Formulation 2 of the Decision Rule: the p-value Rule

What is a Decision Rule?

The decision rule for an hypothesis test is a rule that states when the null hypothesis H_0 is rejected or retained (not rejected) against an alternative hypothesis H_1 at some chosen significance level α.

Formulation 2: Determine if the **p-value for** the ***calculated sample*** **value of the test statistic t_0** or **F_0** under the null hypothesis H_0 **is *smaller* or *larger* than the chosen significance level** α.

Definition: The **p-value** (or **probability value**) associated with the calculated sample value of the test statistic is defined as the ***lowest*** **significance level at which the null hypothesis H_0** ***can be rejected***, given the calculated sample value of the test statistic.

Interpretation

- The **p-value** is the **probability of obtaining a *sample value* of the test statistic *as extreme as* the one we computed *if the null hypothesis H_0 is true***.

- **P-values** serve as ***inverse*** **measures** of **the strength of evidence *against* the *null* hypothesis H_0**.

    - ***Small*** **p-values – p-values *close to zero* –** constitute ***strong*** **evidence against the null hypothesis H_0**.

    - ***Large*** **p-values – p-values *close to one* –** provide only ***weak*** **evidence against the null hypothesis H_0**.

Examples of p-values for common types of hypothesis tests

Two-tail t-tests

♦ For a ***two-tail* t-test**, let the calculated sample value of the t-statistic for a given null hypothesis be t_0. Then the p-value associated with the sample value t_0 is the probability of obtaining an **absolute value of the t-statistic greater than the absolute value of t_0 *if* the null hypothesis H_0 is true**, where the absolute value of t_0 is denoted as $|t_0|$. That is,

$$\textit{two-tail} \text{ p-value for } t_0 = \Pr\left(|t| > |t_0| \mid H_0 \text{ is true}\right)$$

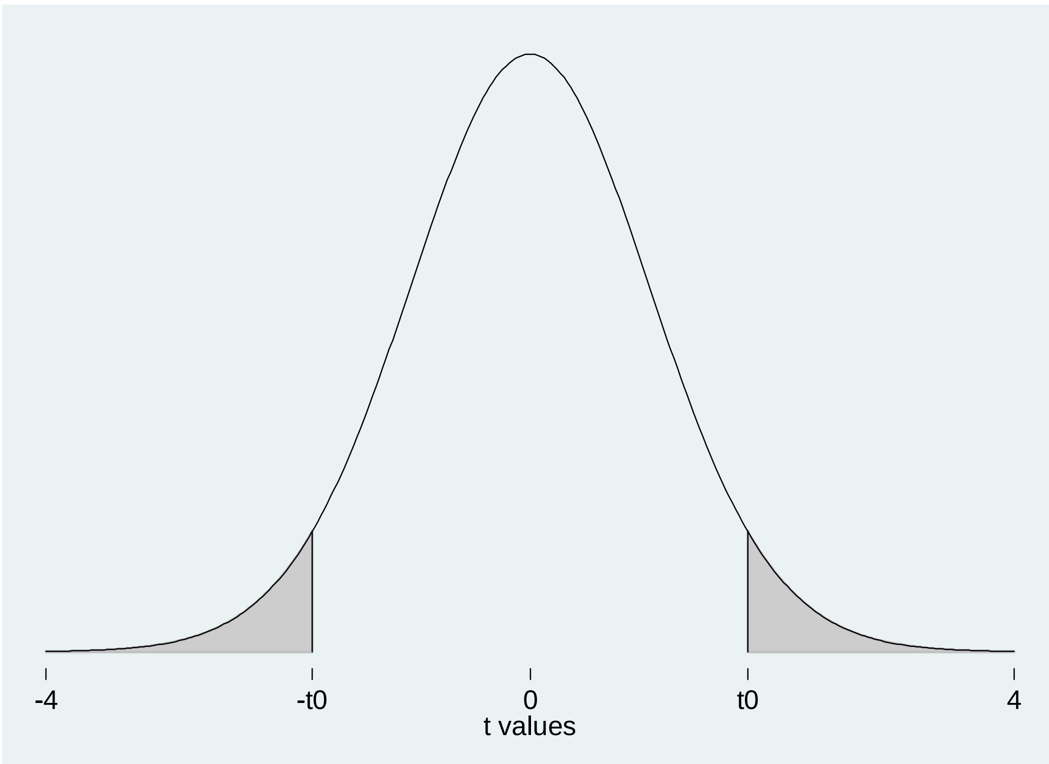
$$= \Pr\left(t > t_0 \mid H_0 \text{ is true}\right) + \Pr\left(t < -t_0 \mid H_0 \text{ is true}\right) = 2 \cdot \Pr\left(t > t_0 \mid H_0 \text{ is true}\right) \quad \text{if } t_0 > 0$$

$$= \Pr\left(t < t_0 \mid H_0 \text{ is true}\right) + \Pr\left(t > -t_0 \mid H_0 \text{ is true}\right) = 2 \cdot \Pr\left(t < t_0 \mid H_0 \text{ is true}\right) \quad \text{if } t_0 < 0$$

Remember: the t-distribution is symmetric about its mean of zero.

Two-tail p-value of t_0 = 1.8 when Null Distribution of t_0 is t[50]

two-tail **p-value for** $t_0 = \Pr\left(|t| > |t_0| \,\middle|\, H_0 \text{ is true}\right) = \Pr\left(|t| > 1.8 \,\middle|\, H_0 \text{ is true}\right) = \mathbf{0.07790}$

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. * TWO-TAIL p-value of t0 = 1.8 when t0 has t[50] distribution
. display 2*ttail(50, 1.8)
.07789525
. display 2*ttail(50, abs(-1.8))
.07789525
```

One-tail t-tests

♦ For a ***one-tail* t-test**, let the calculated sample value of the t-statistic for a given null hypothesis be t_0. Then the p-value associated with the sample value t_0 depends on whether the test is a ***right-tail*** or ***left-tail*** **test**.

(1) For a ***right-tail*** **t-test**, the p-value associated with the sample value t_0 is the **probability of obtaining a t-statistic value *greater than* the calculated sample value t_0 *if* the null hypothesis H_0 is true** – i.e.,

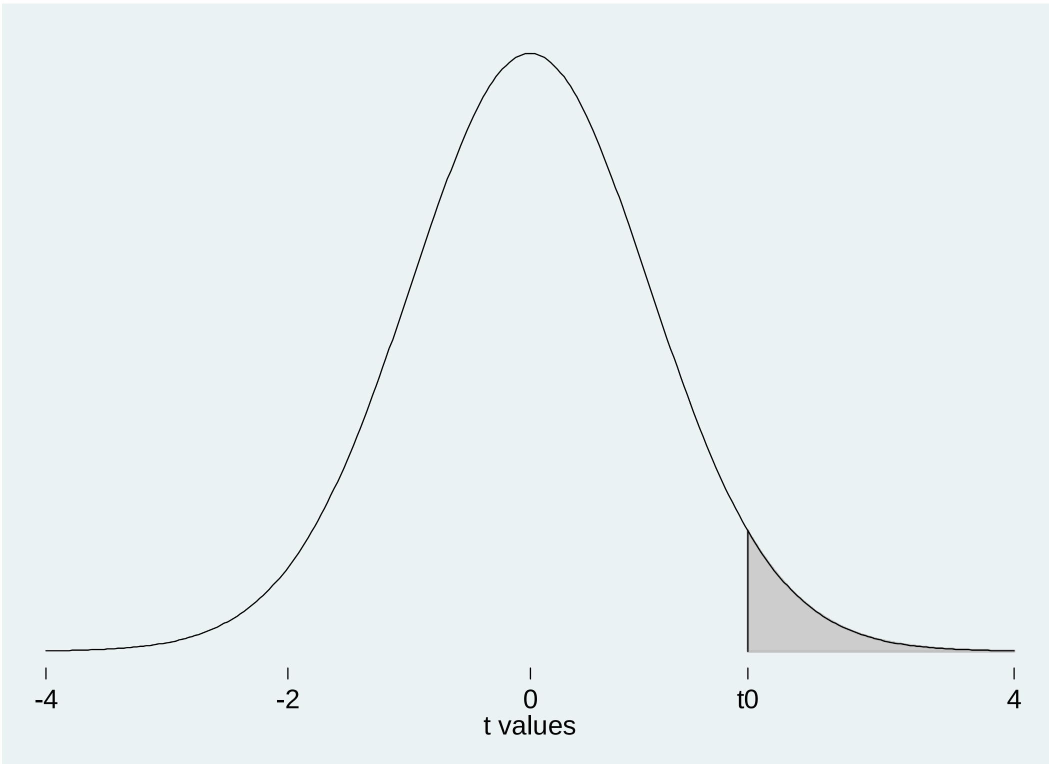
$$\textit{right-tail}\ \textbf{p-value for}\ t_0 = \Pr\left(t > t_0 \mid H_0 \text{ is true}\right).$$

(2) For a ***left-tail*** **t-test**, the p-value associated with the sample value t_0 is the **probability of obtaining a t-statistic value *less than* the calculated sample value t_0 *if* the null hypothesis H_0 is true** – i.e.,

$$\textit{left-tail}\ \textbf{p-value for}\ t_0 = \Pr\left(t < t_0 \mid H_0 \text{ is true}\right).$$

Right-tail p-value of t_0 = 1.8 when Null Distribution of t_0 is t[50]

***right-tail* p-value for t_0** $= \mathbf{Pr\left(t > t_0 \mid H_0 \text{ is true}\right)} = \Pr\left(t > 1.8 \mid H_0 \text{ is true}\right) = \mathbf{0.03895}$

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. * RIGHT-TAIL p-value of t0 = 1.8 when t0 has t[50] distribution
. display ttail(50, 1.8)
.03894762
```

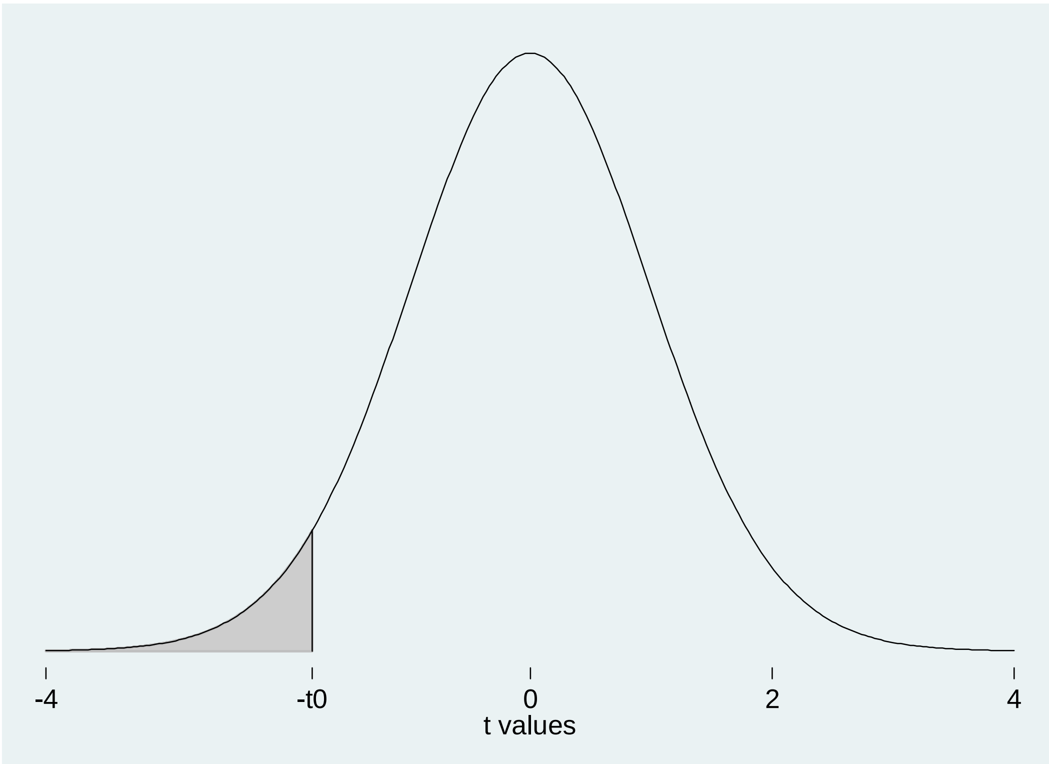
Right-tail p-value of $t_0 = -1.8$ when Null Distribution of t_0 is t[50]

right-tail **p-value for t_0** $= \mathbf{Pr}\left(\mathbf{t > t_0 \mid H_0 \text{ is true}}\right) = \Pr\left(t > -1.8 \mid H_0 \text{ is true}\right) = \mathbf{0.9611}$


```
. * RIGHT-TAIL p-value of t0 = -1.8 when t0 has t[50] distribution
. display ttail(50, -1.8)
.96105238
```

Left-tail p-value of t_0 = –1.8 when Null Distribution of t_0 is t[50]

***left-tail* p-value for t_0** $= \mathbf{Pr\left(t < t_0 \mid H_0 \text{ is true}\right)} = \Pr\left(t < -1.8 \mid H_0 \text{ is true}\right) = \mathbf{0.03895}$

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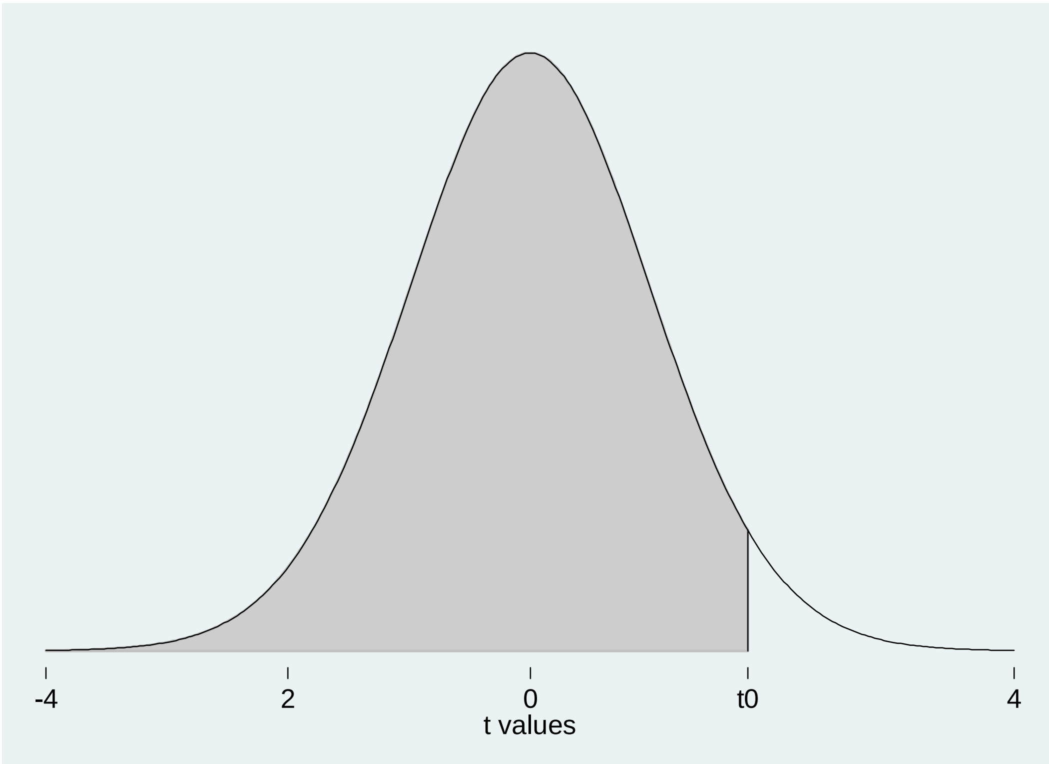
. * LEFT-TAIL p-value of t0 = -1.8 when t0 has t[50] distribution
. display 1 - ttail(50, -1.8)
.03894762
```

Left-tail p-value of t_0 = 1.8 when Null Distribution of t_0 is t[50]

left-tail **p-value for t_0** $= \Pr\left(t < t_0 \,\middle|\, H_0 \text{ is true}\right) = \Pr\left(t < 1.8 \,\middle|\, H_0 \text{ is true}\right) = \mathbf{0.9611}$




```
. * LEFT-TAIL p-value of t0 = 1.8 when t0 has t[50] distribution
. display 1 - ttail(50, 1.8)
.96105238
```

F-tests

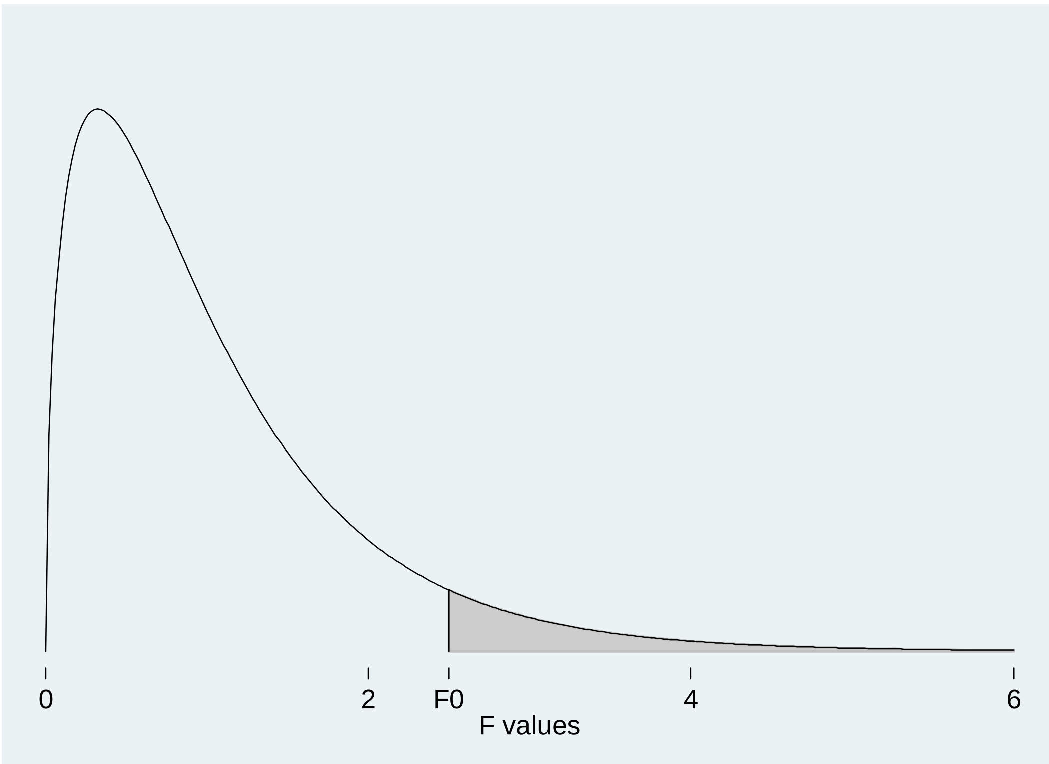
- For an **F-test**, let the calculated sample value of the F-statistic for a given null hypothesis be F_0. Then the p-value associated with the sample value F_0 is the **probability of obtaining an F-statistic value *greater than* the calculated sample value F_0 *if* the null hypothesis H_0 is true** – i.e.,

  $$\textbf{p-value for } \mathbf{F_0} = \Pr\left(\mathbf{F > F_0} \,\middle|\, \mathbf{H_0} \textbf{ is true}\right).$$

  Note that the F-distribution is defined only over non-negative values that are greater than or equal to zero.

P-value for F_0 = 2.5 when Null Distribution of F_0 is F[3, 60]

$$\textbf{p-value of } \mathbf{F_0} = \Pr\left(\mathbf{F} > \mathbf{F_0} \,\middle|\, \mathbf{H_0} \textbf{ is true}\right) = \Pr\left(F > 2.5 \,\middle|\, H_0 \text{ is true}\right) = \mathbf{0.06802}$$

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. * p-value of F0 = 2.5 when F0 has F[3,60] null distribution
. display Ftail(3, 60, 2.5)
.06802185
```
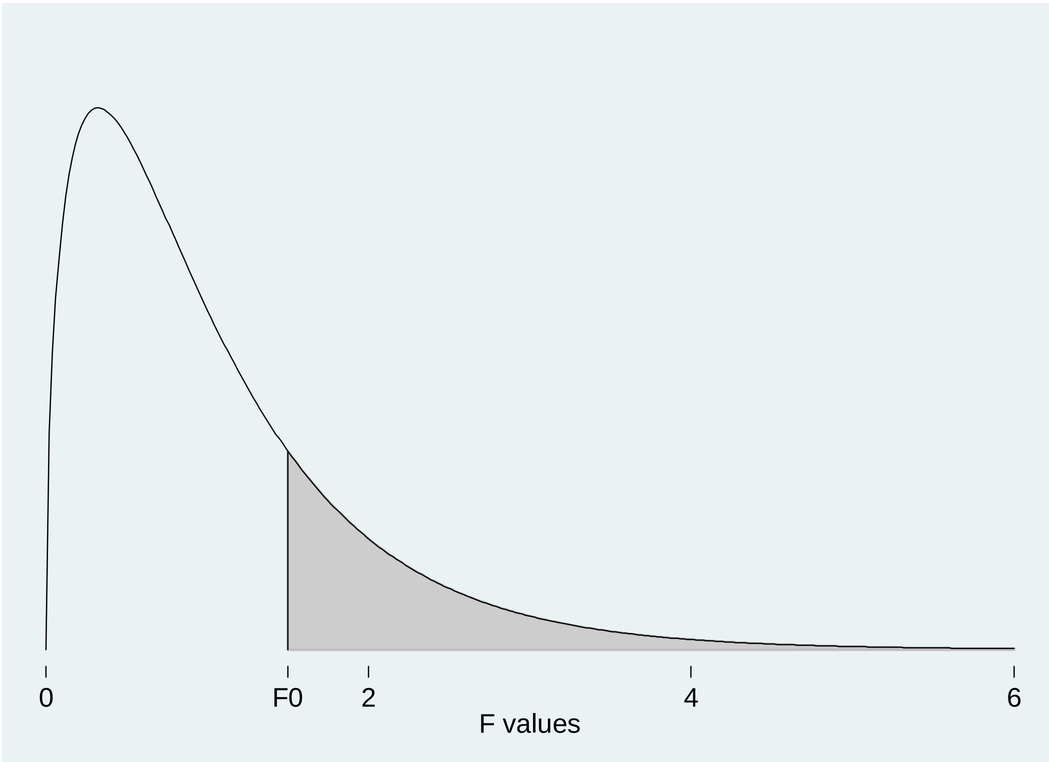

P-value of F_0 = 1.5 when Null Distribution of F_0 is F[3, 60]

p-value for F_0 $= \Pr\left(F > F_0 \mid H_0 \text{ is true}\right) = \Pr\left(F > 1.5 \mid H_0 \text{ is true}\right) =$ **0.2237**




```
. * p-value of F0 = 1.5 when F0 has F[3,60] null distribution
. display Ftail(3, 60, 1.5)
.22372095
```

P-value Decision Rule -- Formulation 2

1. If the **p-value** for the calculated sample value of the test statistic ***is less than*** the chosen **significance level** α, ***reject*** **the null hypothesis** at significance level α.

   p-value $< \alpha \Rightarrow$ ***reject*** $\mathbf{H_0}$ at **significance level** α.

2. If the **p-value** for the calculated sample value of the test statistic ***is greater than or equal to*** the chosen **significance level** α***, retain (i.e., do not reject)*** **the null hypothesis** at significance level α.

   p-value $\geq \alpha \Rightarrow$ ***retain*** $\mathbf{H_0}$ at **significance level** α.