Default, Mortgage Standards and Housing Liquidity*

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Abstract

The influence of households’ indebtedness on their house-selling decisions is studied in a tractable dynamic equilibrium model with search in the housing market and defaultable long-term mortgages. In equilibrium, sellers’ behavior varies significantly with their indebtedness. Specifically, both asking prices and time-to-sell increase with the relative size of sellers’ outstanding mortgages. In turn, the liquidity of the housing market associated with equilibrium time-to-sell determines the mortgage standards offered by competitive banks. When calibrated to the U.S. economy the model generates, as observed, negative correlations over time between both house prices and time-to-sell with downpayment ratios.

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1 Introduction

In this paper we study the influence of households’ mortgage debt on their house-selling decisions and the effects of the resulting housing market liquidity, by which we mean the speed with which houses can be sold on mortgage lending standards. To these ends, we develop a dynamic model with a frictional housing market and defaultable long-term mortgage debt. The model gives rise in equilibrium to distributions of both house prices and debt, as well as to endogenous default probabilities that differ across households. Aggregate shocks drive mortgage standards through their effect on liquidity, and this together with the mortgage terms offered to buyers affect the selling decisions of households.

Our theory demonstrates that (i) house-selling decisions depend critically on sellers’ levels of home equity, and (ii) liquidity in the housing market is an important factor in determining the ease with which households can borrow. A version of the economy calibrated to U.S. data captures, qualitatively, both the observed relationship between households’ mortgage loan-to-value (LTV) ratios and asking prices, and the negative correlation between house prices and mortgage lending standards (measured by down-payments) over time.

Our paper is motivated first by the observation that U.S. house prices are positively correlated with mortgage LTVs at origination (or, equivalently, correlated negatively with down-payment ratios) for first-time home buyers (see Figure 1). This phenomenon is particularly noticeable during the period leading up to the U.S. sub-prime mortgage crisis and subsequent house price collapse. Over this time period, U.S. housing markets were “booming” in the sense that prices were rising, sales volume increasing and time-to-sell declining (Ngai and Sheedy, 2015). At the same time, mortgage lending standards were relaxed, specifically and significantly in the sense of lowered down-payment requirements.\(^2\)

\(^1\) Actual down-payments do not necessarily reflect mortgage standards. First-time buyers, however, are the most likely to be affected by down-payment constraints.

\(^2\) Lending standards may include mortgage approval rates, down-payment ratios, document requirements, interest rates, etc. We focus on down-payment ratios, or LTV at origination. Gerardi et al. (2008) show that many subprime loans in this period were characterized by such high LTV’s. Duca et al. (2011) construct a series for loan-to-value ratios (LTVs) faced by first-time home buyers and show that the overall LTV ratio increased from 2000 to 2005. Barlevy and Fisher (2011) use data compiled for over 200 U.S. cities between 2000 and 2008 to find that interest-only (IO) mortgages were used sparingly in cities in which an elastic housing supply kept housing prices in check, but were common in cities with an inelastic supply in which housing prices rose sharply and then crashed. Dell’Ariccia et al. (2012) document and show that lending standards (denying rates) declined more in areas that experienced larger credit booms (more applicants) and greater price appreciation. Mian and Sufi (2009) find that regions with high latent demand from 2001 to 2005 experienced large relative decreases in denial rates, increases in mortgages originated, and increases in housing price appreciation, despite the fact that the same regions experienced significantly negative relative income and employment growth over this time period.
It has been argued that requirements were lowered to a greater extent than can be explained by explicit changes in regulatory constraints (see e.g. Belsky and Richardson, 2010). Here, we explore the incentive of profit maximizing lenders to relax lending standards in response to a “hot” housing market, by which we mean specifically one in which prices are high and time-to-sell is low by historical standards.

Figure 1: Values and percentage changes (from one year earlier) in average first-time home buyers’ down-payment ratios and S&P/Case-Shiller U.S. National Home Price Index. Source: American Housing Survey (AHS) 2007, 2009, 2011 national data.

An important consideration for lenders determining mortgage standards is borrowers’ default risk. A homeowner in financial distress can in principle avoid foreclosure by selling. Thus, the default risk of an indebted homeowner is to some extent tied to her strategy for selling her house. With this in mind, note that as observed by Genesove and Mayer (1997, 2001) and Anenberg (2011), house sellers’ leverage affects both their asking prices and time-to-sell. Specifically, sellers with high LTVs post higher asking prices, wait longer to sell, and sell ultimately at higher prices. This observation suggests that mortgage debt affects not only prices, but also households’ incentive to sell.

In our theory, we consider a growing population of ex ante identical households, each of which lives either in a single city (on which we focus) or in a largely unmodeled rest-of-the-world. All residents of the city require housing and may either live as a renter or
own one of a large number of identical houses, which are produced and sold initially by a competitive construction industry. Households enter the city when the value of doing so, determined in part by income in the city, exceeds their exogenous outside option. Once there, households remain in the city either as renters or homeowners until they leave as a result of exogenous shocks.

Houses are sold following the protocol of directed search as described by Moen (1997). Sellers offer houses for sale in a variety of sub-markets, within each of which prospective buyers and sellers are randomly matched. Each sub-market is characterized by a unique combination of a posted price and matching probabilities for buyers and sellers. These probabilities determine the expected time-to-buy and time-to-sell for buyers and sellers, respectively. Search is directed in the sense that buyers and sellers choose sub-markets optimally given the trade-off between the posted price and the matching probabilities.

House purchases are financed by mortgages offered by competitive mortgagees (e.g. banks) which control the terms offered. Specifically, mortgagees decide the size of the mortgage to offer, and this determines the LTV at origination or, equivalently, the down-payment ratio. Mortgages are of a fixed length and at an exogenous interest rate, which we model as determined by aggregate conditions rather than those within the city. Households who do not own pay rent each period equal to a fixed and exogenous fraction of income.

Mortgage holders are subject to random financial distress shocks which force them to either sell their homes through the search process or default and face foreclosure. Households in distress are not committed to sell, and based on their specific situations decide whether and how to do so. Thus, they effectively choose optimally their likelihood of default on mortgage debt. If a household defaults, its house is seized by the mortgage company, a foreclosure flag is placed on the its record, and it is prohibited from participating in the housing market until the flag is lifted, which also occurs randomly.

In principle, a homeowner in financial distress can make the probability of foreclosure arbitrarily low by posting a sufficiently low price and driving their likelihood of selling to one. In the equilibrium of our calibrated model, however, they do not do this. Rather, all households choose prices associated with substantial probabilities of default. This probability generally rises with the size of the household’s outstanding mortgage debt. Moreover, some households choose to default outright, making no attempt to sell. This occurs for households with negative home equity. This can occur in equilibrium for households with outstanding mortgages if house prices fall sufficiently.

Because house sellers are heterogeneous, there arises in equilibrium a distribution of
House prices which evolves over time owing to aggregate shocks.³ Home buyers remain identical, however, as we assume that goods are non-storable and rule out household saving. Free-entry of these homogeneous buyers into the housing market gives rise to the aforementioned trade-off between house prices and matching probabilities. Heterogeneous sellers separate themselves optimally into various submarkets based on their individual states. As a result, the individual decision problem is independent of the distribution of sellers and the model is block recursive as in Shi (2009) and Menzio and Shi (2010).

Regardless of the aggregate state, above a certain LTV, the prices posted in equilibrium by selling homeowners are steeply increasing in their outstanding mortgage debt, a result consistent with the empirical findings of Genesove and Mayer (1997). More highly levered sellers thus are more likely to default than are less levered ones. As a result, negative shocks to city-wide income cause particularly severe waves of default and foreclosure if they occur when the economy has a high proportion of highly levered homeowners.

Housing market liquidity affects mortgage lending standards both through the expected default rate and through lenders’ expected losses upon default. The more liquid the housing market, the higher the probability with which indebted households sell and thus the lower the rate of default and foreclosure. Similarly, mortgage companies also sell foreclosed houses more quickly, lowering the expected carrying cost of a foreclosed house and thus the cost of default. Finally, houses typically sell at higher prices in a hotter market, regardless of who sells (including mortgagees) and this further lowers the cost of default. Overall, mortgage companies are willing to offer larger mortgages, as well as allowing for higher LTVs at origination, when the housing market is more liquid and both income and house prices are high. As such, our theory generates a negative correlation over time between house prices and LTVs at origination.

The paper contributes to the growing literature on search frictions in the housing market (see, for example, Diaz and Jerez (2013), Petrosky-Nadeau and Rocheteau (2013), He et al. (2014), Head and Lloyd-Ellis (2012), and Wheaton (1990)). Specifically, we extend the theory of Head et al. (2014) (HLS) which focuses on the dynamics of house prices and construction in an environment with homogeneous buyers and sellers and complete financial markets. Here, we introduce a form of limited commitment which allows households to default under certain circumstances. This generates a role for mortgage debt secured by homes and generates heterogeneity among households, ex post. Also, while HLS focuses mainly on random search, competitive search is integral to our analysis and important for

³Houses are sold by construction firms, mortgagees, and home-owners differentiated by both their reasons for selling and levels of mortgage debt.
For the most part, this literature neither models mortgage contracts nor examines the long-term housing-lending relationship on which we focus. Three papers that do study theoretically the relationship between housing market liquidity and lending activities are Ungerer (2012), Hedlund (2015a) and Hedlund (2015b), with the latter two being the most closely related to ours.

As do we, Hedlund (2015a) and Hedlund (2015b) consider models featuring directed search (i.e., competitive search), long-term mortgages, and limited commitment. The models studied in both of these papers, however, feature a different market arrangement in which buyers and sellers do not interact directly but via the interaction of intermediaries who buy houses from heterogeneous sellers and then sell them, along with newly constructed houses, to heterogeneous buyers. Like ours, this setup renders the model block recursive and tractable. Here, as we focus on the differential decisions of sellers exclusively, find it useful to abstract from heterogeneity on the part of buyers not only for tractability, but also to simplify the calibration, presentation and interpretation of our findings.

In addition to having a different matching environment, our model also differs from those of Hedlund (2015a) and Hedlund (2015b) in that we study finite mortgages at fixed interest rates rather than infinite-horizon mortgage contracts with either fixed rates (as in Hedlund, 2015a) or flexible rates (as in Hedlund, 2015b). Our motivation here is principally realism, as in the U.S. conventional mortgages typically have a 30-year term and about 70% of these mortgages are at fixed interest rates. While finite-horizon contracts add complexity, they enable us to study how optimal house-trading decisions vary across households at different stages of mortgage repayment. Similarly, shocks affect households who have purchased at different times in the past differentially in our environment. Another important difference between our paper and Hedlund’s is that we focus on housing and mortgage markets at the city level, whereas he considers these markets at the national level.

In a New Keynesian model along the lines of Iacoviello (2005) with credit-constrained consumers and housing market frictions, Ungerer (2012) shows that expansionary monetary

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4 HLS does consider a case with competitive search, but only as a robustness check.

5 Directed search and sorting with two-sided heterogeneity is a challenging problem to tackle. There are a handful of papers that characterize the steady state of such an economy under certain conditions (see Shi (2001), Shimer and Smith (2000), Smith (2006), and Eeckhout and Kircher (2010)). Shi (2005) further shows that dynamics of sorting with two-sided heterogeneity can be tractable in some settings. Nevertheless, our model would be intractable for the purposes of the exercise we construct if we allowed for two-sided heterogeneity with endogenous saving alongside mortgage choices.

6 This percentage, however, has declined in recent years.
policy leads to higher leverage among homeowners. In his model, a decrease in mortgage interest rates boosts demand for housing. With more buyers in the frictional market, lenders can liquidate foreclosed houses more quickly, effectively reducing the expected carrying cost of a foreclosed house and making lender more willing to finance larger fractions of house purchases.

Our model differs from that studied by Ungerer (2012) in several respects: First, in Ungerer (2012) there is no default in equilibrium and thus no liquidation of houses. Financial frictions take the form of a Kiyotaki-Moore collateral constraint with lenders offering debt to the extent that default is prevented in equilibrium. In contrast, in our theory lending standards reflect default probabilities and the foreclosure inventory is a component of housing supply. As a result, market liquidity affects both the expected carrying cost of a foreclosed house and the expected default rate. Both factors contribute to the positive correlation between house prices and LTVs at origination. Second, we model debt as long-term rather than one-period. As noted above, we are able to trace out the default decision of indebted households at every stage of the mortgage repayment process. Finally, in Ungerer (2012) houses are divisible, the housing stock is fixed and there is no construction sector.

By focusing on the selling decisions of households, our paper is also related to those of Ngai and Tenreyro (2013) and Ngai and Sheedy (2015). Those papers focus, respectively, on the effect of aggregate conditions and seasonal fluctuations in demand on the decisions of homeowners to put their houses on the market. In contrast, we focus specifically on the selling decisions of heterogeneous homeowners distinguished by their levels of mortgage debt, whether financially distressed or not. As such, while both the models and specific issues studied vary between our paper and theirs, we view our work as complementary.

The remainder of this paper is organized as follows. Section 2 describes the environment for our baseline search economy. Section 3 formalizes the competitive search equilibrium. Section 4 presents an alternative environment with a frictionless housing market, rather than one featuring competitive search. Section 5 describes our baseline calibration and Section 6 characterizes the balanced growth path of our search economy parameterized in this way. Section 7 considers dynamics in response to aggregate shocks. Section 8 concludes.
2 The Environment

Time is infinite and discrete, with time periods indexed by $t$. The economy consists of the city, and the “rest of the world”. The aggregate economy is populated by a measure $Q(t)$ of $ex$ $ante$ identical households, which grows exogenously at net rate $\mu$. Each household lives indefinitely and supplies one unit of labor inelastically every period. In period $t$, the unit of labor supplied earns income $y(t)$, in units of a single date $t$ consumption good. Income follows a stationary stochastic process in log-levels.

Households in the city require housing, and may either rent or own one of a large number of symmetric housing units. Households’ preferences are represented by

$$U = \mathbb{E}(t) \left[ \sum_{t=0}^{\infty} \beta^t [u(c(t)) + z(t)] \right],$$

(1)

where $c(t)$ denotes consumption and $z(t)$ housing in period $t$, respectively. We assume that $z(t) = z_H$ if the household owns the house in which they live and $z(t) = 0$ otherwise. The function $u(\cdot)$ is strictly increasing, strictly concave and twice continuously differentiable, with the boundary properties: $\lim_{c \to \infty} u'(c) = 0$ and $\lim_{c \to 0} u'(c)$ sufficiently large. All households have the common discount factor, $\beta \in (0, 1)$. Both consumption goods and housing services are non-storable and there is no technology for households to save across periods.\textsuperscript{7}

The process by which households enter the city follows HLS. At the beginning of each period, measure $\mu Q(t)$ of new households arrive in the economy. Each of these households has a best alternative value to entering the city, denoted $\varepsilon$. These values are independently and identically distributed across new households via the distribution function $G(\varepsilon)$, with support $[0, \bar{\varepsilon}]$. Households that enter the city are separated randomly and permanently into two groups; those that value home-ownership and those that do not. The former we refer to as buyers and the latter as perpetual renters. Each period there exists a critical alternative value, $\varepsilon_c(t)$, below which a new household strictly prefers to enter the city:

$$\varepsilon_c(t) = \psi V_b(t) + (1 - \psi) V_p(t),$$

(2)

where $V_b(t)$ and $V_p(t)$ are lifetime (from time $t$) values of being a buyer and a perpetual renter, respectively, and $1 - \psi$ is the probability of the entrant becoming a perpetual renter.

New houses are built by a construction industry comprised of a large number of identical

\textsuperscript{7}This assumption renders all buyers in the housing market identical.
and competitive firms which we refer to as developers. Each new house requires one unit of land, which can be purchased in a competitive market at price $q(t) = Q(N(t))$. The developer also incurs construction cost $k(t) = K(N(t))$, where $N(t)$ denotes the total quantity of new houses built in period $t$. Houses require one period to build; those constructed in period $t$ become available for sale at the beginning of period $t+1$.

Home-ownership results immediately in both a utility benefit to the owner-occupier and a per-period maintenance cost. Houses depreciate over time, regardless of whether or not they are occupied. Depreciation is, however, offset by the owner at maintenance cost $d$ each period. Households in the city that do not own houses rent. We abstract from most aspects of the rental market, and assume that rent is equal to a fixed fraction of the city-level income; i.e. $R(t) = \gamma y(t)$. The supply of rental accommodation is totally elastic, and is not considered part of the city’s housing stock.

At the end of each period, all households in the city, regardless of their ownership status, may experience a shock which induces them to leave the city permanently. For perpetual renters and buyers (regardless of whether or not they currently own a house) these shocks occur with probabilities $\pi_p \in (0,1)$ and $\pi_h \in (0,1)$, respectively. All households which exit the city receive continuation utility, $\bar{V}$. Exiting homeowners also have vacant houses that they may want to sell, depending in part on their outstanding mortgage debt, if any. These households also have the option of defaulting.

In the city, the housing market is characterized by competitive search. We imagine it as being characterized by a large variety of potential sub-markets indexed by a price, $p$, and a pair of matching probabilities; one for buyers and one for sellers. Within each sub-market, matching takes place via a matching function, $\mathcal{M}(B,S)$, which is increasing in both arguments and has constant returns to scale. Given this form we can index submarkets by $(\theta, p)$, where $\theta$ denotes the market tightness (i.e., the ratio of the measures of buyers, $B$, and sellers, $S$, present in the sub-market) and $p$ the posted transaction price.

Both buyers and sellers take $(\theta, p)$ for all sub-markets as given and decide which to enter. The matching probabilities for buyers $\gamma(\theta)$ and sellers $\rho(\theta)$ are given by

$$
\gamma(\theta) = \frac{\mathcal{M}(B,S)}{B} = \mathcal{M}\left(1, \frac{1}{\theta}\right)
$$

$$
\rho(\theta) = \frac{\mathcal{M}(B,S)}{S} = \mathcal{M}(\theta, 1) = \theta \gamma(\theta).
$$

Each buyer and seller can enter only a single sub-market in a given period, and there is no cost of entry. Free-entry generates endogenously a trade-off between the house price and
the matching probability across active submarkets. Intuitively, higher-price sub-markets have lower levels of tightness as buyers (who are all identical) are willing to pay a higher price only if they are compensated with higher probability of matching with a seller.

The stock of searching buyers includes both newly entered households and those which have been searching unsuccessfully for some time. As noted above, these households are identical. Sellers, however, are of a number of different types. First, construction firms sell newly built homes. Second, homeowners who receive exit shocks as described above may decide to sell. Note that these buyers are heterogeneous to the extent that they have different outstanding mortgages. Home-owners may also sell as a result of a foreclosure shock (described below), and again they are differentiated by their outstanding mortgage. Finally, mortgagees sell foreclosed houses (see below).

In our calibration, prices typically exceed per period income and as there is no saving, households must borrow to finance house purchases.\(^8\) Mortgages are provided by a large number of perfectly competitive firms owned by risk-neutral investors who consume all profits and losses \textit{ex post}.\(^9\) To finance their loans, these mortgagees trade one-period risk-free bonds at an exogenous interest rate, \(i\), in an external bond market. They also incur a proportional service cost, \(\phi\), per period associated with the administration of mortgages.

The debt contract is a fixed-rate mortgage with finite maturity \(T\). Let \(m^t\) and \(r^t\) represent the size and interest rate on a mortgage loan issued in period \(t\), respectively. Contract, \((m^t, r^t, T)\), specifies a constant payment per period:

\[
x(m^t, r^t) = \frac{r^t}{1 - (1 + r^t)^{-T}} m^t.
\] (5)

As the homeowner makes payments, the principle balance on a period \(t\) mortgage, \(m^t_n\), evolves via

\[
m^t_{n+1} = (1 + r^t) m^t_n - x(m^t, r^t)
\] (6)

where \(n \in \{0, T - 1\}\) and \(m^t_0 = m^t\). Since \(T\) is fixed exogenously and \(r^t\) is constant over the life of the contract, \(x(\cdot)\) is unrelated to \(t\) after origination, and \(m^t_n\), for \(n \in \{0, T - 1\}\), represents the mortgage balance at the beginning of the period in which the \(n+1\)st payment

\(^8\)In the absence of saving, households would prefer to borrow to smooth consumption even if the house price were less than period income.

\(^9\)Alternatively, these firms could be owned by households to whom they would transfer their \textit{ex post} profits and losses lump-sum. This formulation would, however, complicate the computation without changing our results significantly.
will take place.\textsuperscript{10}

A borrower can terminate his/her mortgage contract at any time by paying off the remaining balance. Mortgages, however, are issued only on new home purchases.\textsuperscript{11} A mortgage termination is a default if the borrower does not repay all of the outstanding mortgage balance. Default leads to foreclosure, whereby the mortgage company takes control of the house, remitting to the borrower any surplus value of the house in excess of the outstanding loan balance. Mortgages are non-recourse, in that lenders do not have direct access to homeowners' current and/or future income in the event of a default.

Homeowners with outstanding mortgage debt receive, with probability $\pi_d$ each period, a financial distress shock. We interpret these shocks as representing circumstances such as accidents or unexpected illnesses that render the household unable to continue mortgage payments. Recipients of such shocks are referred to as distressed owners. They must terminate their current mortgage contract at the end of the current period and either pay their outstanding debt or default.

In the event of default, a borrower’s mortgage balance is set to zero and a foreclosure flag is placed on his/her credit record. The mortgage company repossesses the borrower’s house, puts it in real-estate-owned (REO) inventory, and decides whether and how to sell it starting the following period. As noted above, the defaulting homeowner receives the difference between the value of a house in REO inventory and the outstanding mortgage balance, if positive. Upon a successful sale, the mortgage company loses a fraction $\chi \in (0, 1)$ of the revenue to cover an exogenous cost, which we think of as representing, for example, legal fees. As a penalty for defaulting, buyers with foreclosure flags lose access to the mortgage market and are thus excluded from the housing market. Beginning with the following period, the foreclosure flag either remains on a buyer’s record, with probability $\pi_f \in (0, 1)$, or is removed.\textsuperscript{12}

In equilibrium, the mortgage rate is given by $r^t = i + \phi + \varrho$, where $i$ and $\phi$ are exogenously given as described above. The component $\varrho$ represents a moderate risk premium, which compensates for the risk of default.\textsuperscript{13}

Each period consists of two sub-periods. At the beginning of sub-period 1, new house-

\textsuperscript{10}That is, at the beginning of period $t$, $m^t_n$ represents the remaining balance on a mortgage issued in period $t - n - 1$, for $n = 0, \ldots, T - 1$.

\textsuperscript{11}In the absence of saving, a limitation on refinancing is required to generate a distribution of outstanding mortgage debts on the balanced growth path. In the absence of such a restriction homeowners would have incentive to refinance every period in order to smooth consumption.

\textsuperscript{12}According to the policies of Fannie Mae and Freddie Mac, foreclosure filings stay on a borrower’s credit record for a finite number of years.

\textsuperscript{13}Without $\varrho$, a mortgage contract earns strictly negative expected profit due to the positive probability of default.
holds with $\varepsilon \leq \varepsilon_c(t)$ enter the city. Income shocks, financial distress shocks and shocks on the foreclosure flag are all revealed. Immediately thereafter, the housing market opens: Buyers and sellers decide on the submarkets, $(p, \theta)$, in which to search and list their houses for sale, respectively. After the housing market closes, the mortgage sector becomes active: New owners take out mortgages to finance their purchases and current mortgage holders decide whether or not to default.

In sub-period 2, households receive income, make payments (for maintenance, on new house purchases, mortgages and/or rents), and consume the remainder. At the end of the period, moving shocks are revealed for all households and those who receive them leave the city immediately. Figure 2 provides an illustration of the timing of decisions.

3 Equilibrium

We begin by describing in detail the behavior of agents and then define an equilibrium for the environment described above, which we refer to as our baseline search economy.

3.1 Households

Consider households’ value functions sequentially for the two sub-periods of a typical time period $t$. Throughout $V(t)$’s will be used to denote agent and house values at the beginning of period $t$; while $W(t)$’s will be used to denote agents’ values at the beginning of the second sub-period of $t$. Sub-scripts will be used to distinguish agent and house states. In general, household values at the beginning of the second sub-period also depend on intra-period asset holdings, $a$. For agents who are home-owners, their values, $V_o(m_n, t)$ and $W_o(m_n, t)$,
depend also on their outstanding mortgage balance. Given that house purchases take place in the first sub-period, new homeowners values depend also on the price at which they purchased their home, \( W_0(p, m_0, t) \). From this point we will suppress the dependence of values on time where possible. In so doing, primes \( (e.g., \, V'_o) \) will be used to denote future values.

### 3.1.1 The first sub-period

House trading and mortgage default decisions are both made in the first sub-period. Let \( V_b \) denote the value function for a buyer. These households are either new entrants or those not owning a house and without a foreclosure flag, \( i.e. \) for whom \( f = 0 \):

\[
V_b = \max_{(p, \theta)} \left[ \gamma(\theta)W_o(p, m_0) + (1 - \gamma(\theta))W_b(0) \right].
\]

In sub-period 1, a buyer will search for a house to buy, choosing optimally to enter sub-market \( (p, \theta) \). The buyer is matched with a seller with probability \( \gamma(\theta) \), in which case she proceeds to sub-period 2 as a new owner with value \( W_0(p, m_0) \). The price paid and initial loan balance \( m_0 \) (offered by the mortgage company and specified below) determine the homeowner’s down-payment.\(^\text{14}\) With probability \( 1 - \gamma(\theta) \), the buyer fails to get a match, remains a buyer, and proceeds to sub-period 2 with value \( W_b(0) \). As noted above, the argument of \( W_b(\cdot) \) indicates the intra-period asset balance, \( a \). This will be non-zero only in the event that the buyer has sold a house in the sub-period that has just ended. A buyer who enters the current period without a house, necessarily enters the second sub-period with \( a = 0 \).

With buyers free to choose among them, all active sub-markets must offer the same value, \( V_b \). Using (7) yields

\[
\theta = \gamma^{-1} \left( \frac{V_b - W_b(0)}{W_0(p, m_0) - W_b(0)} \right) \equiv \theta(p). \quad (8)
\]

Thus, free-entry of buyers determines the relationship between the transaction price and market tightness across sub-markets.

Let \( V_o(m_n) \) denote the value for a resident owner with mortgage \( m^{t-n-1} \) at origination,

\(^{14}\)We measure loan-to-value (LTV) ratios by the mortgage balance relative to the value of a house in REO inventory. Thus, while home buyers make different down-payments depending on the sub-market in which they purchase, at each point in time the LTV at origination is the same for all mortgages.
who has made \( n - 1 \) payments and is not in financial distress:

\[
V_o(m_n) = \max_{p_s} \left\{ \rho(\theta(p_s)) W_b(\max[0, p_s - m_n]) + \left[ 1 - \rho(\theta(p_s)) \right] \max_{D_n \in \{0,1\}} \left\{ (1 - D_n) W_o(m_n) + D_n W_f(\max[0, \beta E[V'_{REO}] - m_n]) \right\} \right\}.
\]

(9)

This household decides whether and in which sub-market to sell her house. This can be represented by choice of asking price alone, given (8). If the household enters sub-market \( p_s \), then with probability, \( \rho(\theta(p_s)) \), the house is successfully sold. In this case, this individual repays as much outstanding debt as possible, and keeps the remaining profit, if any, to that \( a = \max[0, p_s - m_n] \). The household then proceeds to the second sub-period as a buyer without the foreclosure flag and value \( W_b(a) \). Note that the constraint \( p_s \geq m_n \) is not imposed on sellers. That is, the choice of selling price is not constrained to meeting/exceeding the outstanding debt. Short sales are an option in the sense that mortgage lenders allow indebted homeowners to clear their debt (without the consequence of foreclosure) with an amount lower than the outstanding balance, as long as the owner makes the effort to list the house and successfully sells at the listed price.\(^{15}\)

If the household chooses not to sell her house, or has failed to sell it, she then decides whether or not to default on her current mortgage contract. Here, for \( n = 0, \ldots, T - 1 \), \( D_n = 1 \) if a household who has made \( n - 1 \) payments on a mortgage (issued in period \( t - n - 1 \)) chooses to default rather than making the \( n \)th payment; and \( D_n = 0 \) otherwise. The value of a homeowner who has not defaulted at the beginning of the second sub-period is \( W_o(m_n) \). A homeowner who has defaulted has value \( W_f(a) \), where \( a = \max[0, \beta E[V'_{REO}] - m_n] \), at the beginning of the second sub-period. Such a homeowner effectively “sells” their house to the mortgage company for the expected discounted value of a vacant house in the mortgage company’s inventory at the beginning of the next period, \( \beta E[V'_{REO}] \). If this

\(^{15}\)For quantitative exercises, we have computed the seller decision problems both with and without the constraint \( p^* \geq m_n \). The constraint does not make a difference quantitatively. In particular, we find this constraint non-binding for all types of indebted sellers in both the steady state and the dynamics given the parameters and income shocks we adopt in the paper. Theoretically, it is straightforward to see why an indebted seller may find it optimal to list at a price \( p^* > m_n \), even when short sale is permitted. In the seller’s problem (9), for any choice of \( p^* \leq m_n \) the problem amounts to choosing \( p^* \) such that \( \rho(\theta(p^*)) \) is maximized. Since \( \rho(\theta(p^*)) \) is decreasing in \( p^* \), this is equivalent to minimizing the choice of price. Say, \( p^* \) is chosen such that the resulting selling probability is one, ignoring whether such price exists or not. For any \( p^* \leq m_n \), the maximized value of the right-hand side of (9) can be no greater than that of the value given \( \rho = 1 \). Given that \( W_b \) increases with \( p^* \), the seller will optimally choose to list the house at a price higher than the outstanding debt as long as there exists some \( \tilde{p}^* > m_n \) such that the maximized value is greater than the seller’s.

We allow for short sales of this type to illustrate that the positive relationship between LTV and asking prices does not depend on short sales being ruled out.
value is less than the household’s outstanding mortgage debt, \( m_n \), the household’s assets are set to zero. The expectation here is taken with respect to aggregate shocks which affect the value of vacant houses. If the value of the vacant house exceeds the debt, the defaulting homeowner keeps the residual value. In either case, the home-owner acquires a foreclosure flag.

Next, consider a resident owner who receives a financial distress shock at the beginning of period \( t \).\(^\text{16}\) Such a homeowner must terminate her mortgage contract within the same period. If the house is sold, the homeowner receives the residual value net of debt and then becomes a buyer without a foreclosure flag, \( W_b(\max[0, p_{sd} - m_n]) \). If the house is not sold, the owner defaults, the foreclosure flag is placed on her credit record. In this case, the homeowner receives the residual value of the house net of the debt and enters the next sub-period with value \( W_f(\max[0, \beta E [V^\prime_{REO}] - m_n]) \).\(^\text{17}\) Thus the value of a distressed resident owner with debt \( m_n \) is given by:

\[
V_f(m_n) = \max_{p_{sd}} \left\{ \rho(\theta(p_{sd})) W_b(\max[0, p_{sd} - m_n]) + [1 - \rho(\theta(p_{sd}))] W_f(\max[0, \beta E [V^\prime_{REO}] - m_n]) \right\}. \tag{10}
\]

A resident homeowner without a mortgage decides whether and how to sell her house. If the house is successfully sold, the owner moves on as a buyer with value \( W_b(p_{nd}) \). Otherwise, she moves onto the next sub-period as an owner without debt \( W_{nd} \). Such a homeowner has value

\[
V_{nd} = \max_{p_{nd}} \{ \rho(\theta(p_{nd})) W_b(p_{nd}) + [1 - \rho(\theta(p_{nd}))] W_{nd} \}. \tag{11}
\]

Next, consider homeowners who have left the city. Such households become irrelevant once they are no longer homeowners. As long as they are, however, they still make sales

\(^{16}\)In the event of financial distress, it is always in an owner’s best interest to attempt to sell if they have positive equity. If the housing market is liquid, distressed owners with positive equity would never default because they could sell immediately pay their mortgage debt. According to the RealtyTrac report, however, less than 50% of homeowners who go into foreclosure have negative equity. In our model, time-consuming search and matching account for this feature of the housing market.

\(^{17}\)Note that distressed resident owners can use proceeds from sales, but not labor income, to pay off outstanding mortgage debt. Relaxing this constraint would complicate the model without affecting the results significantly.
and default decisions. Such a homeowner has value:

\[
V_{Lw}(m_n) = \max_{p_{Lw}} \left\{ \rho(\theta(p_{Lw})) \left\{ u(p_{Lw} + y_L - R_L - d) \right\} + \beta E[V'_{REO}(m_n+1)] \right\} + \max_{D_{Ln}} \left\{ u(y_L - R_L - x_n - d) \right\} + \beta V + \left[ 1 - \rho(\theta(p_{Lw})) \right] \{ u(y_L - R_L - d) + \beta E[V'_{Lw}] \}. \tag{13} \]

Here, \( y_L, R_L, \) and \( d \) are income, rent and maintenance costs paid by the exiting household while lives outside the city.\(^{18}\) Also, \( x_n = x(m^t, r^t) \) denotes households’ \( n \)th payment on their mortgage issued \( n + 1 \) periods prior. Once the homeowner has either sold her house or defaulted, she receives exogenous continuation value, \( V' \).

The value of an owner who has left the city \textit{without} debt prior to moving is given by:

\[
V_{Lw} = \max_{p_{Lw}} \left\{ \rho(\theta(p_{Lw})) \left\{ u(p_{Lw} + y_L - R_L) + \beta V \right\} + \left[ 1 - \rho(\theta(p_{Lw})) \right] \{ u(y_L - R_L - d) + \beta E[V'_{Lw}] \} \right\}. \tag{13} \]

Such a household’s only decision is with regard to whether and at what price to sell.

### 3.1.2 Vacant Houses

At the beginning of the current period, the values of vacant houses in construction firms’ inventories, \( V_c \), and of foreclosed houses, \( V_{REO} \) are given, respectively, by

\[
V_c = \max_{p_c} \left\{ \rho(\theta(p_c)) \{ p_c + \left[ 1 - \rho(\theta(p_c)) \right] [ -d + \beta E[V'_{c}] ] \} \right\} \tag{14} \]

\[
V_{REO} = \max_{p_{REO}} \left\{ \rho(\theta(p_{REO})) (1 - \chi) \{ p_{REO} + \left[ 1 - \rho(\theta(p_{REO})) \right] [ -d + \beta E[V'_{REO}] ] \} \right\} \tag{15} \]

Note that in (15), it can be seen that the mortgage company loses fraction \( \chi \) of the proceeds of its sales as a cost of foreclosure.

### 3.1.3 Households in sub-period 2

As there is no saving, households’ behaviour in sub-period 2 effectively is trivial: They consume their income net of rent and mortgage payments, as well as whatever assets with

\(^{18}\)These quantities are necessary as long as the household remains a homeowner, because they impinge on its default and pricing decisions.
which they enter the sub-period. Here, we establish the value functions for the various household states at the beginning of this sub-period, which were used in the expressions above.

A perpetual renter remains a renter (never seeking to purchase a house) the entire time she stays in the city. Such a household’s value is given by:

$$V_p = W_p = u(y - R) + \pi_p \beta V + (1 - \pi_p) \beta E [W_p'] .$$

(16)

With probability $\pi_p$, the perpetual renter receives a moving shock, leaves the city immediately and receives the continuation value $\beta V$. Otherwise, she moves onto the next period as a renter. Her consumption is simply income net of rent.

A buyer with the foreclosure flag on her credit record has access neither to credit nor the housing market. She will remain renting until she moves out of the city or the foreclosure flag is lifted from her record. As above, $W_f(a)$ is the value of such a buyer with asset $a$ at the beginning of sub-period 2. Thus,

$$W_f(a) = u(y + a - R) + \pi_h \beta V + (1 - \pi_h) \beta \left\{ \pi_f E [W_f'(0)] + (1 - \pi_f) E [V_b'] \right\} .$$

(17)

Conditional on staying in the city, with probability $\pi_f$ the foreclosure flag remains and the household moves onto the following period with expected value $W_f'(0)$ (such households are inactive in the first sub-period of next period, as the foreclosure flag prevents them from purchasing a house). With probability $1 - \pi_f$, the foreclosure flag is lifted and this household will enter the next period as a buyer with value $V_b'$.

A buyer without the foreclosure flag at the beginning of sub-period 2 is either a resident owner who just successfully sold her house or a buyer who has failed to purchase a house in sub-period 1. Such a buyer may have a positive intra-period asset balance, $a$, coming from sale proceeds net of the outstanding mortgage debt in the previous sub-period. She will move on with value $V_b'$ and participate in the housing market in the next period if not hit by the moving shock at the end of the current period. The value of such a buyer is given by

$$W_b(a) = u(y + a - R) + \pi_h \beta V + (1 - \pi_h) \beta E [V_b'] .$$

(18)

A resident homeowner with a mortgage has the principle balance $m_n$. The owner’s periodic income is used to cover repayment, maintenance cost and consumption. Let
\[ W_o(m_n) = u(y - x_n - d) + z_H + \pi_h \beta E[V'_L(m_{n+1})] + (1 - \pi_h) \{ \pi_d \beta E[V'_f(m_{n+1})] + (1 - \pi_d) \beta E[V'_o(m_{n+1})]\}. \quad (19) \]

If the owner receives a moving shock, she exits the city immediately and continues with value \( V'_L(m_{n+1}) \). Note that her mortgage debt does not vanish because she has relocated. Conditional on not relocating, in the next period the owner receives a financial distress shock with probability \( \pi_d \). In this case, she continues as a distressed resident owner with debt \( V'_f(m_{n+1}) \). Otherwise, she enters the next period as a non-distressed owner with value \( V'_o(m_{n+1}) \).

For \( n = T - 1 \), a resident homeowner with a mortgage has value
\[ W_o(m_{T-1}) = u(y - x_{T-1} - d) + z_H + \pi_h \beta V'_Lw + (1 - \pi_h) \beta E[V'_{nd}]. \quad (20) \]

In this case, the current mortgage payment is the homeowner’s last. Thus, she will continue on with value \( V'_Lw \) if hit by the moving shock (in which case she leaves the city owning a house but having no debt) and with value \( V'_{nd} \) if she remains in the city.

A new owner who has purchased a house in the preceding sub-period pays the difference between the purchase price and total debt \( m_0 \); that is, the down-payment. The periodic mortgage payment begins from the following period. Let \( W_o(p, m_0) \) denote the value of a new homeowner:
\[ W_o(p, m_0) = u(y - (p - m_0) - d) + z_H + \pi_h \beta V'_L(m_0) + (1 - \pi_h) \{ \pi_d \beta E[V'_f(m_0)] + (1 - \pi_d) \beta E[V'_o(m_0)]\}. \quad (21) \]

Finally, homeowners without mortgage debt do not suffer financial distress shocks. They remain in the city until they experience a moving shock. The value of such an owner is given by
\[ W_{nd} = u(y - d) + z_H + \pi_h \beta V'_Lw + (1 - \pi_h) \beta E[V'_{nd}]. \quad (22) \]

### 3.2 Developers

As noted above, the construction industry is comprised of a large number of competitive firms. Free entry into the industry ensures that in equilibrium the cost of building a house
equals the expected value of a vacant house for sale in period $t + 1$:

$$Q(N) + K(N) = \beta E [V_c'] .$$

### 3.3 Mortgagees

Because a mortgagee has access to funds at a fixed cost, it issues mortgages until it earns zero profit on each contract. In particular, the expected return net of expected foreclosure costs on mortgages will equal the opportunity cost of funds, that is, the interest rate $i$ of the external bonds plus the servicing cost $\phi$. Houses are identical, households cannot save over time, and regular repayments of all new mortgages start from the period following that in which the house is purchased and the mortgage initiated. As such, all new borrowers are identical to the mortgage company at the point of loan origination. Therefore, mortgagees loan the same amount, $m'$, to all new borrowers in period $t$, regardless of the price they pay for their house.

Again suppressing time super-scripts. Let $P(m_n)$, for $n \in \{0, \cdots, T-1\}$, be the present at the beginning of the current sub-period 2 of a mortgage issued in $n + 1$ periods before and held by a resident homeowner. That is, the value of a mortgage of original size $m^{t-n-1}$ after $n - 1$ payments have been made. Correspondingly, let $P_L(m_n)$, be the present value of such a mortgage held by an owner that has relocated.\(^{19}\) Then, for $n \in \{0, 1, \cdots, T - 1\}$,

$$P(m_n) = x_n \mathbb{I}_{\{n \neq 0\}} + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1 + i + \phi} \times$$

$$\pi_h \left[ \rho(\theta(p'_L)) \min \left[ p'_L, m'_{n+1} \right] + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1 + i + \phi} \times \right.$$  

$$\pi_d \left[ \rho(\theta(p'_s)) \min \left[ p'_s, m'_{n+1} \right] + [1 - \rho(\theta(p'_s))] \min \left[ \beta V'_{REO}, m'_{n+1} \right] \right]$$

$$\right]$$

$$+ \left[ 1 - \rho(\theta(p'_s))] \min \left[ \beta V'_{REO}, m'_{n+1} \right] \right]$$

$$\left] \right.$$

$$\right]$$

\(^{19}\)For such owners, we have $n \geq 1$ as one repayment has already been made by the beginning of the first sub-period 2 following the household’s relocation.
and for all \( n \in \{1, \ldots, T - 1\} \),

\[
P_L(m_n) = x_n + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1 + i + \phi} E \left[ \frac{\rho(\theta(p'_L)) \min[p'_L, m_{n+1}] + D'_{L_{n+1}} \min[\beta V_{REO}^{''}, m_{n+1}]}{1 - \rho(\theta(p'_L))} \right] \]

(25)

where \( p_s, p'_{sd}, p'_L, D'_{n+1}, D'_{L_{n+1}} \) are household policies (sales prices and default decisions) in period \( t + 1 \) contingent on having mortgage balance \( m_{n+1} \). Also, \( I_{\{n \neq 0\}} \) and \( I_{\{n \neq T-1\}} \) are index functions, with

\[
I_{\{n \neq 0\}} = \begin{cases} 0, \text{ if } n = 0 \\ 1, \text{ otherwise} \end{cases}
\]

(26)

\[
I_{\{n \neq T-1\}} = \begin{cases} 0, \text{ if } n = T - 1 \\ 1, \text{ otherwise} \end{cases}
\]

(27)

Here, \( I_{\{n \neq 0\}} \) indicates a mortgage on which the borrower is making regular repayments beginning with the period after origination, while (27) indicates a mortgage that matures after the current repayment is made. The present value, \( P(m_n) \), equals the current period payment \( x_n \) on a mortgage originated \( n + 1 \) periods ago, plus the discounted expected value of the mortgage in period in the next. The latter is affected by the probabilities of the borrower receiving a moving and/or financial distress shock, and the decisions the household will make regarding pricing and/or default in the event that such shocks are realized. Note again that these decisions do not depend on the price at which the homeowner originally purchased.

To compute the present value of a mortgage contract at origination, we proceed recursively. First, we compute \( P(m_{T-1}) \), and then use backward induction to obtain \( P(m_n) \) for \( n \in \{0, \ldots, T - 2\} \). The value \( P_L(m_n) \) is determined in a similar way except that once relocated borrowers experience neither moving nor distress shocks.

If a borrower sells her house in period \( t + 1 \), the amount that the mortgage company will receive is the minimum of the sale proceeds and the outstanding debt \( m_n \). The equilibrium mortgage loan size, \( m^t \), is then determined using the mortgage lender’s zero-profit condition:

\[
P(m^t) - m^t = 0.
\]

(28)
3.4 Laws of motion

We now describe the evolution of the distributions of households and houses across states. We express all in per capita terms (i.e. divided by the total economy population, $Q_t$). At the beginning of period $t$, the households are divided into renters; perpetual renters, $F(t)$; buyers without a foreclosure flag, $B(t)$; “buyers” with a foreclosure flag, $B_f(t)$, and homeowners. This latter group are either residents in the city or have relocated and still own a house. Within each type, households are further differentiated by their mortgage balance, if any. For $n = 0, \ldots, T-1$, let $H_n(t)$ denote the period $t$ measure of city resident homeowners who were issued a mortgage in period $t - n - 1$ and thus have made $n - 1$ payments prior to period $t$. Similarly, $H_{Ln}(t)$ denotes the measure of homeowners who have relocated by period $t$ holding such a mortgage. Let $H_0(t)$ and $H_{L0}(t)$, respectively denote resident and re-located homeowners who no longer have an outstanding mortgage. Note that there is no need to keep track of relocated households once they cease to be homeowners (because they have already either sold or defaulted).

Houses for sale are either held vacant by relocated homeowners, construction firms and mortgage lenders or offered by distressed homeowners. Denote the inventories of construction firms and mortgage lenders by $H_c(t)$ and $H_{REO}(t)$ respectively. Suppressing the time indicator, the total stock of houses for sale in the current period is then given by:

$$H_s = H_{L0} + \sum_{n=1}^{T-1} H_{Ln} + H_c + H_{REO} + \pi_d \sum_{n=0}^{T-1} H_n.$$  

(29)

Similarly, the total current period measure of buyers searching to trade in the housing market, $B_{sum}$, can be written:

$$B_{sum} = \sum_{n=0}^{T-1} \left[ (1 - \pi_d) \theta(p_s) H_n + \pi_d \theta(p_{nd}) H_n + \theta(p_L) H_{Ln} \right]$$

$$+ \theta(p_{nd}) H_0 + \theta(p_{Lnd}) H_{L0} + \theta(p_c) H_c + \theta(p_{REO}) H_{REO}.$$  

(30)

In particular, the measure of buyers in an active sub-market equals the measure of sellers in that sub-market multiplied by the the corresponding market tightness. For example, the measure of buyers searching for foreclosed houses sold by a mortgagee equals the measure of REO houses, $H_{REO}$, multiplied by the tightness of the mortgagee’s optimally chosen submarket, $\theta(p_{REO})$.

Continuing to suppress the dependence of variables on time, we now write out the laws
of motion for the stocks households and houses in the various states. To begin with, the *per capita* measure of permanent renters *next period* consists of those remaining from the current period and those who will have newly entered:

\[
(1 + \mu)F' = (1 - \pi_p)F + (1 - \psi)G(\varepsilon'_c)\mu. \tag{31}
\]

Similarly, the measure of buyers with foreclosure flags next period includes those remaining from the current period who have neither moved nor had their flag removed randomly. To this is added the measure of resident homeowners who default this period. These homeowners may either have received a financial distress shock and failed to sell or have defaulted strategically. Thus, we have

\[
(1 + \mu)B'_f = (1 - \pi_h) \left\{ \frac{\pi_f B_f + \sum_{n=0}^{T-1} (1 - \rho(\theta(p_{sd}))) H_n}{\pi_d + \sum_{n=0}^{T-1} (1 - \rho(\theta(p_s))) D_n H_n} \right\}, \tag{32}
\]

where as above, \( p_{sd}, p_s, \) and \( D \) represent optimal pricing and default decisions. Note that in general these depend on homeowners’ outstanding mortgages.

The measure of buyers *without* foreclosure flags at the beginning of next period consists of newly-entering buyers, previously flagged buyers whose flag has been removed, and non-relocating buyers from the current period who fail to buy a house. Note that the measure of buyers who successfully match in the current period equals the sum of the measures of the prospective sellers of various types multiplied by their corresponding matching probabilities. Thus, we have

\[
(1 + \mu)B' = \psi G(\varepsilon'_c)\mu + (1 - \pi_f)B_f + \left(1 - \pi_h\right) \left\{ B - \rho(\theta(p_{Lw})) H_{L\theta} - \rho(\theta(p_c)) H_c - \rho(\theta(p_{REO})) H_{REO} - \sum_{n=1}^{T-1} \rho(\theta(p_L)) H_{Ln} \right\}. \tag{33}
\]

The measure of indebted owners who have made \( n \) periodic payments by the beginning of period \( t + 1 \) on a mortgage of size \( m^{t-n-1} \) at origination evolves (for \( n > 0 \)) via:

\[
(1 + \mu)H'_n = (1 - \pi_h)(1 - \pi_d) (1 - \rho(\theta(p_s)) (1 - D_n) H_{n-1}. \tag{34}
\]
That is, the indebted owners with an ongoing mortgage going into the next period are the indebted owners from the current period who do not move, experience financial distress, successfully sell their house, or default strategically.

For \( n = 0 \), \( H'_{0} \) is the measure of resident homeowners who successfully purchase a house, remain in the city and do not experience financial distress. This measure can be recovered from the number of sales in the current period. Thus we have

\[
(1 + \mu)H'_{0} = (1 - \pi_{n}) \left\{ \begin{array}{c}
(1 - \pi_{d}) \sum_{n=0}^{T-1} \rho \left( \theta(p_{s}) \right) H_{n} \\
+ \pi_{d} \sum_{n=0}^{T-1} \rho \left( \theta(p_{sd}) \right) H_{n} \\
+ \sum_{n=1}^{T-1} \rho \left( \theta(p_{L}) \right) H_{Ln} \\
+ \rho \left( \theta(p_{nd}) \right) H_{0} + \rho \left( \theta(p_{Lw}) \right) H_{L0} \\
+ \rho \left( \theta(p_{c}) \right) H_{c} + \rho \left( \theta(p_{REO}) \right) H_{REO} \\
\end{array} \right\}. \quad (35)
\]

Finally, the measure of resident owners without a mortgage evolves via

\[
(1 + \mu)H'_{0} = (1 - \pi_{n}) \left\{ \begin{array}{c}
(1 - \pi_{d})(1 - \rho \left( \theta(p_{d}) \right))(1 - D_{T-1})H_{T-1} \\
+(1 - \rho \left( \theta(p_{nd}) \right))H_{0} \\
\end{array} \right\}. \quad (36)
\]

This group is comprised of its previous members who have neither move nor sell plus resident homeowners who make their last mortgage payment in the current period.

Proceeding similarly for relocated homeowners, \( H'_{Ln} \) is the measure who made their \((n + 1)\)st payments in the period \( t \). Again, the loan volume at origination is \( m^{t-n-1} \) and \( H_{L0} \) is the current period measure of relocated owners without debt:

\[
(1 + \mu)H'_{Ln} = (1 - \rho \left( \theta(p_{L}) \right))(1 - D_{Ln-1})H_{Ln-1} \quad (37)
\]

\[
\begin{array}{c}
+ \pi_{h}(1 - \pi_{d})(1 - \rho \left( \theta(p_{s}) \right))(1 - D_{n-1})H_{n-1}; \\
\end{array}
\]

\[
(1 + \mu)H'_{L0} = \pi_{h} \left\{ \begin{array}{c}
(1 - \pi_{d}) \sum_{n=0}^{T-1} \rho \left( \theta(p_{s}) \right) H_{n} \\
+ \pi_{d} \sum_{n=0}^{T-1} \rho \left( \theta(p_{sd}) \right) H_{n} \\
+ \sum_{n=0}^{T-1} \rho \left( \theta(p_{L}) \right) H_{Ln} \\
+ \rho \left( \theta(p_{nd}) \right) H_{0} + \rho \left( \theta(p_{Lw}) \right) H_{L0} \\
+ \rho \left( \theta(p_{c}) \right) H_{c} + \rho \left( \theta(p_{REO}) \right) H_{REO} \\
\end{array} \right\}. \quad (38)
\]
\[(1 + \mu)H'_{L0} = \pi_h \left\{ (1 - \rho(\theta(p_{nd}))H_0 \right. \\
+ (1 - \pi_d)(1 - \rho(\theta(p_s)))(1 - D_{T-1})H_{T-1} \\
+ (1 - \rho(\theta(p_L)))(1 - D_{LT-1})H_{LT-1} \\
+ (1 - \rho(\theta(p_{Lnd})))H_{L0} \right\} \] (39)

As depreciation is offset by maintenance, the per capita city housing stock evolves via

\[(1 + \mu)H' = H + N, \] (40)

where \(N\) is the measure houses built in the current period and available for sale in the next.

The per capita stock of houses in construction firms’ inventory at the beginning of the next period includes houses held by construction firms that go unsold in the current period plus those that are newly built:

\[(1 + \mu)H'_c = (1 - \rho(\theta(p_c)))H_c + N. \] (41)

Finally, the stock of houses in the REO inventory at the beginning of the next period, \(H'_{REO}\), includes those that go unsold in the current period plus the new foreclosures:

\[(1 + \mu)H'_{REO} = (1 - \rho(\theta(p_{REO})))H_{REO} \right. \\
+ \pi_d \sum_{n=0}^{T-1} (1 - \rho(\theta(p_{sd})))H_n \\
+ (1 - \pi_d) \sum_{n=0}^{T-1} (1 - \rho(\theta(p_s)))D_nH_n \\
+ \sum_{n=1}^{T-1} (1 - \rho(\theta(p_L))D_{Ln}H_{Ln}. \right\} \] (42)

3.5 A Directed Search Equilibrium

Definition. Given a mortgage interest rate, \(r\); rent level, \(R\); terminal continuation value, \(\bar{V}\); and a stochastic process for city-level income, \(y\), a directed search equilibrium is, for all
periods, a collection of

1. Household value functions:

\[ V_b, W_b; V_o, W_o; V_f, W_f; V_{nd}, W_{nd}; V_L, V_{Lw}, V_p, W_p \]  \hspace{1cm} (43)

with associated policy functions (choices of sub-market to enter and whether to default):

\[ p_s, p_{sd}, p_{nd}, p_L, p_{Lw}, D_n, D_{Ln}, \quad n = 0, \ldots, T - 1; \]  \hspace{1cm} (44)

2. House values:

\[ V_c, V_{REO} \]  \hspace{1cm} (45)

with associated policies for developers and mortgage firms:

\[ p_c, p_{REO}; \]  \hspace{1cm} (46)

3. An entry cut-off and mortgage contract:

\[ \varepsilon_c, m_0; \]  \hspace{1cm} (47)

4. And per capita measures of households and houses

\[ \{F, B, B_f, H_n, H_{Ln}, \underbrace{H_\emptyset, H_{L\emptyset}}_{\text{households}}, n = 0, \ldots, T - 1; \} \quad \{H, N, H_c, H_{REO} \} \] \hspace{1cm} (48)

Such that:

1. New households enter the city optimally so that (2) holds;

2. All agents optimize such that the value and policy functions listed in (43) - (46)
satisfy (7), (9) - (22);

3. Free entry of construction firms: \( N \) satisfies (23);

4. Free entry of mortgage companies: \( m_0 = m^t \) satisfies (28);

5. The stocks of households and inventories of houses evolve according to (31) - (40);
6. “Market clearing”: \( B = B_{\text{sum}} \).

Requirements 1-5 in the above definition are standard and have been described in detail above. Requirement 6 states that in equilibrium the measure of buyers without foreclosure flags must be consistent with the total measure of buyers actively participating in housing search. Alternatively, it means that all buyers enter some sub-market.

As has been mentioned, sellers are heterogeneous, and each period their distribution is characterized by \((H_n, H_{Ln}, H_\emptyset, H_{L\emptyset}, H_c, H_{REO})\). The decision problems faced by households, developers and mortgage companies are, however, not affected by this distribution. In fact, as can be seen from (7), and (9) - (28), all of the value and policy functions listed in (43) - (46), together with the mortgage contract \(m_0\), are independent of the stocks listed in (48). This is true despite the fact that the stocks themselves depend on individual decisions, and that the distribution of sellers does affect aggregate statistics.

Thus, the model is block recursive in the sense of Shi (2009). As discussed there, block recursivity arises because heterogeneous sellers select themselves optimally into separate submarkets through the directed search mechanism. In doing so, they take the trade-off between the price and the matching probability as given. Given a particular target transaction price, the only factor that matters for a seller’s trading decision is the probability with which it will be matched with a buyer; the distribution of sellers over other price targets is irrelevant. Vice versa, for a given matching probability a seller cares only about the price at which it can sell.

Block recursivity greatly aids tractability by eliminating the role of the distribution of sellers in individual decisions. It is especially useful here as it enables us to examine the dynamics of the model in response to aggregate shocks.

4 An Economy without Search

To illustrate the role of search frictions in our model, we consider an environment in which the housing market is perfectly competitive. Here, houses are perfectly liquid in that buyers (without a foreclosure flag) and sellers are able to trade immediately and neither construction firms nor mortgage companies hold houses in inventory.

In this setting, financial distress is extreme — at the beginning of period \( t \), with probability \( \pi_d \) an indebted resident owner may experience a default shock which forces her to default immediately. A borrower not hit by such a shock may choose to default only in
the case in which their housing equity becomes negative.\textsuperscript{20}

4.1 Value functions

Household decisions in sub-period 2 are identical to those in the search economy. Subperiod 1 household values here are distinguished by the superscript \( w \). A buyer without the foreclosure flag purchases a house at competitive price \( p_t \) and immediately becomes an owner with value \( V_{o}^w(m_0) \). An indebted resident owner who does not receive a default shock decides whether and how to sell and whether or not to default. As before, let \( D^w \in \{0, 1\} \) be the default indicator. If the owner sells, she repays as much of her outstanding debt as possible, keeps any remaining profit, and becomes a buyer without the foreclosure flag. If she decides not to sell, then she decides whether to default:

\[
V_{o}^w(m_n) = \max \left\{ \begin{array}{c}
W_b^w(\max \{0, p - m_n\}) \;
\text{sell} \\
(1 - D^w)W_b(m_n) + D^wW_f^w(\max\{0, \beta E[V_{REO}^w] - m_n\}) \;
\text{don’t sell}
\end{array} \right\}
\] (49)

where \( V_{REO}^w = (1 - \chi)V_c^w \) is the value of a vacant house in the next period, net of the foreclosure cost, \( \chi \).

An indebted owner who experiences a distress shock immediately defaults. Such an owner has the value:

\[
V_f^w(m_n) = W_f^w(\max\{0, \beta E[V_{REO}^w] - m_n\}).
\] (50)

A resident owner without debt decides whether or not to sell and has value:

\[
V_{nd}^w = \max \{W_b^w(\max \{0, p - m_n\}), W_o^w(m_n)\}.
\] (51)

Relocated owners with and without mortgage debt make similar selling and default

\textsuperscript{20}Note, however, that as default is costly, not all owners with negative equity will default.
decisions and have values $V^w_L(m_n)$ and $V^w_{Lnd}$, respectively:

$$V^w_L(m_n) = \max \left\{ \begin{array}{l}
u(\max [0, p - m_n] + y^L - R^L) + \beta V + \\
\max_{D_L^w \in \{0, 1\}} \begin{cases}
(1 - D^w_L)(u(y^L - R^L - x_n - d) + \beta E[V^w_L(m_{n+1})]) + \\
+ D_L^w(u(\max [0, \beta E[V^w_{REO}] - m_n] + y^L - R^L) + \beta V)
\end{cases}
\end{array} \right. \quad (52)$$

$$V^w_{Lnd} = \max \left\{ \begin{array}{l}
u(p + y^L - R^L) + \beta \overline{V}, u(y^L - R^L - d) + \beta E[V^w_{REO}] \end{array} \right. \quad (53)$$

The values of vacant houses to both construction firms and mortgage companies are given respectively by:

$$V^w_c = p$$

$$V^w_{REO} = (1 - \chi)p. \quad (54)$$

For mortgage contract $t = (m^t; r^t)$, the present mortgage values at the beginning of sub-period 2 after $n-1$ payments are given, for relocated and resident homeowners, respectively, are given by:

$$P_L(m_n) = x_n + \frac{1}{1 + i + \phi} \times$$

$$E \left\{ \begin{array}{cl}
\begin{array}{l}
\min \{p', m_{n+1}\}, \\
\max_{D^w_{L_{n+1}}} \left[ [D^w_{n+1} \min \{\beta V^w_{REO}, m_{n+1}\} + (1 - D^w_{n+1})P^t_L(m_{n+1}) \} \right]
\end{array}
\end{array} \right. \quad (56)$$
for \( n \in \{1, \ldots, T-1\} \), and

\[
P_t^i(m_n) = \frac{x_n \mathbb{I}_{\{n \neq 0\}}}{1 + \iota + \phi} + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1 + \iota + \phi} \times E \left\{ \pi_h \max \left\{ \min[p, m_{n+1}], \max_{D_{Ln+1}^w} \left( D_{Ln}^w \min [\beta V_{REO}^w, m_{n+1}] + (1 - D_{Ln}^w) P_L^t(m_{n+1}) \right) \right\} \right\}
\]

\[
\times E \left\{ \pi_d \min [\beta V_{REO}^w, m_{n+1}] + (1 - \pi_d) \max \left\{ \min[p', m_{n+1}], \max_{D_{Ln+1}^w} \left( D_{n+1}^w \min [\beta V_{REO}^w, m_{n+1}], (1 - D_{n+1}^w) P_t^{n+1}(m_{n+1}) \right) \right\} \right\}
\]

for all \( n \in \{0, \ldots, T-1\} \), where, \( D_{n+1}^w \) and \( D_{Ln+1}^w \) are households default choices in the next period, conditional on the aggregate shocks and having mortgage balance, \( m_{n+1} \).

### 4.2 Equilibrium

The definition of equilibrium is similar to that for the search economy except that the housing market now clears each period in the Walrasian sense. All households (other than permanent renters) who begin the period without a house are buyers. If the measure of buyers exceeds the sum of the measures of new and foreclosed houses, then the price of housing adjusts until the appropriate measure of current homeowners chooses to sell. A shortage of buyers (and thus \( p = 0 \)) is avoided by the continual entry of buyers without homes driven by population growth. Finally, the per capita laws of motion for households are listed in Appendix A.

### 5 Calibration

We now choose parameters for both the baseline search and non-search economies to match selected facts of the U.S. economy along a balanced growth path. In this steady-state, the housing stock grows at the rate of population growth and all other components of the equilibrium, including the distribution of agents across states and real house values, remain constant.
For the baseline search economy, we choose the following functional forms:

\[ u(c) = \ln(c) \]
\[ M(B, S) = \varpi B^n S^{1-n} \]
\[ k = \frac{1}{\kappa} N^{\frac{1}{\xi}} \]
\[ q = \bar{q} N^{\frac{1}{\xi}} \]  

(58)

where \( \eta \) is the elasticity of the measure of matches with respect to the measure of buyers and \( \xi \) represents the elasticity of new land supply with respect to land prices.

Table 1: Calibration Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.96</td>
<td>Annual interest rate</td>
<td>4.0%</td>
</tr>
<tr>
<td>( \pi_p )</td>
<td>0.120</td>
<td>Annual mobility of renters</td>
<td>12%</td>
</tr>
<tr>
<td>( \pi_h )</td>
<td>0.032</td>
<td>Annual mobility of owners</td>
<td>3.2%</td>
</tr>
<tr>
<td>( \xi )</td>
<td>1.75</td>
<td>Median price-elasticity of land supply</td>
<td>1.75</td>
</tr>
<tr>
<td>( i )</td>
<td>0.040</td>
<td>International bond annual yield</td>
<td>4.0%</td>
</tr>
<tr>
<td>( T )</td>
<td>30</td>
<td>Fixed-rate mortgage maturity (years)</td>
<td>30</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.012</td>
<td>Annual population growth rate</td>
<td>1.2%</td>
</tr>
<tr>
<td>( \pi_f )</td>
<td>0.80</td>
<td>Average duration (years) of foreclosure flag</td>
<td>5</td>
</tr>
<tr>
<td>( \bar{q} )</td>
<td>0.96</td>
<td>Average land-price-to-income ratio</td>
<td>30%</td>
</tr>
<tr>
<td>( m )</td>
<td>0.08</td>
<td>Residential housing gross depreciation rate</td>
<td>2.5%</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>5</td>
<td>Median price elasticity of new construction</td>
<td>5</td>
</tr>
<tr>
<td>( \varsigma )</td>
<td>0.16</td>
<td>Rent-price ratio</td>
<td>5%</td>
</tr>
</tbody>
</table>

**Parameters determined independently**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \chi )</td>
<td>0.440</td>
<td>Loss severity rate</td>
<td>27%</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.0246</td>
<td>Average down-payment ratio</td>
<td>20%</td>
</tr>
<tr>
<td>( \varrho )</td>
<td>0.0074</td>
<td>Average annual FRM-yield</td>
<td>7.20%</td>
</tr>
<tr>
<td>( \psi )</td>
<td>0.570</td>
<td>Fraction of households that rent</td>
<td>33.3%</td>
</tr>
<tr>
<td>( \pi_d )</td>
<td>0.060</td>
<td>Annual foreclosure rate</td>
<td>1.6%</td>
</tr>
<tr>
<td>( z_H )</td>
<td>0.3280</td>
<td>Average loan-to-income ratio at origination</td>
<td>2.72</td>
</tr>
<tr>
<td>( \varpi )</td>
<td>0.56</td>
<td>Average fraction of delinquent loans repossessed</td>
<td>33.5%</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>0.137</td>
<td>Average housing price relative to annual income</td>
<td>3.2</td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.1880</td>
<td>Relative volatility of sales growth</td>
<td>1.32</td>
</tr>
<tr>
<td>( \alpha_p )</td>
<td>6.200</td>
<td>Relative volatility of population growth</td>
<td>0.17</td>
</tr>
</tbody>
</table>

**Parameters determined jointly**

Table 1 lists parameter values for the baseline search economy. Parameters above the separating line are set to match the corresponding targets directly, while those below the
line are determined jointly to match the targets in the right-most column. A time period is defined as one year.\textsuperscript{21} The discount factor $\beta$ is set to reflect an annual interest rate of 4%. Income in the steady state is normalized to one. Thus, all present values and prices are measured relative to steady-state per capita income. The terminal continuation value, $V$, is equal to the steady-state value of being a perpetual renter, $V_p$. To determine the mortgage rate $r$, the annual yield on international bonds $i$ is set at 4%. The values of $\phi$ and $\varphi$ are determined jointly in calibration.

The following parameters and targets are chosen following HLS: The rate $\mu$ is chosen to match the annual population growth during the 1990s. The value of $\pi_p$ is set to match the annual fraction of renters that move between counties and $\pi_h$ to match the annual fraction of home owners who move between counties according to the Census Bureau.

The supply elasticity parameter is set to $\xi = 1.75$ following Saiz (2010). There, the supply elasticity for 95 U.S. cities is estimated for the period between 1970 and 2000. The estimates vary from 0.60 to 5.45 with a population-weighted average of 1.75 (2.5 unweighted). The steady-state unit price of land $\bar{q}$ is set such that the relative share of land in the price of housing is 30% (see Davis and Palumbo (2008) and Saiz (2010)). The elasticity of new construction with respect to the price of housing, $\zeta$, is set equal to the median elasticity for the 45 cities studied by Green et al. (2005), $\zeta = 5$.

The maintenance cost $d$ is chosen to be 2.5% of the steady-state housing price according to Harding et al. (2007). Moreover, the average house price is 3.2 times annual income. The value of $\psi$ is calibrated so that the ownership rate in the city $H/(H+B+F) = 66.7\%$, where

$$H = \sum_{n=0}^{T-1} H_n + H_0$$

denotes the total measure of homeowners in the steady state. The Census Bureau reports the ownership rate among households whose head is between age 35 and 44 is roughly 66.7%.

We set the rent-to-income ratio to $\zeta = 0.16$ as follows. Based on empirical findings of the Lincoln Institute of Land Policy, the average rent-to-price ratio is around 5% prior to the most recent housing boom leading up to the 2008 financial crisis. Then we compute $\zeta$ as the product of rent-to-price ratio and the average house price.

\textsuperscript{21}Setting a time period as one year is due to specifics of our model. In this setting, households cannot save (so that all buyers are homogeneous) and thus house buyers can only finance the down payment with their periodic labor income. Empirically, the average housing price is about 12.8 times of quarterly income. If one period takes a quarter, then a buyer’s periodic income is too small to afford a typical down payment of 15% - 20%. In fact, borrowers could only afford a down payment less than 7.9% of the average housing price if the time period was set as a quarter.
The remainder of the parameters listed in Table 1 are determined to match jointly a number of targets based on the model. First, we set the average length of time following a foreclosure until a borrower is again allowed to access the mortgage market to five years. This time frame is consistent with the policies of Fannie Mae and Freddie Mac, which guarantee most U.S. mortgages. Thus we set the probability that a foreclosure flag remains on a borrower’s credit record to $\pi_f = 0.8$.

According to the Federal Housing Finance Board, the average contract rate on conventional, fixed-rate mortgages between 1995 and 2004 was 7.2%. We target an average down-payment ratio of 20% and an annual default rate of 1.6%, which is close to the average annual foreclosure rate among all mortgages during the 1990s according to the National Delinquency Survey by the Mortgage Bankers Association.

Foreclosed houses tend to cause losses to lenders due to associated transaction and time cost. Moreover, foreclosed houses are usually sold at a discount relative to other houses with similar properties. The loss severity rate is defined as the present value of all losses on a given loan as a fraction of the balance on the default date. Economists have found the loss severity rates range from as small as 2% during calm period over 1995-1999 (Pennington-Cross, 2003) to as much as over 75% during the Great Recession (Andersson and Mayock, 2014). In this paper, we choose parameters so that in the event of a default, the value of

$$\min\{\beta\nabla_{REO}, m_n\} = 0.73$$

on average. This implies an average loss severity rate of 27%.

Phillips and Vanderhoff (2004) find that 30% of defaulted conventional fixed-rate loans and 50% of defaulted conventional adjustable-rate loans transition to REO and Ambrose and Capone (1996, 1998) report that 32% to 38% of defaulted FHA loans transition to foreclosure. Based on these numbers, we choose parameters such that in the event of financial distress, the average probability of a successful sale is 66.5%. That is, 33.5% of the homeowners who experience financial distress ultimately end up in foreclosure in the steady-state.

Evidence available from the American Housing Survey (AHS) suggests that prior to 2003 the ratio of the original loan size to yearly income averages 2.72. Accordingly, we choose parameters such that the steady-state loan-to-income ratio at origination is given by $m_0/\bar{y} = 2.72$.

Finally, dynamics of our model depend crucially on two elasticities: the elasticity of $G(\cdot)$, evaluated at $\varepsilon_c$, $\alpha_p = \varepsilon_c g(\varepsilon_c)/G(\varepsilon_c)$ (here $g$ is the density of $G$) and the elasticity...
of the matching function with respect to the number of buyers, \( \eta \). These two parameters are calibrated jointly by using estimates of the relative standard deviations of population growth and housing sales growth in response to income shocks as in HLS.

For the non-search economy, all parameters remain at their values in Table 1 except for \( \pi_d, z_H \) and \( \psi \), which are adjusted so that the steady-state statistics match the relevant targets again. In the steady-state of the non-search economy, the construction cost parameter is adjusted so that \( P^* = 3.2 \) given the rate of population growth. Appendix B provides lists of parameter values re-calibrated to the non-search economy.

6 The Balanced Growth Path

We now characterize the steady-state of the baseline search economy. Along the balanced growth path, \( \text{per capita} \) income remains constant over time. The steady-state is defined based on the definition of equilibrium established in Section 3.5, plus the requirement that all functions and values listed in (43) - (48) are time invariant.

In the steady-state, all owners have strictly positive home equity\(^{22}\). Resident owners who receive neither moving nor financial distress shocks do not attempt to sell their houses regardless of their outstanding mortgage balances. All relocated owners continue to make repayments until a successful sale occurs or their mortgage is completely paid off. Finally, all distressed owners attempt to sell their houses. As such, there are no strategic defaults, in the sense that foreclosure occurs only as the result of financial distress shocks followed by unsuccessful attempts to sell.

Figure 3 depicts the steady-state distribution of house sellers across types. Nearly half of the sellers in the market are there as a result of financial distress. Note that this is consistent with the mobility and default rate targets from the calibration.

Figure 4 presents the distributions of resident homeowners (upper panel) and sellers (lower panel) by mortgage status. In the steady-state, the distribution of homeowners is driven solely by exogenous shocks. The measures of owners decrease with the number of payments for \( n = 1, \cdots, 29 \), owing to the effects of both moving and financial distress shocks which affect homeowners at constant rates over time. The large bin at \( n = 30 \) represents the stock of homeowners who have repayed their entire mortgage before experiencing either shock. While these homeowners no longer face a risk of financial distress, they remain subject to moving shocks and exit the city eventually with probability one.

\(^{22}\)Here home equity represents the difference between the average housing price and outstanding mortgage debt.
Similarly, the measure of distressed sellers decreases with the number of payments fulfilled, for \( n = 1, \cdots, 29 \), although there are no such sellers with \( n = 30 \) by construction.

The distribution of relocated sellers, in contrast, is driven by households’ choice of selling probability. These households are not required to sell, and they are no longer hit by relocation shocks. The fact that some enter sub-markets with high prices and low sales probabilities accounts for the hump-shape of the distribution. The spike at \( n = 30 \) arises from the fact that resident homeowners who have paid off their mortgages are still subject to moving shocks, at which point they become relocated sellers without a mortgage.

Figure 5 illustrates the distribution of housing prices in the steady state.

![Figure 3: Composition of house sellers](image)

### 6.1 Leverage and seller behavior

Figure 6 illustrates the relationship between a seller’s optimal choice of sub-market (which determines both her asking price and sales probability) and her debt position. Overall, a distressed seller is more eager to sell than a relocated seller and therefore, conditional on debt position (represented here by the LTV ratio), posts a lower price and sells with a higher probability. The cost of failing to sell is higher for distressed sellers for two reasons. First, a distressed seller has no choice but default if she fails to sell her house within the
period, while a relocated seller does not, and moreover retains the choice of whether to default in the next period. Second, a relocated seller receives continuation value $V$, which is independent of her credit record, while a distressed seller who has defaulted on her mortgage and remains in the city is excluded from the housing market until her foreclosure flag is lifted (five periods on average).

Note also the relationship between the posted asking price and LTV. For both types of seller, the posted price is initially (very) weakly decreasing in LTV. At some point, and this is more dramatic for distressed sellers, the relationship becomes strongly increasing. For distressed sellers in particular, the relationship resembles closely that reported by Genesove and Mayer (1997) (see Figure 2, p. 267). In their empirical study of condominium sales in Boston during the 1990’s and is also consistent with the findings of Anenberg (2011). Figure 7 combines their results with ours in common units.\textsuperscript{23} The closeness of the relationship is striking, given that none of the quantities depicted for our economy are

\textsuperscript{23}In our economy, the posted (asking) price is proportional to the mark-up, as all vacant (non-foreclosure) houses have a common value.

Figure 4: Steady-state distributions of mortgage status respectively among resident owners (upper panel) and household sellers (lower panel)
To understand the leverage-price relationship illustrated in Figures 6 and 7, consider the case of a distressed seller. Using (10), in the steady state, for any given \( p > d \) (see footnote 15), the gain from trade for such a seller as a function of her outstanding debt, \( m_n \), is given by:

\[
\Psi (m_n) = W_b (p - m_n) - W_f (\max [0, \beta V_{REO} - m_n])
\]  

\[
= \begin{cases} 
W_b (p - m_n) - W_f (\beta V_{REO} - m_n), & \text{if } m_n < \beta V_{REO} \\
W_b (p - m_n) - W_f (0), & \text{if } m_n \geq \beta V_{REO}
\end{cases}
\]  

\[
= \pi_f (1 - \pi_h) \beta [W_f (0) - V_b] + \\
\begin{cases} 
u (y - R + p - m_n) - u (y - R + \beta V_{REO} - m_n), & \text{if } m_n < \beta V_{REO} \\
u (y - R + p - m_n) - u (y - R), & \text{if } m_n \geq \beta V_{REO}
\end{cases}
\]  

\[\tag{62}\]

24 It is reasonable to believe that the curvature of the price choice of relocated sellers would resemble more of the Genesove-Mayer result if the continuation value, \( L \), did depend on one’s credit record in the city.
Figure 6: Leverage and seller behavior. The top panel shows the choices of selling probability by distressed and relocated sellers. Correspondingly, the bottom panel demonstrates the choices of selling price by the two types of sellers.

Differentiating (62) with respect to the level of debt, $m_n$, we have

$$
\Psi'(m_n) = \begin{cases} 
    u'(y - R + \beta V_{REO} - m_n) - u'(y - R + p - m_n), & \text{if } m_n < \beta V_{REO} \\
    -u'(y - R + p - m_n), & \text{if } m_n \geq \beta V_{REO}
\end{cases}
$$

where here $\Psi'(\cdot), u'(\cdot)$ denote differentiation. Given that $u' > 0$ and $u'' < 0$, we have:

**Proposition 1** Conditional on $p > m_n$, we have:

(i) If $m_n \geq \beta V_{REO}$, then $\Psi'(m_n) < 0$;

(ii) If $m_n < \beta V_{REO}$, $\Psi'(m_n) > 0$ for any given $p > \beta V_{REO}$ and $\Psi'(m_n) < 0$ for any given $p < \beta V_{REO}$.

In the steady-state a distressed seller chooses a sub-market to maximize her expected gain from trade. Given the matching function and free-entry of buyers, the optimal sub-
Figure 7: The red dot-dash curve (left axis) depicts the ratio of asking price to assessed value as measured by Genesove and Mayer (1997) plotted against sellers’ LTV. The blue curve (right) depicts the same relationship for the ratio of the posted price to the value of a house in REO inventory for our baseline search economy.

The market decision in (10) is equivalent to

$$\max_{p, \theta} \rho(\theta) \Psi(m_n; p)$$

(64)

where

$$\theta(p) = \gamma^{-1} \left( \frac{V_b - W_b(0)}{V_o(p, m_0) - W_b(0)} \right)$$

(65)

follows directly from (8) evaluated at the steady-state. It is straightforward to show that if \( p > m_n \), (i) \( \rho(\theta(p)) \) is strictly decreasing in \( p \) given the properties of the matching function listed in (3) and (4); and (ii) the gain from trade \( \Psi(m_n; p) \) increases with price \( p \) for any debt level \( m_n \), since \( u' > 0 \). Thus, a higher selling price raises the gain from trade, but reduces selling probability. The optimal sub-market choice reflects this trade-off.

The shape of the relationship depicted in Figures 6 and 7 can be understood using Proposition 1. When a seller is sufficiently indebted \( (m_n \geq \beta V_{REO}) \), the gain from trade \( \Psi(m_n; p) \) is strictly decreasing in debt \( m_n \), for any \( p > m_n \). Essentially, heavily indebted sellers with more debt worry less about the likelihood that they successfully sell than about
the gain they receive if they do. The reason for this is that they receive residual profit only if they sell at a sufficiently high price. To them, the foreclosure cost is fixed; the marginal cost of defaulting on a larger debt is borne entirely by the lender.

A less indebted seller (i.e., one with \( m_n < \beta V_{REO} \)) has greater incentive to sell, as failure to do so results in the loss of residual profit as well as the cost of the foreclosure tag. Moreover, for \( p > \beta V_{REO} \), the gain from trade \( \Psi \) is strictly increasing in debt \( m_n \). As such, a more indebted seller (but with \( m_n < \beta V_{REO} \)) will chose a lower price/higher sales probability. Overall, for \( m_n < \beta V_{REO} \), the effect of debt on the gain from trade (i.e., \( \Psi'(m_n) \)) is likely to be small in that \( m_n \) affects symmetrically the returns both to selling and failing to do so (see (63)). Thus the relationship is essentially flat for lower LTVs but rapidly increasing for higher LTVs.

It is worthwhile clarifying that the condition \( p > \beta V_{REO} \) is not particularly restrictive. For example, in our baseline calibration, the steady-state value of \( \beta V_{REO} = 1.79 \), while the minimum selling price chosen by a seller is 3.04. In general, \( \beta V_{REO} \) tends to be much lower than the choice of selling price by any seller due to the foreclosure and carrying costs associated with houses in REO inventory.

Note that while the proof of Proposition 1 relies on the assumption that consumption goods are non-storable, the result is in fact more general. In particular, the derived properties of \( \Psi'(d) \) require only that \( W_f'(p - m_n) > 0 \) and \( W_f' (\beta V_{REO} - m_n) - W_f'(p - m_n) > 0 \) for \( m_n < \beta V_{REO} \). The former condition requires that the value of a buyer without the foreclosure flag is a strictly increasing function of her asset holdings. The latter requires that the slope of the value of a buyer with the flag at asset position \( \beta V_{REO} - m_n \) exceed that of the value of an unflagged buyer at \( p - m_n \) for lower levels of debt. Observing that \( p > \beta V_{REO} \) in general, this requirement is not overly restrictive for value functions such as \( W_f(\cdot) \) and \( W_b(\cdot) \), which are strictly concave.

### 6.2 Matching and lending standards

We now conduct two comparative statics exercises to illustrate the role of specific assumptions regarding matching. First, holding all other parameters constant, we consider the effects of changing the matching coefficient \( \varpi \) and the elasticity \( \eta \). In our calibration, these parameters determine the fundamental trading conditions of the housing market. Our goal here is to examine how search frictions, which determine in part the liquidity of housing, affect mortgage lending standards directly.

The top two panels of Figure 8 illustrate the effect of changes to \( \varpi \) on the average LTV
Figure 8: Effects of search frictions on average down-payment ratios and default rates in the steady state. In the left column, the three curves represent the following: the average down-payment ratio in solid blue, the maximum LTV ratio at origination in dashed black and the minimum LTV ratio at origination in dot-dash red.

(or alternatively, the down-payment ratio) at origination and the probability of a mortgage ending in foreclosure, respectively. Consider a mortgage issued in the current period. The probability of such a mortgage ending in foreclosure, \(\Pi_d\), is given by

\[
\Pi_d = \sum_{n=1}^{T} (1 - \pi_h)^n (1 - \pi_d)^{n-1} \pi_d [1 - \rho(\theta(p_{sd}))] m_{n-1} \\
+ \sum_{n=1}^{T} (1 - \pi_h)^{n-1} \pi_h [1 - \rho(\theta(p_{Ld}))] m_{n-1} \tag{66}
\]

where \(\rho(\theta(p_{sd}))\) and \(\rho(\theta(p_{Ld}))\) are the trading probabilities in the optimal sub-markets chosen at period \(t + i\) for resident and relocated borrowers, respectively, who default after having made \(n - 1\) payments. The first term is the summation of probabilities of default over the entire duration of mortgage conditional on staying in the city. Similarly, the second term is the summation of probabilities of default conditional on having relocated.

\(25\)In equilibrium, households differ in LTV at origination because they purchase houses at different prices but are advanced loans of the same size. Both of the left-hand panels of Figure 8 depict the effects of matching parameters on average, maximum and minimum LTVs at origination, respectively.
elsewhere.

As is shown in Figure 8, LTVs at origination are increasing and default probabilities decreasing with the value of $\varpi$. Ceteris paribus, the higher the value of $\varpi$, the more likely a seller is to match, or equivalently, the more liquid the housing market. Thus, the expected default rate is lower because more homeowners experiencing distress successfully sell their houses. At the same time, houses in REO inventory also sell more quickly. Overall, with both the likelihood and cost of default and foreclosure reduced, mortgage firms are willing to advance larger loans, resulting in higher LTVs at origination.

The bottom two panels in Figure 8 demonstrate similar results for varying the value of $\eta$. Intuitively, the surplus resulting from housing transactions that accrues to buyers increases with $\eta$, the elasticity of the measure of matches with respect to the measure of buyers.$^{26}$ A higher $\eta$ thus implies a higher value of being a buyer, which in turn increases the value of living in the city, lowering the entry cutoff, $\varepsilon_e$. Similarly the return to construction is lower as firms receive less of the surplus associated with new houses. Overall, the housing market is tighter in the steady-state, and all houses sell with relatively higher probability. Again, this lowers both the expected rate and cost of default, leading mortgage firms to issue larger loans.

Overall, these exercises demonstrate that mortgage lending standards are lower the more liquid is the housing market. We now turn to the effects of aggregate shocks which induce liquidity to vary endogenously over time.

### 7 Equilibrium Dynamics

We now consider the dynamics resulting from aggregate shocks in equilibrium. We also compare the baseline search economy to the alternative *non-search* (NS) economy described in Section 4.

To begin with, we posit an $AR(1)$ process for the log of income, $\ln y_t$:

$$\ln y(t) = \lambda \ln y(t-1) + \epsilon(t), \quad \epsilon \sim N(0, \sigma_\epsilon).$$

We set $\lambda = 0.96$ and $\sigma_\epsilon = 0.02$.

$^{26}$See Moen (1997) and Head et al. (2014).
7.1 Population growth, house prices and construction

Figure 9 illustrates the responses of city population growth, the average house price, and construction to a shock to local income which evolves via (67). For each of the three endogenous variables, the responses to the shock in both the baseline and NS economy are similar to those reported by HLS. This is not surprising as our baseline economy has been constructed in part to preserve the dynamics of basic housing market variables generated in that paper. For this reason we discuss them only briefly here before moving on to a discussion of seller behaviour and the mortgage market, which are the focus of this paper.27

Briefly, a positive shock to local income induces immediate entry of households to the city and the population growth rate rises. The response of population growth is, however, much larger in the search economy.28 The responses of housing prices and construction rates differ both qualitatively and quantitatively across the two economies. The search model generates serial correlation in both price growth and construction. In contrast, the non-search economy does not generate such dynamics, rather, the house price jumps immediately, by a large amount, and then returns monotonically to its steady-state level. This initial jump in house prices, followed by a long and slow decline, effectively limits the entry of households to the city.

7.2 Market tightness and matching probabilities.

In the baseline search economy, serial correlation in both house price growth and the construction rate is driven by the change in housing market liquidity due to search and matching. To illustrate this, Figure 10 depicts responses of overall market tightness, and respective average matching probabilities of buyers and sellers.29 The figure shows that changes in housing market liquidity leads to serial correlation in both tightness and the matching probabilities. Following a positive shock to city income, the increase in household entry leads to an increase in housing search by prospective buyers. Construction, however, takes time and so overall market tightness (i.e., the ratio of the total numbers of buyers to sellers across all sub-markets) increases immediately. Tightness continues to rise for a prolonged period for two reasons: First, there is further entry of prospective buyers due to the persistence of the income shock. Second, buyers who do not match initially remain

27For a detailed discussion, see Section V.A. of HLS.
28In their experiment HLS adjusted the elasticity of alternative values so that the variance of population growth in the search and non-search economies was equal. Here we do not do this, in order to highlight the difference in the responses of house prices in the two models.
29These phenomena do not occur in the NS economy. This accounts for the lack of momentum in the impulse responses.
Figure 9: Impulse responses to a positive income shock: population, prices and construction in the market. Although construction results in a persistent increase in the measure of sellers in the market, the former effect dominates and tightness both rises and remains above the steady-state for an extended period of time.

Given the matching function, higher market tightness implies higher (lower) matching probabilities for sellers (buyers) at any given trading price. The top-right and bottom-left panels in Figure 10 demonstrate these relationships very clearly. As houses become increasingly more liquid in the sense that it takes increasingly less time to sell them, their values, and thus their sales prices continue to rise. This leads to serial correlation both in house price growth and construction, as the latter is driven by the value of new houses. As income returns to its steady-state level, entry of households to the city slows. As fewer households enter, searching buyers match, and new houses come on the market, tightness falls. Eventually, house prices and construction return to their steady-state values.

7.3 The default rate, mortgage size, and LTV at origination

In the search economy, the average selling probability for sellers increases on impact and continues to rise for several periods before gradually reverting to its long-run level. An increase in the selling rate benefits distressed sellers substantially by lowering the probabil-
Figure 10: Impulse responses to a positive income shock: matching

ity with which they face foreclosure. As such, the default rate moves opposite the selling rate, as shown in the first panel of Figure 11. The default rate for the NS economy is exogenous and thus does not deviate from its steady-state level.

The responses of loan size at origination (i.e., $m_0$) for the baseline differ qualitatively in the two economies (see Figure 11). Reconsidering the bottom-left panel of Figure 9, it is clear that the response of loan size largely follows that of the house price. For example, in the baseline search economy, loan size increases on impact and exhibits momentum following the house price. In the NS economy, loan size also tracks the house price. Recall that loan size, $m_0$, is determined by (28) and in general depends on the expected default rate and carrying costs in addition to house prices. The close tracking of equilibrium loan size to the house price illustrates that ultimately the value of houses must be reflected in mortgage size, in both economies. Thus, as discussed above, it is movements in housing demand relative to construction (supply) that drive home values, including the component associated with default risk.

The responses of LTV at origination differ significantly depending on whether there is search. In the baseline, the initial LTV rises immediately following the shock. Several forces contribute to this result: First, the expected default rate on new mortgages declines and remains low for an extended time as houses become increasingly liquid. Similarly,
lenders’ exposure to risk associated with mortgages issued in earlier periods declines as well. Since the mortgage market is competitive and the interest rate fixed, in equilibrium lower risk translates into loans being larger relative to the purchase price. We refer to this as the market tightness effect.

Second, borrowers holding mortgages at the time of the shock experience a relatively large increase in home equity (and a corresponding reduction in LTV) as a result of the increase in house values. As illustrated earlier, a decline in LTV is associated with lower asking prices and higher sales probabilities, especially for sellers in financial distress. This home equity effect also lowers the default rate and hence the riskiness of lenders’ portfolios of outstanding mortgages. Again, competition results in this being passed through to buyers in the form of larger mortgages.

Third, the proceeds of foreclosure sales rise and remain high for several periods reflecting the increases in both house values and the selling rate. This increases the value of houses in REO inventory ($V_{REO}(t)$) and thus lowers the cost of default to lenders resulting in greater returns to lending and larger mortgages in equilibrium. See the bottom-right panel of

Figure 11: Impulse responses to a positive income shock: mortgage
Overall, reductions of both the expected default rate and the expected loss upon default motivate the mortgage company to relax lending requirements in the sense that increase the size of the loan they offer at origination. These effects are enhanced because mortgages are long-term in duration, and thus the LTVs on all existing mortgages are instantaneously and persistently reduced. Eventually, as tightness and the selling rate return to their steady-states, LTVs at origination do as well, following the path of the average house price.

In contrast, for the NS economy the LTV at origination falls significantly in response to the shock, and gradually returns to the steady-state monotonically thereafter. In this model the expected default rate is exogenously given and the mechanism discussed above for the baseline economy is not operative. As house prices rise in response to the shock and are expected to fall monotonically back to their steady-state levels in the future, on origination the mortgage company’s expected loss upon default of a mortgage is higher. Given that the default rate cannot fall to compensate, the mortgage company must require a higher down-payment to cover the increase in default risk.

7.4 The co-movement of house prices and LTVs at origination

Figure 12 depicts co-movements between the average house price and the LTV at origination for both economies. The baseline model generates a clearly positive co-movement between the two variables while the non-search economy predicts a strong negative relationship. While not reported here, similar differences between the two economies are apparent in their responses to negative shocks to income. The responses of both economies to negative income shocks are reported in Appendix C.

7.5 The pricing decisions of indebted sellers

Figure 13 depicts the responses of the sellers’ asking prices at four different stages of the mortgage-repayment process. The first three panels depict choices of distressed sellers and the last for newly relocated sellers. The figure depicts responses only for the baseline

---

30 As the response of $V_{REO}(t)$ is almost identical to that of housing prices, it is driven mostly by changes in house prices, as opposed to the higher selling rate. This is not surprising given that the carrying cost (e.g., maintenance costs) is less than 1.5% of the average house price in the calibration.

31 For example, the first panel of the figure depicts the pricing choice of a seller who has not yet made her first payment ($n = 0$), $t$ periods following the shock. That is, it depicts the pricing decisions of a cross-section of sellers at the same stage of repayment but with loans originating at different times.
search economy as the NS economy has no counterparts for these measures. For the baseline economy, all four panels demonstrate a pattern consistent with an average sales probability as shown in the bottom panel of Figure 10.\textsuperscript{32} That is, all panels display patterns consistent with the time-paths of tightness and the average sales rate.

The main departure from this pattern involves distressed sellers who have just purchased and taken out a mortgage in the period before the shock occurs (n = 0). As described above, the raises the value of houses and reduces these households’ LTVs substantially. When such a household receives a financial distress shock, it faces the prospect of losing this potential capital gain if it fails to sell and ends up in foreclosure. The household thus has a strong incentive to sell, and so posts a low price and, equivalently, sells with a relatively high probability.\textsuperscript{33}

Buyers who purchase following the shock experience no unanticipated capital gain, as current and future house prices, and future matching rates all are taken into account when

\textsuperscript{32}Despite the connection, note that Figure 13 displays a panel where as Figure 10 depicts a time-series relationship. Each point in the last panel of Figure 10 represents a weighted average of the corresponding points in Figure 13 together with those for all sellers with n’s not shown in the figure, construction firms, and mortgage companies holding REO inventories.

\textsuperscript{33}The increase in the sales probability is particularly significant given the entry of buyers.
a new mortgage is issued. This explains the large rise in asking price (and drop in the selling-probability) for distressed sellers with \( n = 0 \) in subsequent periods. The choice of relocated sellers with \( n = 0 \) also displays a similar initial responses, albeit of smaller magnitude. These sellers neither face imminent foreclosure nor experience such a large capital gain because they are on average less levered than new homeowners.

![Selling Prices of Distressed Sellers: n=0](image1)

![Selling Prices of Distressed Sellers: n=15](image2)

![Selling Prices of Distressed Sellers: n=29](image3)

![Selling Prices of Relocated Sellers: n=1](image4)

**Figure 13: Impulse responses to a positive income shock: house-selling choices**

Consider next the case of sellers one period away from paying off their mortgage (\( n = 29 \)) in the period before the shock. In the figure it is clear that these sellers *raise* their asking prices and thus experience a *lower* probability of a sale. This, of course, raises their default probability. Recall that in the event of a default, the mortgage company keeps the outstanding mortgage balance and returns any remaining sale proceeds when the foreclosed house is. For sellers with \( n = 29 \), the outstanding balance is low precisely because the mortgage has been nearly repaid in full. Thus, the cost of default is low because these households can still recover a large portion of their equity after a default.

The responses of sellers in the middle of their mortgage repayment term (\( n = 15 \) in the figure) lie between those of sellers at the beginning and end of their terms. The effects discussed above combine for these sellers and largely cancel, leaving the response to reflect largely the movements of the average sales probability.
Responses of these variables to a negative income shock are contained in Appendix C. For the baseline, the dynamics are nearly symmetric to the responses to a positive income shock. One significant exception is that immediately following a negative income shock, non-distressed owners may experience such a large increase in LTV that they have negative home equity and thus choose to default on their loans strategically. Therefore, the set of homeowners selling in response to a negative income shock also includes some non-distressed homeowners, hoping to sell and avoid defaulting.

Tables 2 and 3, respectively, contain the choices of sales probabilities associated with optimal pricing decisions and the implied default rates following a positive income shock. Similarly, Tables 4 and 5 list the corresponding probabilities and rates following a negative income shock. Overall, sellers with relatively high leverage are much more likely to default on mortages than those with less leverage. Out of the steady-state, the distribution of indebted sellers matters for the response of the economy to shocks. All else equal, a negative shock occurring when the economy has a high proportion of high-leverage home owners will cause much more severe defaults at the aggregate level than will one occurring when leverage is lower overall.
Table 2: Sales probability of borrowers made $n$ payments upon a positive shock

<table>
<thead>
<tr>
<th></th>
<th>n=0</th>
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<th>15</th>
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<tbody>
<tr>
<td>t=1</td>
<td>0.7995</td>
<td>0.8056</td>
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Table 3: Default probability of borrowers made $n$ payments upon a positive shock

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8 Conclusion

We develop a tractable dynamic equilibrium model of housing transactions in which purchases financed by long-term defaultable mortgages. The model is used to study (i) the effect of sellers’ degree of leverage on their pricing behavior and likelihood of default; and (ii) the effects of housing market liquidity on mortgage standards. The model generates endogenous responses of house prices, market liquidity, mortgage standards, and default probabilities in response to income shocks.

We find that sellers’ asking prices are decreasing in and relatively insensitive to increase in leverage when LTVs are low, but become steeply increasing in leverage at higher debt ratios. This result matches well the curvature of the leverage-price relationship estimated by Genesove and Mayer (1997), Anenberg (2011) and others. Moreover, seller behavior...
Table 4: Sales probability of borrowers made n payments upon a negative shock

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Table 5: Default probability of borrowers made n payments upon a negative shock

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<td>3</td>
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<td>0.2045</td>
<td>0.2076</td>
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<td>0.1437</td>
<td>0.1259</td>
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<td>4</td>
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<td>0.2070</td>
<td>0.2101</td>
<td>0.1796</td>
<td>0.1465</td>
<td>0.1288</td>
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<td>0.2098</td>
<td>0.2098</td>
<td>0.2129</td>
<td>0.1825</td>
<td>0.1496</td>
<td>0.1319</td>
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<tr>
<td>6</td>
<td>0.2124</td>
<td>0.2093</td>
<td>0.2154</td>
<td>0.1852</td>
<td>0.1523</td>
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<td>7</td>
<td>0.2139</td>
<td>0.2109</td>
<td>0.1957</td>
<td>0.1867</td>
<td>0.1540</td>
<td>0.1364</td>
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<tr>
<td>8</td>
<td>0.2153</td>
<td>0.2123</td>
<td>0.1972</td>
<td>0.1882</td>
<td>0.1555</td>
<td>0.1380</td>
</tr>
<tr>
<td>9</td>
<td>0.2166</td>
<td>0.2136</td>
<td>0.1986</td>
<td>0.1896</td>
<td>0.1570</td>
<td>0.1395</td>
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<tr>
<td>10</td>
<td>0.2175</td>
<td>0.2145</td>
<td>0.1994</td>
<td>0.1905</td>
<td>0.1579</td>
<td>0.1405</td>
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also differs in leverage along the dynamic path. Second, housing market liquidity influences mortgage standards significantly. In particular, the theory generates a consistent positive co-movement between house prices and LTVs at origination. This observation is qualitatively in line with observations regarding lending standards both during the period leading up to the recent house price collapse in the U.S., and during the current and ongoing period of house price growth in Canada. An alternative model without search (i.e., a frictionless housing market) fails to capture these phenomena.

References


A  Laws of motion for the non-search economy

For the non-search economy, we have the following laws of motion:

\[(1 + \mu)F' = (1 - \pi_p)F + (1 - \psi)G(\varepsilon'_c)\mu.\]  \hspace{1cm} (68)

\[(1 + \mu)B'_f = (1 - \pi_h)\left\{ fB_f + \pi_d \sum_{n=0}^{T-1} H_n \\
+ (1 - \pi_d) \sum_{n=0}^{T-1} (1 - I_n) D_n H_n \right\} \]  \hspace{1cm} (69)

where \(I_n = 1\) if the owner chooses to sell, and 0 otherwise.

\[(1 + \mu)B' = \psi G(\varepsilon'_c)\mu + (1 - \pi_f)B_f + (1 - \pi_h) \sum_{n=0}^{T-1} I_n H_n.\]  \hspace{1cm} (70)

\[(1 + \mu)H'_n = (1 - \pi_h)(1 - \pi_d)(1 - I_{n-1})(1 - D_{n-1})H_{n-1};\]  \hspace{1cm} (71)

\[(1 + \mu)H'_0 = (1 - \pi_h)(1 - \pi_d)B;\]  \hspace{1cm} (72)

\[(1 + \mu)H'_\emptyset = (1 - \pi_h)\left\{ (1 - \pi_d)(1 - I_{T-1})(1 - D_{T-1})H_{T-1} \\
+ (1 - I_T)H_\emptyset \right\}.\]  \hspace{1cm} (73)

\[(1 + \mu)H'_{Ln} = (1 - I_{Ln-1})(1 - D_{Ln-1})H_{Ln-1} \\
+ \pi_h(1 - \pi_d)(1 - I_{n-1})(1 - D_{n-1})H_{n-1};\]  \hspace{1cm} (74)

\[(1 + \mu)H'_{L0} = \pi_h(1 - \pi_d)B;\]  \hspace{1cm} (75)

\[(1 + \mu)H'_{L\emptyset} = \pi_h\left\{ (1 - \pi_d)(1 - I_{T-1})(1 - D_{T-1})H_{T-1} \\
+ (1 - I_T)H_\emptyset \right\} \\
+ (1 - I_{LT-1})(1 - D_{LT-1})H_{LT-1} + (1 - I_L)H_{L\emptyset}.\]  \hspace{1cm} (76)

\[(1 + \mu)H'_c = N.\]  \hspace{1cm} (77)

\[(1 + \mu)H'_{LREGO} = \pi_d \sum_{n=0}^{T-1} H_n + \sum_{n=1}^{T-1} (1 - I_{Ln})D_{Ln}H_{Ln} \\
+ (1 - \pi_d) \sum_{n=0}^{T-1} (1 - I_n)D_n H_n.\]  \hspace{1cm} (78)
## Calibration parameters for the non-search economy

Table 6: Calibration Parameter Values: Non-Search Economy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameters determined independently</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.96</td>
<td>Annual interest rate</td>
<td>4.0%</td>
</tr>
<tr>
<td>$\pi_p$</td>
<td>0.120</td>
<td>Annual mobility of renters</td>
<td>12%</td>
</tr>
<tr>
<td>$\pi_h$</td>
<td>0.032</td>
<td>Annual mobility of owners</td>
<td>3.2%</td>
</tr>
<tr>
<td>$\xi$</td>
<td>1.75</td>
<td>Median price-elasticity of land supply</td>
<td>1.75</td>
</tr>
<tr>
<td>$i$</td>
<td>0.040</td>
<td>International bond annual yield</td>
<td>4.0%</td>
</tr>
<tr>
<td>$T$</td>
<td>30</td>
<td>Fixed-rate mortgage maturity (years)</td>
<td>30</td>
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<tr>
<td>$\mu$</td>
<td>0.012</td>
<td>Annual population growth rate</td>
<td>1.2%</td>
</tr>
<tr>
<td>$\pi_f$</td>
<td>0.80</td>
<td>Average duration (years) of foreclosure flag</td>
<td>5</td>
</tr>
<tr>
<td>$\bar{q}$</td>
<td>0.96</td>
<td>Average land price-income ratio</td>
<td>30%</td>
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<tr>
<td>$m$</td>
<td>0.08</td>
<td>Residential housing gross depreciation rate</td>
<td>2.5%</td>
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<tr>
<td>$\zeta$</td>
<td>5</td>
<td>Median price elasticity of new construction</td>
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<tr>
<td>$\varsigma$</td>
<td>0.16</td>
<td>Rent-price ratio</td>
<td>5%</td>
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<tr>
<td><strong>Parameters determined jointly</strong></td>
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<tr>
<td>$\chi$</td>
<td>0.460</td>
<td>Loss severity rate</td>
<td>27%</td>
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<tr>
<td>$\varphi$</td>
<td>0.0246</td>
<td>Average down-payment ratio</td>
<td>20%</td>
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<tr>
<td>$\varrho$</td>
<td>0.0074</td>
<td>Average annual FRM-yield</td>
<td>7.20%</td>
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<tr>
<td>$\psi$</td>
<td>0.570</td>
<td>Fraction of households that rent</td>
<td>33.3%</td>
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<td>$\pi_d$</td>
<td>0.016</td>
<td>Annual foreclosure rate</td>
<td>1.6%</td>
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<td>$z_H$</td>
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<td>Average loan-to-income ratio at origination</td>
<td>2.72</td>
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<td>$\kappa$</td>
<td>0.137</td>
<td>Average price of a house</td>
<td>3.2</td>
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<tr>
<td>$\alpha_p$</td>
<td>6.200</td>
<td>Relative volatility of population growth</td>
<td>0.17</td>
</tr>
</tbody>
</table>
C IRFs upon a negative local income shock

Figure 15: Impulse responses to a negative income shock: population, prices and construction

Figure 16: Impulse responses to a negative income shock: matching
Figure 17: Impulse responses to a negative income shock: mortgage

Figure 18: Impulse responses to a positive income shock: house-selling choices
Figure 19: Impulse responses to a positive income shock: house-selling choices (probability)

Figure 20: Co-movements between average down-payment ratio and average housing price in baseline and non-search economies.