The intermediary will consume the hidden resources, \( E(\phi)(s - \bar{s}) \) units of type-1 goods. We have the following proposition for the intermediary’s contract design problem:

**Proposition 4** The contract design problem given by (6) has a fixed point \( \varphi^*_\text{in} \).

**Proof.** (i) First prove \( \varphi^*_\text{in} \leq T\varphi^*_\text{in} \). Suppose for some \( v \in D \), we have \( \varphi^*_\text{in}(v) > T\varphi^*_\text{in}(v) \). Then there exists some \( (f^0, g^0) \in A \) and an arbitrary
\[ \varphi_{in}(v) - \max_{s \in S} \{ \max \{-\gamma^{-1} \log [-f^0(v, s)], \varphi_{in}^s [g^0(v, s)]\} \} > \delta. \]  \hspace{1cm} (9)

Let \( v_1 = g^0(v, \hat{s}) \), where \( \hat{s} = \arg \max_s \{ \max \{-\gamma^{-1} \log [-f^0(v, s)], \varphi_{in} [g^0(v, s)]\} \} \). Since \( \varphi_{in}^*(v) > \varphi_{in}^*(v_1) \), then \( \varphi_{in}^*(v_1) \) must be finite. By definition, \( \varphi_{in}^*(v_1) \) is the minimum cost of attaining utility \( v_1 \) for the intermediary. Then there is a contract \( \sigma^1 = \{ f^1_t, g^1_t \}_{t=0}^{\infty} \) that attains \( v_1 \) with resources \( \varphi_{in}^*(v_1) + \delta/2 \).

Define the contract \( \sigma^0 = \{ f^0_t, g^0_t \}_{t=0}^{\infty} \) by setting \( (f^0_0, g^0_0) = (f^0, g^0) \) and \( (f^0_{t+1}, g^0_{t+1}) = (f^1_t, g^1_t) \) for \( t \geq 0 \). Let \( A \equiv -\gamma^{-1} \log [-f^0(v, \hat{s})] \). Condition (9) implies that \( A < \varphi_{in}^*(v) - \delta \) and \( \varphi_{in}^*(v) < \varphi_{in}^*(v) - \delta \), where the latter implies that \( \varphi_{in}^*(v_1) + \delta/2 < \varphi_{in}^*(v) - \delta/2 \). Suppose \( A \leq \varphi_{in}^*(v_1) + \delta/2 \).

Then the contract \( \sigma^0 \) attains \( v \) with \( \varphi_{in}^*(v_1) + \delta/2 < \varphi_{in}^*(v) - \delta/2 \). This is a contradiction. Now suppose \( A > \varphi_{in}^*(v_1) + \delta/2 \). Then the contract \( \sigma^0 \) attains \( v \) with \( A < \varphi_{in}^*(v) - \delta \), which is also a contradiction. Thus it must be the case that \( \varphi_{in}^* \leq T \varphi_{in}^* \).

(ii) Secondly, prove \( \varphi_{in}^* \geq T \varphi_{in}^* \). Suppose for some \( v \in D \) there is a \( \delta > 0 \) such that for all \((f, g) \in A\),

\[ \max_{s \in S} \{ \max \{-\gamma^{-1} \log [-f(v, s)], \varphi_{in}^s [g(v, s)]\} \} - \varphi_{in}^*(v) > \delta. \]

The above is impossible if \( \varphi_{in}^*(v) = +\infty \). Suppose \( \varphi_{in}^*(v) \) is finite. Then by definition of \( \varphi_{in}^* \), there is a contract \( \sigma^0 = \{ f^0_t, g^0_t \}_{t=0}^{\infty} \) that attains \( v \) with
resources $\varphi_{in}^*(v) + \delta/2$. Let $v_1 = g^0(v, \hat{s})$, where
\[
\hat{s} = \text{arg max}_s \left\{ \max \left\{ -\gamma^{-1} \log [-f^0(v, s)], \varphi_{in}^* \left[ g^0(v, s) \right] \right\} \right\}.
\]

Consider another contract $\sigma^1 = \{f^1_t, g^1_t\}_{t=0}^\infty$, where $(f^1_t, g^1_t) = (f^0_{t+1}, g^0_{t+1})$ for all $t \geq 0$. The contract $\sigma^1$ attains $v_1$ with resources $\varphi_{in}^*(v) + \delta/2$. Thus $\varphi_{in}^*(v_1) \leq \varphi_{in}^*(v) + \delta/2$ and it follows that
\[
\max_{s \in S} \left\{ \max \left\{ -\gamma^{-1} \log [-f^0(v, s)], \varphi_{in}^* \left[ g^0(v, s) \right] \right\} \right\} \leq \varphi_{in}^*(v) + \delta/2.
\]

This is a contradiction to the supposition. Thus $\varphi_{in}^* \geq T\varphi_{in}^*$. Parts (i) and (ii) imply that $\varphi_{in}^* = T\varphi_{in}^*$. ■

Finally, let us consider a type-2 individual’s decision on whether to accept the contract in $t = 0$. Given the contract, at each $\tau = 1$ a type-2 individual receives $y_t$ units of type-1 goods from the intermediary; and at each $\tau = 2$ this individual submits $\varphi^* (\psi_0) - \gamma^{-1} \log [-f(v_t, s_t)]$ units of type-2 goods to the intermediary. As a result, every period a type-2 individual consumes $y_t$ units of type-1 goods and $\bar{y} - \varphi^* (\psi_0) + \gamma^{-1} \log [-f(v_t, s_t)]$ units of type-2 goods. Note that according to the proof of Proposition 2, the total amount of type-2 goods allocated to type-1 agents are constant over time and is given by $\varphi^* (\psi_0)$. 

Ex ante a type-2 individual’s expected consumption every period is greater than $E(y_t) = \bar{y}$ since $\bar{y} - \varphi^* (\psi_0) + \gamma^{-1} \log [-f(v_t, s_t)] \geq 0$. It follows that all type-2 individuals have the incentive to accept the intermediary’s
contract in \( t = 0 \).

Define \( \varphi^*(\psi_0; \sigma^*_{nm}) \equiv \varphi^*_{nm}(\psi_0, v_0(\psi_0)) \) as the minimum expected cost achieved by the optimal contract given any initial distribution \( \psi_0 \), when there is aggregate uncertainty. Note that \( v_0(\psi_0) \) is the equilibrium initial utility entitlement for the intermediary given any initial distribution of promises \( \psi_0 \). We have the following proposition:

**Proposition 5** For any given initial distribution \( \psi_0 \), \( \varphi^*(\psi_0; \sigma^*_{nm}) > \varphi^*(\psi_0; \sigma^*) \).

**Proof.** For the optimal contract under aggregate uncertainty, it must be incentive compatible for the intermediary as well as for individual type-1 agents. Recall that \( \varphi^*(\psi_0; \sigma^*_{nm}) = \varphi^*_{nm}(\psi_0, v_0(\psi_0)) = \varphi^*(\psi_0) + \varphi^*_{in}(v_0(\psi_0)) \), where \( \varphi^*(\psi_0) \) is the minimum expected cost of incentive compatible allocations across type-1 agents and \( \varphi^*_{in}(v_0(\psi_0)) \) is the minimum expected cost of incentive compatible allocations for the intermediary. As is explained before, the first component of the intermediary’s problem is the same as the constrained efficiency problem. Thus \( \varphi^*(\psi_0) = \varphi^*(\psi_0; \sigma^*) \). To solve the intermediary’s incentive problem, it must be the case that \( \varphi^*_{in}(v_0(\psi_0)) > 0 \). Otherwise the intermediary has no incentives to report the true aggregate states. It follows that \( \varphi^*(\psi_0; \sigma^*_{nm}) > \varphi^*(\psi_0; \sigma^*) \).

Proposition 5 shows that when there is aggregate uncertainty, the optimal contract cannot achieve the constrained efficiency achieved under no aggregate uncertainty. This is due to the costly incentive problems on the intermediary’s side. I now move on to consider the intermediary’s contract.
design problem when monetary instruments are allowed to implement the contract. After characterizing the optimal contract with money, I will compare it with the optimal contract studied in the current section.

5 Efficient Allocation with Fiat Money

Now I study credit arrangements under aggregate uncertainty where the financial intermediary executes contracts with money. That is, the intermediary allocates money instead of real goods among individuals. In particular, at the beginning of time, each type-2 agent is endowed with $M > 0$ units of fiat money. The money stock is constant and is public information. In this section, assume that the intermediary is not allowed to issue private money. At $t = 0$, the intermediary writes contracts with both type-1 and type-2 agents. The contract specifies that (i) at all $\tau = 1$ of $t \geq 1$ the intermediary allocates $M$ units of money to each type-2 agent; (ii) at all $\tau = 1$ each type-1 agent must sell all his endowments to type-2 agents for money. Then the type-1 agent reports his endowments to the intermediary and submits all money receipts to the intermediary; (iii) at all $\tau = 2$ the intermediary allocates money to each type-1 agent based on the latter’s reported history of endowment shocks; (iv) at all $\tau = 2$ each type-2 agent must sell all his endowments to type-1 agents for money. The terms of the contract are public information. All markets are perfectly competitive.

Recall that the intermediary has access to a technology that keeps track
of financial histories. However, there is no record keeping for market transactions. Individuals trade anonymously in perfectly competitive markets of goods for money. Consequently, type-1 individuals have two ways to deviate: (i) by not selling their endowments and hiding these goods from the intermediary; and (ii) by selling their endowments but hiding the money receipts from the intermediary. Incentive compatibility requires that the type-1 individuals do not have incentives to deviate in either or both of the above two ways.

I adopt the same notations used to characterize the constrained efficient contract. A type-1 agent is identified with $w_t$, which is his entitlement to expected utility from period $t$ on. Utility entitlements are distributed according to $\psi_t$ across all type-1 agents. At $\tau = 2$ of all $t$, the intermediary assigns $m_t$ units of money to a type-1 agent with current entitlement $w_t$ and who has announced the current shock $\phi_t$. The agent can buy $c_t$ units of type-2 goods with $m_t$ in period $t$. The intermediary’s choice function $h_t$ is the agent’s utility from consuming $c_t$, that is, $h_t(w_t, \phi_t) = -\exp(-\gamma c_t)$. Moreover, the choice function $k_t$ specifies this agent’s expected utility entitlement from period $t + 1$ on, that is, $w_{t+1} = k_t(w_t, \phi_t)$. The intermediary seeks the optimal contract $\sigma = \{h_t, k_t\}_{t=0}^\infty$ that attains a given utility distribution $\psi_0$ with resources $\bar{y}$. Let $\varphi_m^*(\psi_t)$ be the minimum cost for the intermediary to attain a utility distribution $\psi_t$.

In the following, I first define the competitive equilibrium with financial intermediation and then I characterize and compare the optimal contract
with money, $\sigma_{m}^{*}$, with the efficient contract $\sigma^{*}$ and the optimal contract without money $\sigma_{nm}^{*}$. Let $P_{t}^{i}$ denote the date-$t$ market price of goods for money in village $i = 1, 2$.

**Definition 6** An equilibrium with fiat money and credit consists of a contract $\sigma = \{h_{t}, k_{t}\}_{t=0}^{\infty}$, price sequences $\{P_{t}^{1}, P_{t}^{2}\}_{t=0}^{\infty}$, an initial distribution of utility $\psi_{0}$ and resources $\bar{y}$ such that (i) given $\{P_{t}^{1}, P_{t}^{2}\}_{t=0}^{\infty}$, the contract $\sigma$ attains $\psi_{0}$ with resources $\bar{y}$; (ii) $\varphi_{m}^{*}(\psi_{0}) = \bar{y}$; (iii) all markets clear.

Note that the contract in the above definition does not involve any allocations to the intermediary to solve the intermediary’s incentive problem. It will become clear later that the intermediary is perfectly disciplined when the contract is executed with money. Given the contract, the date-$t$ market-clearing prices are given by

\[ P_{t}^{1} = \frac{M}{s_{t} \int_{\Phi} \phi_{t} d\mu}, \quad P_{t}^{2} = \frac{M}{\bar{y}}. \]  

(10)

### 5.1 The intermediary’s incentive problem

Even with aggregate uncertainty, the intermediary’s incentive problem is fully solved in the current credit arrangement with money. To see this, recall that the market price in village 1 is given by $P_{t}^{1} = M / [s_{t} \int_{\Phi} \phi_{t} d\mu]$, which implies that

\[ s_{t} = \frac{M}{P_{t}^{1} \int_{\Phi} \phi_{t} d\mu}. \]
The market price $P^1_t$ fully reveals the aggregate state $s_t$, which leaves the intermediary on no ground to misrepresent the aggregate information (as in Sun, 2007a). Therefore, in a credit arrangement with money, one does not need to worry about how to discipline the intermediary in telling the true aggregate information. Note that this result applies as long as money is used to execute the contract. What kind of money does not matter. That is, it can be either fiat money or private money, which will be introduced in the next section. In what follows, I characterize the contract with fiat money and focus on the individuals’ incentive problems.

### 5.2 The individual incentive problem

Provided that the contract is incentive compatible for type-1 agents, they sell all their endowments in the markets. All type-2 agents receive the same amount of money at all $\tau = 1$, which allows them to purchase equal shares of the aggregate type-1 endowment. Thus by selling $\bar{y}$ units of type-2 goods every period, each type-2 agent is entitled to consumption of type-1 goods with an expected value of $E(y) = \bar{y}$. Given their preferences, type-2 agents have no incentives to deviate.

The intermediary seeks the optimal contract $\sigma = \{h_t, k_t\}_{t=0}^{\infty}$ that attains a given utility distribution $\psi_0$ with resources $\bar{y}$. Given market prices $P^1_t$ and $P^2_t$ and the current distribution of promised expected utilities $\psi_t$, the
intermediary solves the following recursive contract design problem:

$$T \varphi(\psi) = \inf_{h, k \in B} \max \left\{ -\gamma^{-1} \int_{D \times \Phi} \log[-h(w, \phi)] d\mu d\psi, \varphi(\psi_{+1}) \right\} \quad (11)$$

where the optimization is subject to the following constraints

$$\int_{\Phi} [h(w, \phi) + \beta k(w, \phi)] d\mu = w, \quad \forall \ w \in D \quad (12)$$

$$h(w, \phi) + \beta k(w, \phi) \geq h(w, \tilde{\phi}) \exp\left\{-\gamma s(\phi - \tilde{\phi}) \max_{\rho} \left\{ \varepsilon \rho + (1 - \rho) \rho^2 \right\}\right\} + \beta k(w, \tilde{\phi}), \quad (13)$$

$$\forall \ s \in S, \phi, \tilde{\phi} \in \Phi; \tilde{\phi} \neq \phi.$$

The above is the same contract design problem as in (1) except for the incentive compatibility constraint (13). Note that $h(w, \phi) = -\exp(-\gamma m/P^2)$, where $P^2 = M/\bar{y}$. Thus individuals know exactly the amount of type-2 goods they can get using the $m$ units of money assigned by the intermediary. The right-hand side of (13) is the agent’s value of deviation. If the agent reports $\tilde{\phi}$, then he will have $s(\phi - \tilde{\phi})$ units of endowments at his own disposal. As is mentioned before, the agent can either consume these goods or sell them in the market and then keep the money receipt to himself. Let $\rho$ be the fraction of the hidden goods that the agent chooses to consume. Then $(1 - \rho)s(\phi - \tilde{\phi})$ is the amount of endowments for which the agent hides the money receipts from the intermediary. The individual can spend the hidden money in the following period on type-1 goods.\footnote{For simplicity, I assume that the individual spends the hidden money all at once in the next period, if he chooses to deviate by hiding money. Conceivably, the individual could}
the individual’s optimal strategy of deviation is:

\[
\rho^*_t \begin{cases} 
0, & \text{if } s_t < \frac{\bar{y}}{E(\phi)} \\
\in [0, 1], & \text{if } s_t = \frac{\bar{y}}{E(\phi)} \\
1, & \text{if } s_t > \frac{\bar{y}}{E(\phi)}
\end{cases} \tag{14}
\]

Note that the incentive compatibility constraint for the planner’s problem (3) is indeed a special case of (13), where \( \rho_t = 1 \) for all \( t \). The optimal strategy (14) implies that \( \rho_t^* = 0 \) given \( \varepsilon < \bar{y}/[s^1 E(\phi)] \). Given the constrained efficient contract, the individual does not have a choice but to consume all hidden endowments if he decides to deviate. The reason is that agents do not have access to market trades because no money is used to implement allocations. Given that \( \rho_t^* = 0 \) for all \( t \), (13) reduces to:

\[
h(w, \phi) + \beta k(w, \phi) \geq h(w, \tilde{\phi}) \exp\left\{-\gamma(\phi - \tilde{\phi}) \frac{\bar{y}}{E(\phi)}\right\} + \beta k(w, \tilde{\phi}), \quad \forall \phi, \tilde{\phi} \in \Phi; \tilde{\phi} \neq \phi. \tag{15}
\]

All else equal, the right-hand side of (13) has a higher value than (3). In other words, the gain from deviation is higher in the current credit arrangement with money than that of the constrained efficiency problem. Therefore, the incentive compatibility constraint (13) of the intermediary’s problem is more stringent than of the constraint (3). The involvement of money in the benefit from spending the hidden money gradually over time to smooth consumption. In that case, the value of hiding money would be even higher than the one shown and the individual incentive problem would get even worse when money is involved in settlement of credit. Hence all main results would still apply if one chose to consider a full-fledged strategy to spend the hidden money.
current credit arrangement ends up making the individual incentive problem more severe, although it helps solve the intermediary’s incentive problem. Let $\sigma_m^*$ denote the intermediary’s optimal contract given by (11) subject to constraints (12) and (13). We have the following proposition:

**Proposition 7** There exists a unique equilibrium with fiat money and credit. For any given initial distribution $\psi_0$, $\varphi^*(\psi_0; \sigma_m^*) > \varphi^*(\psi_0; \sigma^*)$.

**Proof.** First recall that the contract design problem (11) is the same as in (1) except for the incentive compatibility constraint (13). It is straightforward to show that Proposition 1 also applies to the intermediary’s contract design problem (11), except that the following equations hold for part (iv) of Proposition 1:

$$h_t(w_t, \phi_t) + \beta k_t(w_t, \phi_t) = h_t(w_t, \tilde{\phi}_t) \exp\{-\gamma(\phi_t - \tilde{\phi}_t) \frac{y}{E(\phi)}\} + \beta k_t(w_t, \tilde{\phi}_t),$$

where $\phi_t = \phi^j$ and $\tilde{\phi}_t = \phi^{j+1}$ for all $j = 1, \cdots, J$. Following parts (i) and (ii) of Proposition 1, the contract design problem has a fixed point and the optimal contract $\sigma_m^*$ is unique. The existence and uniqueness of the equilibrium follows.

If $\rho_t = 1$ for all $t$, the incentive constraint (13) is equivalent to (3), which then implies that the contract design problems (11) and (1) are identical. Now suppose $\varphi^*(\psi_0; \sigma_m^*) < \varphi^*(\psi_0; \sigma^*)$. It follows immediately that $\sigma^*$ cannot be the optimal contract for (1) because $\sigma_m^*$ is implementable for
the intermediary’s problem and it achieves a lower expected cost. Therefore, it must be the case that \( \varphi^*(\psi_0; \sigma^*_m) \geq \varphi^*(\psi_0; \sigma^*) \). Then suppose \( \varphi^*(\psi_0; \sigma^*_m) = \varphi^*(\psi_0; \sigma^*) \). Since the optimal contract is unique, the supposition implies that \( \sigma^*_m \) induces \( \rho_t = 1 \) for all \( t \), which contradicts \( \rho^*_t = 0 \) for all \( t \). Therefore, it must be true that \( \varphi^*(\psi_0; \sigma^*_m) > \varphi^*(\psi_0; \sigma^*) \).

Proposition 7 shows that the optimal contract with fiat money cannot achieve constrained efficiency. This is because the intermediary is faced with a more severe individual incentive problem than the problem he would face when there is no aggregate uncertainty. Having access to fiat money causes stronger individual incentives to deviate because money is a store of value. That is, money allows individuals to purchase their preferred goods, which generate higher utilities than consuming their own endowments. In the current arrangement, fiat money plays two opposing roles: on one hand, requiring settlements in money induces individuals to engage in market trades, which generates information-revealing prices that solve the intermediary’s incentive problem. On the other hand, money as a store of value worsens the individual incentive problems.

Recall from Proposition 5 that \( \varphi^*(\psi_0; \sigma^*_{nm}) > \varphi^*(\psi_0; \sigma^*) \) and from Proposition 7 that \( \varphi^*(\psi_0; \sigma^*_m) > \varphi^*(\psi_0; \sigma^*) \). Given aggregate uncertainty, if the intermediary is allowed to choose whether to execute contracts with fiat money or with real goods, he will compare \( \varphi^*(\psi_0; \sigma^*_{nm}) \) with \( \varphi^*(\psi_0; \sigma^*_m) \). If \( \varphi^*(\psi_0; \sigma^*_{nm}) < \varphi^*(\psi_0; \sigma^*_m) \), the intermediary will choose to implement the contract with allocations of goods. This occurs when the severity of the
augmented individual incentive problems outweighs that of the intermediary’s incentive problem. If $\varphi^*(\psi_0; \sigma_{nm}^*) = \varphi^*(\psi_0; \sigma_m^*)$, then intermediary is indifferent between allocating money and allocating goods. Finally, if $\varphi^*(\psi_0; \sigma_{nm}^*) > \varphi^*(\psi_0; \sigma_m^*)$, the intermediary will choose to execute the contract with fiat money. This is the case when the intermediary’s incentive problem is so severe that fiat money is used to make payments, although at the cost of augmenting the individual incentive problems. In this case, fiat money is essential in that it helps resolve the intermediary’s incentive problem and save the overall cost of incentive compatibility.

In the next section, I examine the credit arrangement with private money, which allows the intermediary to achieve constrained efficiency given by $\sigma^*$.

6 Efficient Allocation with Private Money

Now assume that the intermediary is allowed to use the record-keeping technology to issue private money, i.e., banknotes. Every unit of banknote bears a financial history. In particular, the history contains full records of this particular unit of banknote being paid to and paid from the intermediary. If a banknote has been repaid to the intermediary at every $\tau = 1$ up to the current period, it is called legitimate money, or legitimate banknote. The total supply of banknotes is $M > 0$ units, where $M$ is public information. It is worthwhile clarifying that record keeping for market trades is still not available. Individuals trade anonymously in competitive markets.
At $t = 0$, the intermediary writes contracts with both type-1 and type-2 agents. The contract specifies that (i) at all $\tau = 1$ the intermediary allocates $M$ units of banknotes to each type-2 agent; (ii) at all $\tau = 1$ each type-1 agent must sell all his endowments to type-2 agents for banknotes. Then the type-1 agent reports his endowments to the intermediary and submits his current money receipts to the intermediary. Only legitimate banknotes will be accepted by the intermediary; (iii) at all $\tau = 2$ the intermediary allocates banknotes to each type-1 agent based on the latter’s reported history of endowment shocks; (iv) at all $\tau = 2$ each type-2 agent must sell all his endowments to type-1 agents for banknotes. The terms of the contract are public information. All markets are perfectly competitive. Since the contract involves monetary payments to and from the intermediary, market prices at $\tau = 1$ will fully discipline the intermediary’s announcement of the aggregate states. Provided that the above contract is also incentive compatible for type-1 agents, type-2 agents have no incentives to deviate from this arrangement.

Part (ii) of the above contract implies that an individual does not get credit for trying to submit any money that is not legitimate. This is critical in that it allows the intermediary to implement the efficient contract $\sigma^*$. Given the contract, type-1 agents can only gain credit by paying the intermediary with legitimate money. Accordingly, type-1 agents will sell their endowments only for legitimate banknotes. As a result, type-2 agents have no incentive to accept illegitimate banknotes. If a type-1 agent chooses to deviate by hiding banknotes from the intermediary, the hidden money will not be accepted
by anyone in the future. Such a deviation generates no value for the agent but wasting his endowments. Therefore, when money has a “memory” as described, an individual will only choose to hide endowments for his own consumption if he chooses to deviate. In this case the intermediary is faced with the same contract design problem as that of the case without aggregate uncertainty, which is given by (1) subject to constraints (2) and (3). Needless to say, the intermediary can implement efficient allocations given by $\sigma^*$, when dynamic credit meets private money. Private money is useful in solving the intermediary’s incentive problem and reducing individuals’ incentives to deviate.

7 Conclusion

I have constructed a dynamic model of money and credit with private information and aggregate uncertainty. Individuals form a long-term credit relationship with the intermediary. Individuals have private information about their random endowments and the intermediary has private information about the random aggregate states. I show that using money to settle debts helps solve the incentive problems on both the intermediary’s side and the individual side. Requiring settlements in money can induce market trades, which generate information-revealing prices to discipline the intermediary. However, money as a store of value poses another problem in contract design because it gives individuals stronger incentives to deviate.
If private money is prohibited, designing the optimal contract involves evaluating the cost of achieving individual incentive compatibility and the cost of achieving the intermediary’s incentive compatibility. If the severity of the augmented individual incentive problem outweighs that of the intermediary’s incentive problem, it is optimal to require settlements made in real goods. Otherwise, it is optimal to require settlements made in fiat money.

If private money is allowed, the optimal credit arrangement involves the intermediary issuing private money that can record its own history of being paid to the intermediary. Payments are required to be made with only money that has a “full record” (i.e., has been returned to the intermediary every period.) In the equilibrium, previously hidden money has no value because it will not be accepted as a means of settlement or a medium of exchange. According to this arrangement, money stores value only if it is properly used. This effectively reduces individuals’ incentives to deviate and allows intermediation to achieve the efficient allocations.
References


